1. Consider an 1D bar element with $E = 1$, $L = 1$, and $A(x) = 1 + x$.
   (a) Obtain the stiffness matrix for finite element method, using equation (373) in section 2.2 of course notes.(**10 Points**)
   (b) Obtain the stiffness matrix using the direct method using the methodology described on pages 307-308 of section 2.2 (equations 375-376). Include all the steps in equation 307 in your derivation. (**20 Points**)
   (c) Comparison of the two approaches: i) Compare the stiffness matrices; ii) Assume that $u_1 = 0, u_2 = 1$ (the end point displacements). Plot the displacement field, strain, and stress for the two methods; iii) (extra credit) compare internal energies $\mathcal{E} = \int_0^1 \frac{1}{2} E(x)\epsilon^2(x)\,\mathrm{d}x$ ($\epsilon = \frac{\mathrm{d}u}{\mathrm{d}x}$). (**30 Points**) + (**20 Points**) (extra credit)

2. For the truss shown in figure the prescribed dofs are: $\bar{U}_{\bar{1}} = \bar{U}_{\bar{2}} = \bar{U}_{\bar{4}} = \bar{U}_{\bar{5}} = 0; \bar{U}_{\bar{3}} = \frac{1}{10}$. For all truss members $E = 1, A = 1$. Other information is provided in the figure 2.

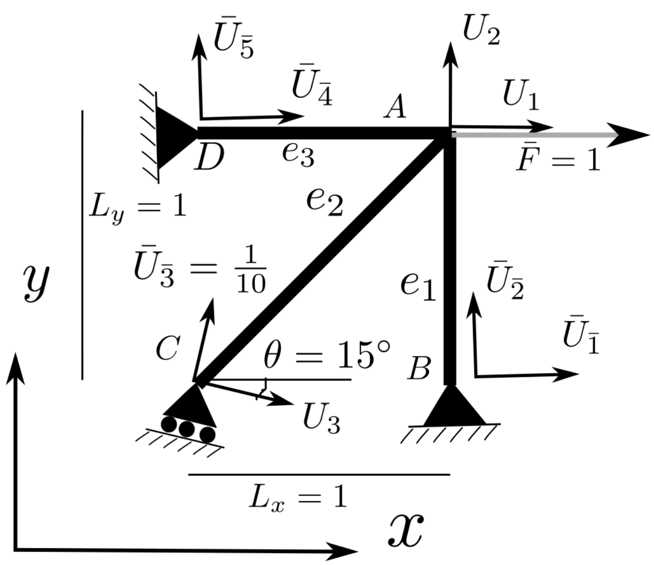

Figure 1: 3 dof truss with an angled support

   (a) Form element force vectors and stiffness matrices. Note that for element e_2 two different coordinate systems are used at the two ends and an equation of the form (394) in section 2.3 should be employed. Also, the angle θ in the figure is the angle that the bar makes with x axis and is different from θ_1 and θ_2 in the figure used for equation (394). (**20 Points**)
   (b) Form global stiffness matrix, total force and solve for global $\mathbf{U}_f$.(**20 Points**)
   (c) Summarize element local displacements and obtain their axial force. (**20 Points**)
   (d) Obtain support reactions at B (2), C (1), and D (2). (**20 Points**)

3. In figure 3 **Frame element** e_2 is hinged to **truss element** e_1. For the frame element, a concentrated moment $\bar{M} = 1$ is applied at $x = 0.75$ and a distributed load $q = x$ is applied over the length of the frame.
   (a) Number free and prescribed dofs. (**10 Points**)
   (b) Form element force vectors and stiffness matrices. (**20 Points**)
   (c) Form global stiffness matrix, total force and solve for global $\mathbf{U}_f$.(**20 Points**)

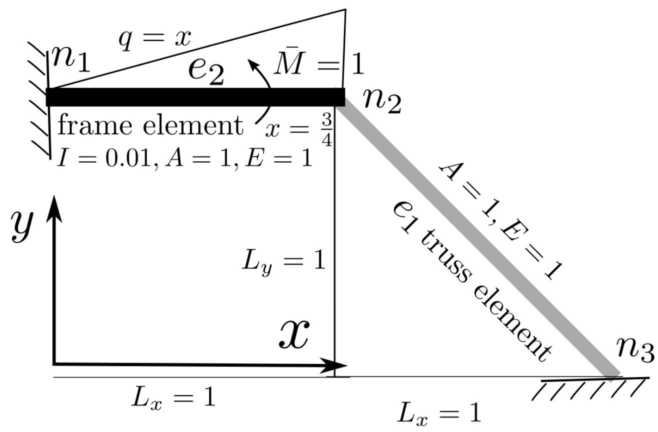

Figure 2: Frame and truss example.

(d) Obtain displacement (y), rotation ($\theta = \frac{\mathrm{d}y}{\mathrm{d}x}$), and moment ($M = EI\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$) for the frame element at $x = 0.5$. Note that $y(\xi) = \sum_{i=1}^{4} N_i^e(\xi)a_i^e$. Also, since $\mathbf{B}^e = \frac{\mathrm{d}^2\mathbf{N}^e}{\mathrm{d}x^2} \Rightarrow M = EI\sum_{i=1}^{4} B_i^e(\xi)a_i^e$. (**30 Points**)