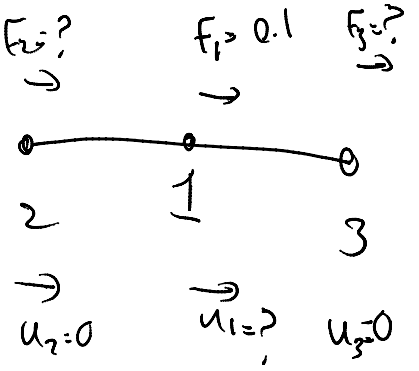
2016/09/22

Thursday, September 22, 2016
10:09 AM

Continuing from the last class

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

unknowns

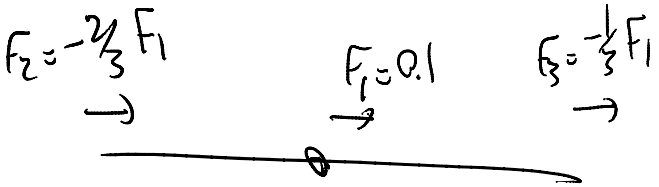


$$F_1 = [3]\,u_1 + [-2 \quad -1]\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \Rightarrow F_1 = 3u_1 - 0.1 = 3u_1$$

$$\Rightarrow \boxed{u_1 = \frac{0.1}{3} = 0.0\bar{3}}$$

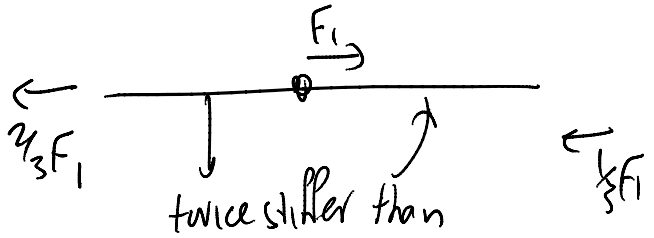
$$\begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix} u_1 + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \rightarrow \begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}\frac{0.1}{3} \Rightarrow$$

now we have $u_1 = \frac{0.1}{3}$



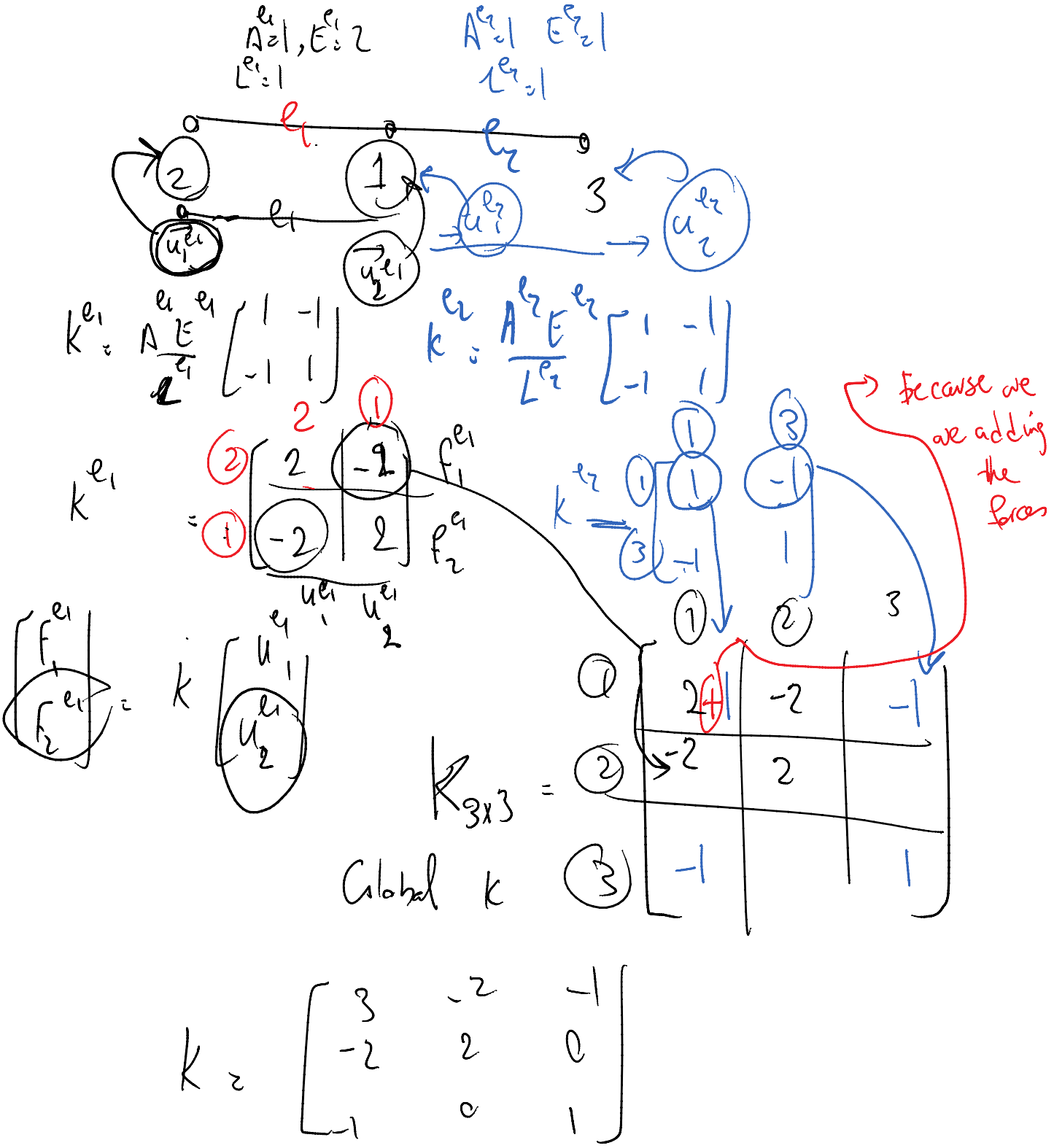
$$F_2 = -\frac{2}{3} \times 0.1$$

$$F_3 = -\frac{1}{3} \times 0.1$$



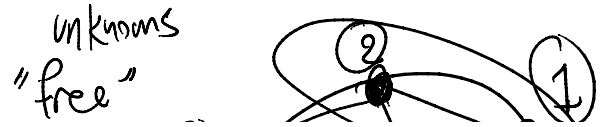
How can we do the assembly easier?

$A^e = 1, E^e = 2$ $\quad A^e = 1 \quad E^e = 1$

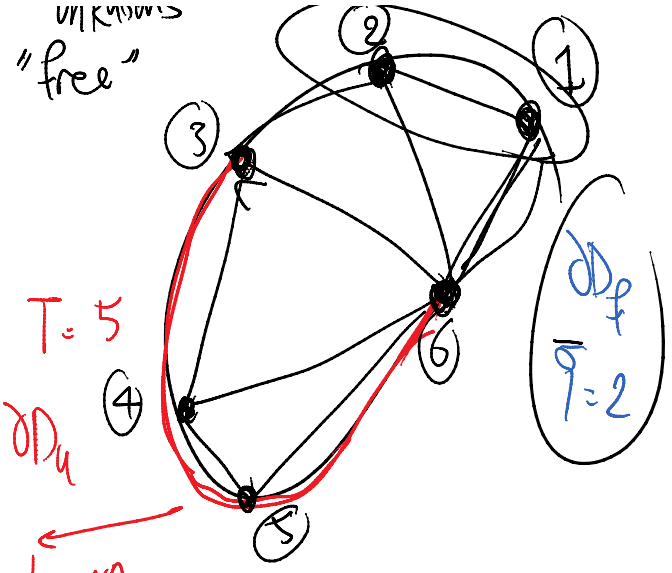


This is the automated way to assemble stiffness matrix

How do we solve the unknowns from F = K U system

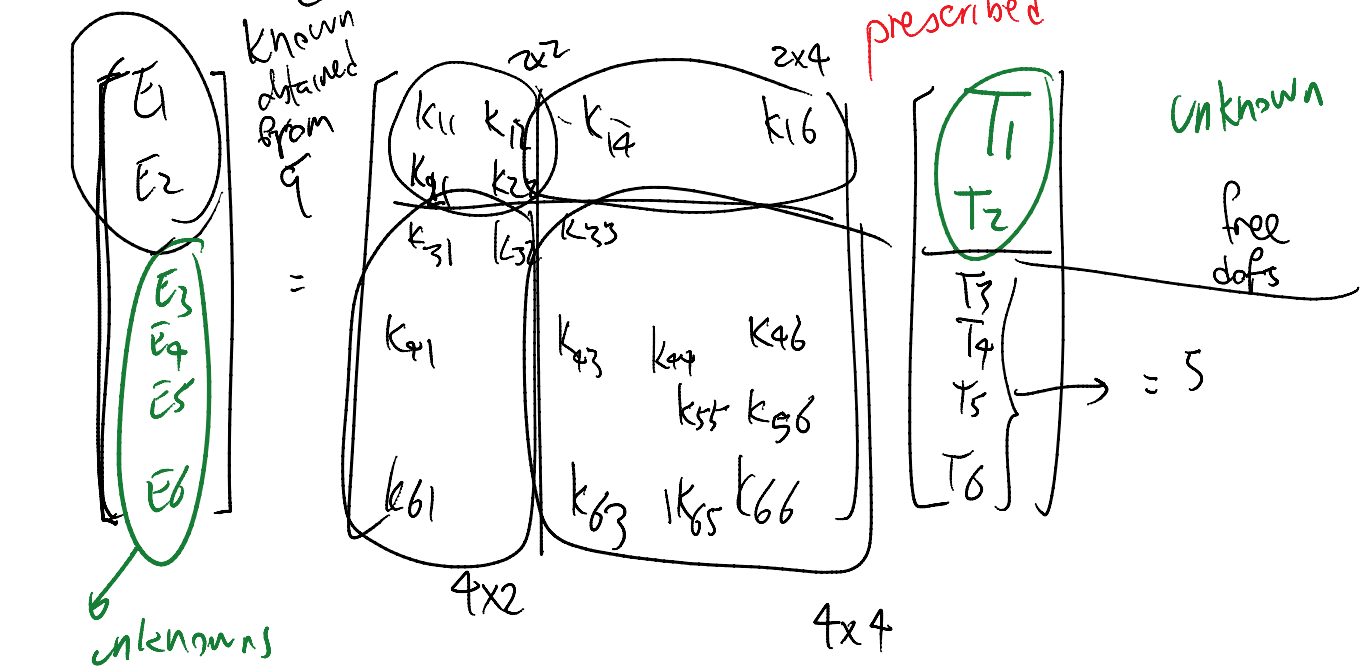


How do we solve the unknowns from F = K U system



We always number the free dofs first

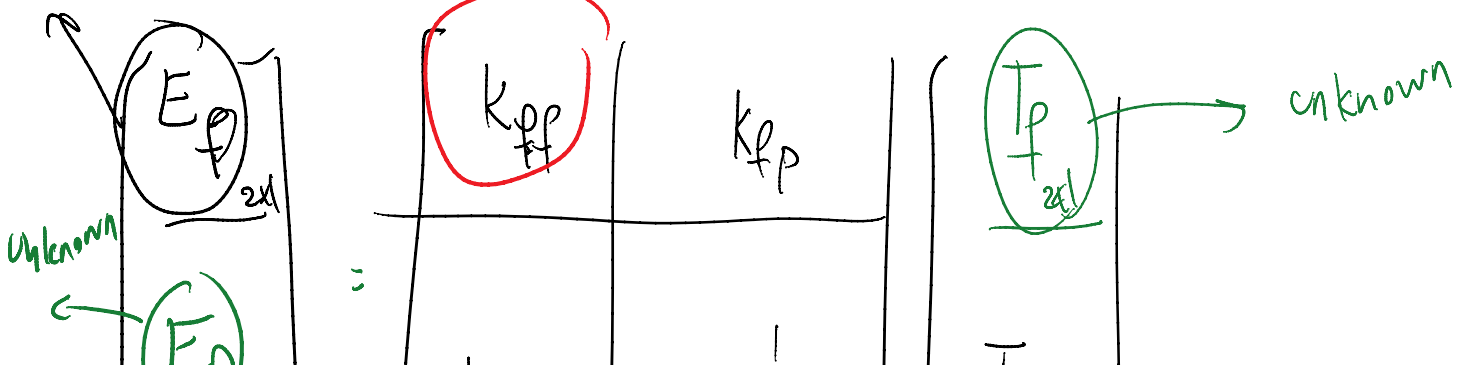
point-wise thermal energy { dealing with balance of energy }



known

we only assemble this

$$\begin{bmatrix} k_{33} & k_{36} \\ k_{63} & k_{66} \end{bmatrix}$$



$$\begin{bmatrix} E_p \\ 4\times1 \end{bmatrix} = \begin{bmatrix} K_{pf} & K_{pp} \end{bmatrix} \begin{bmatrix} T_p \\ 4\times1 \end{bmatrix} \rightarrow \text{known}$$

$$E_{f_{2\times1}} = K_{ff_{2\times2}} T_{f_{2\times1}} + K_{fp_{2\times4}} T_{p_{4\times1}}$$

$$E_{p_{4\times1}} = K_{pf_{4\times2}} T_{f_{2\times1}} + K_{pp_{4\times4}} T_{p_{4\times1}}$$

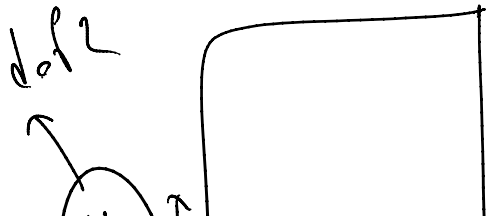
Green unknowns

$$T_f = K_{ff}^{-1}\left(E_f - K_{fp} T_p\right)$$

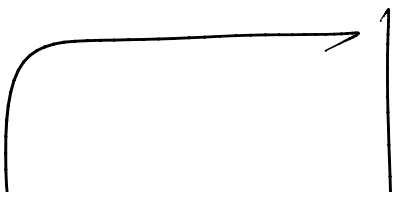
plug into second equation to get E_p

$$E_p = K_{pf} K_{ff}^{-1}\left(E_f - K_{fp} T_p\right) + K_{pp} T_p$$

2D plate



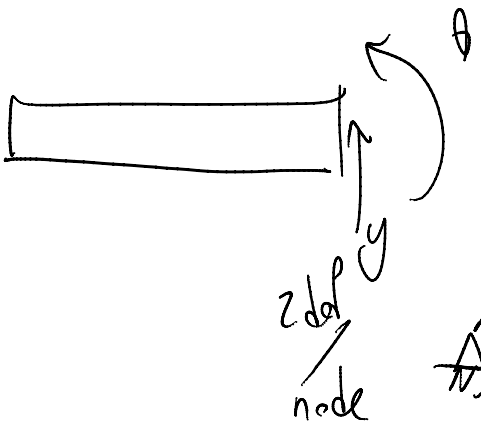
2D thermal

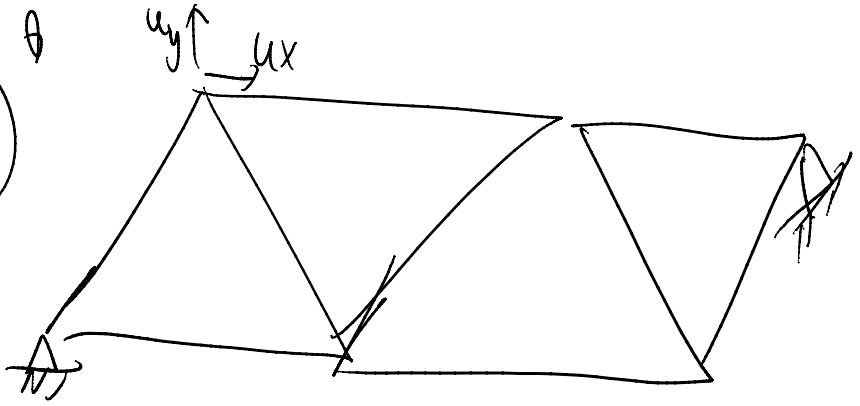


2 dof per node

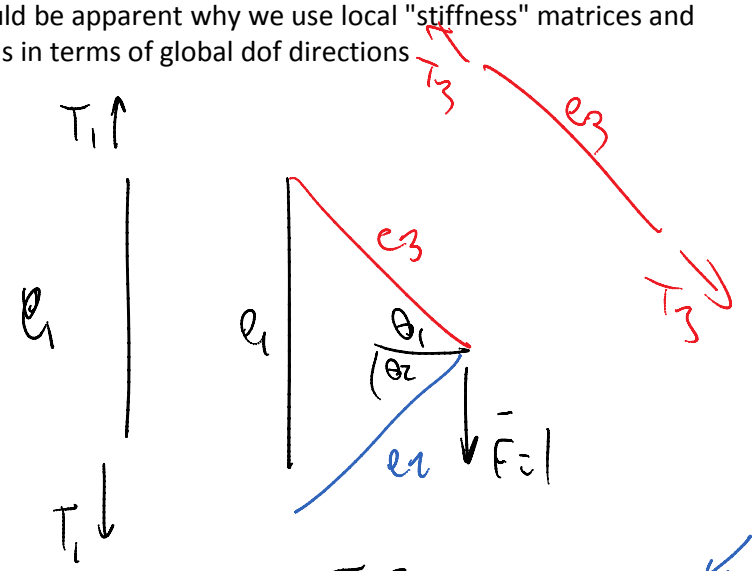
beam

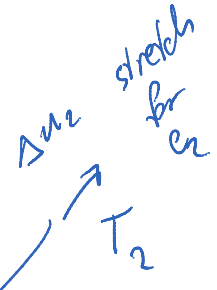
1 dof (unknown) /node





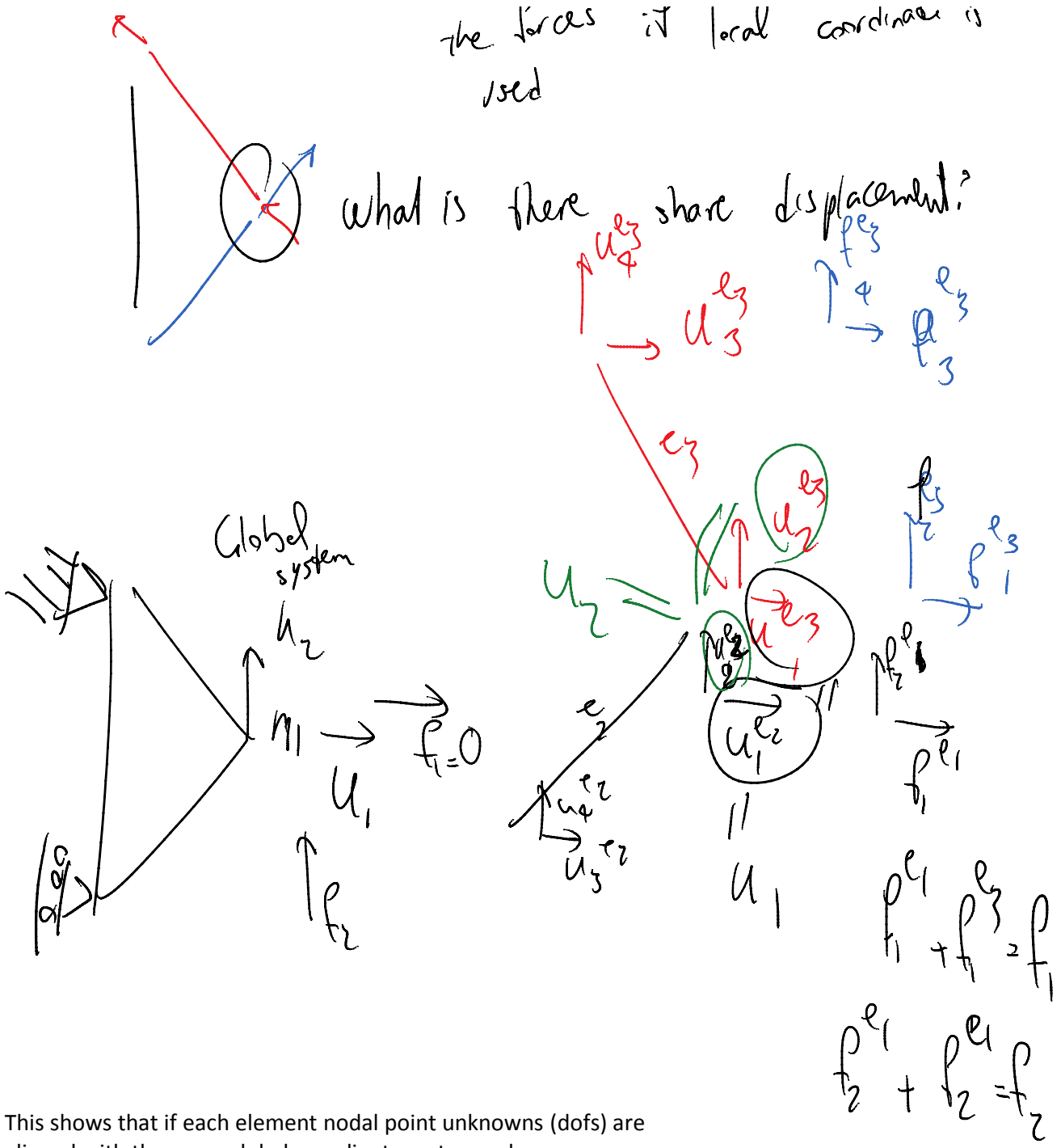
Now it should be apparent why we use local "stiffness" matrices and everything is in terms of global dof directions





$T_3 \sin\theta_1 - T_2 \sin\theta_2 \bar{F}$ T_2

difficulty in "assembling" summing the forces if local coordinate is



This shows that if each element nodal point unknowns (dofs) are aligned with the same global coordinate system unknowns we can easily do the assembly and form the stiffness matrix as we did for the bar problem

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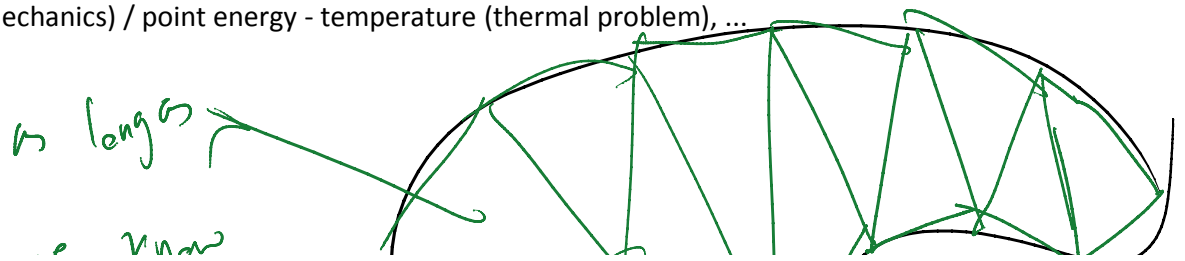
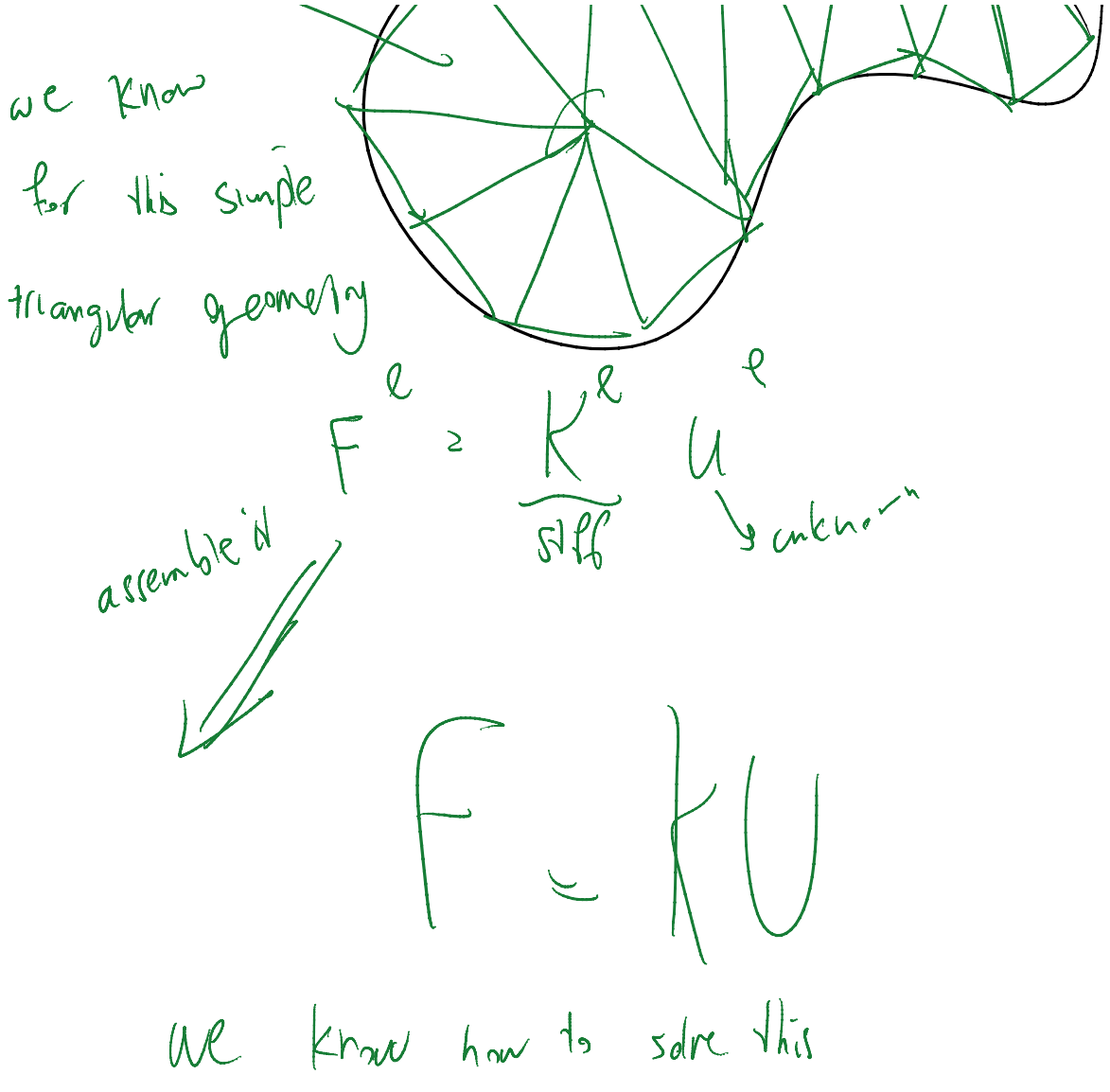
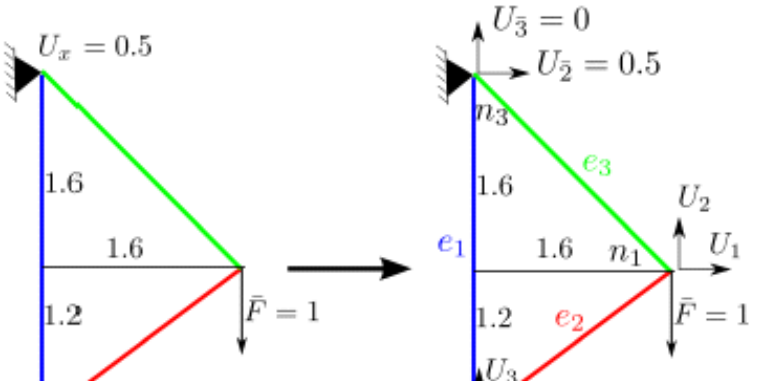
Once we understand this process we can solve any problem as long as we have relations for the local force displacement relations (solid mechanics) / point energy - temperature (thermal problem), ...

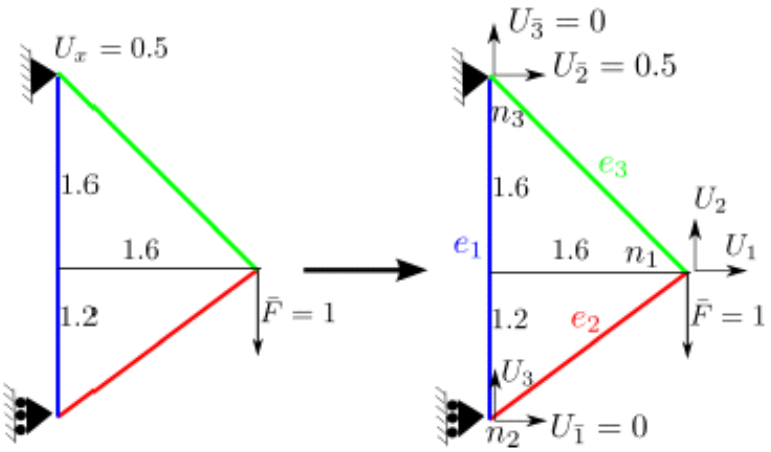




The generality of FEMs come from the fact that knowing the force/displacement, … relations for simple geometries (which we call elements) such as triangles, squares, etc. we can model very complex geometries.

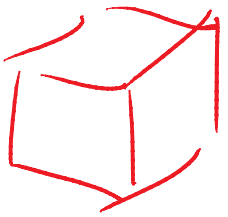
We are going to solve the following simple truss problem with FEM

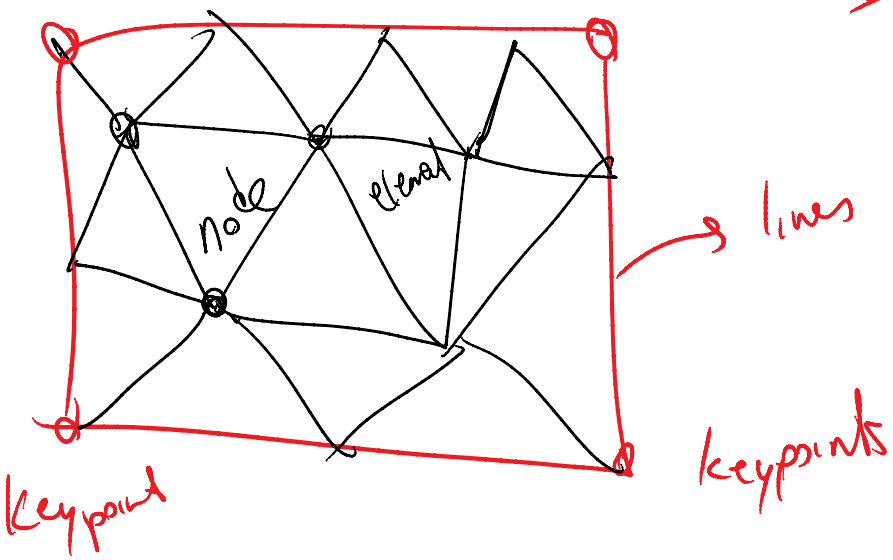




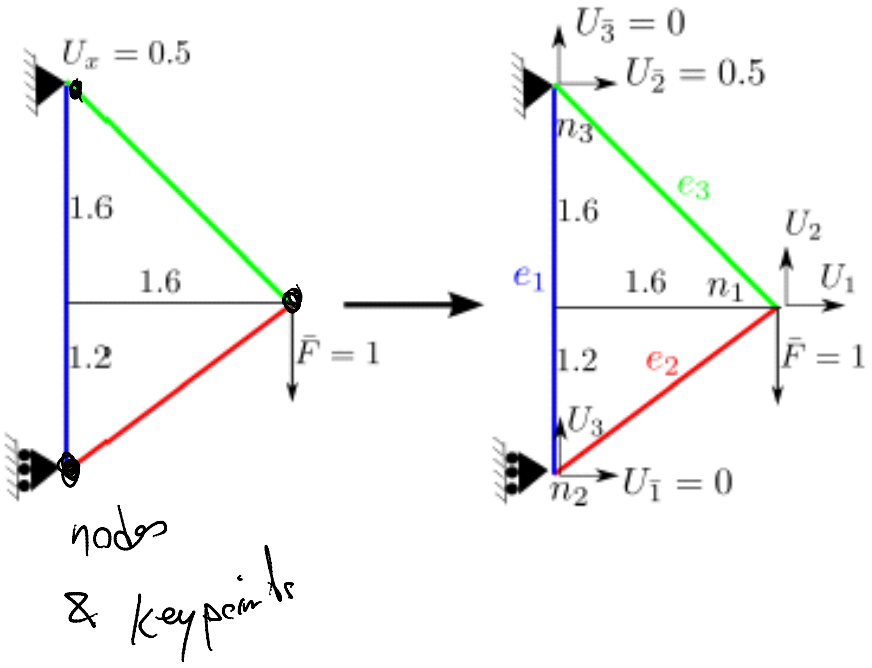
Geometric objects		FE object
0D	vertices / keypoint	node
1D	lines	bar (boundary of triangular element)
2D	areas / surfaces	Tri. element
3D	volumes	brick element

Not the same thing



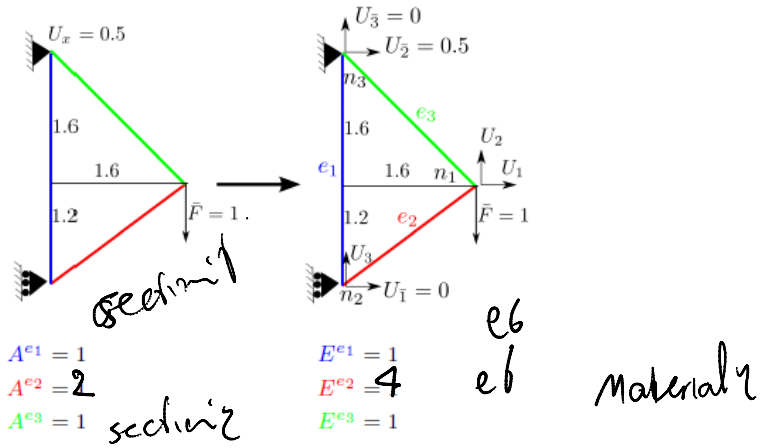


Truss example

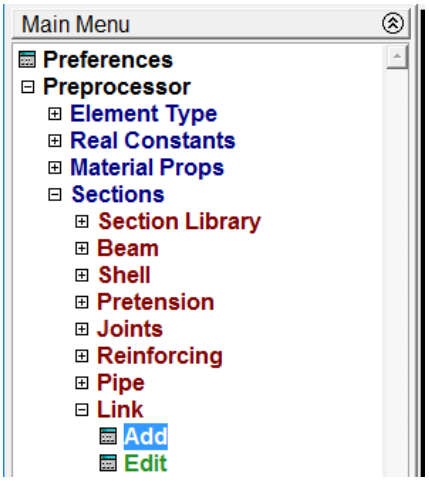


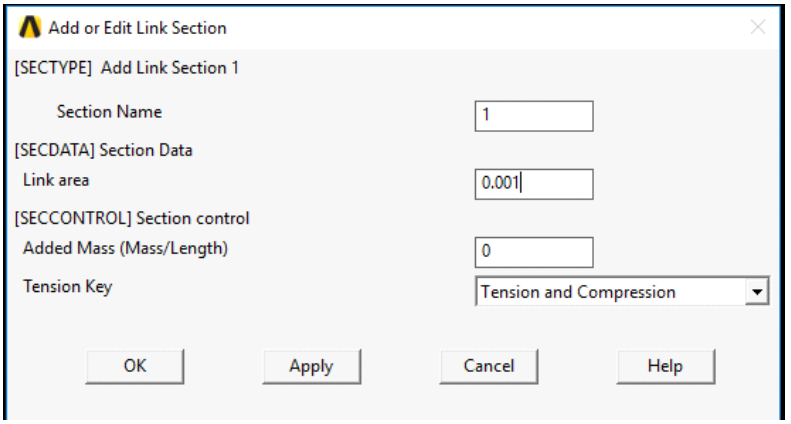
In FEMS almost always the user creates geometry objects first ->
Use the meshing tool to create elements / nodes based on that

Almost never we directly create nodes and element: One exception can be a truss example that we can directly enter nodes and elements



Note:
In Ansys 17.x area section of truss elements is entered through "section" section rather than "constants" section:


Main Menu
Preferences
Preprocessor
Element Type
Real Constants
Material Props
Sections
Section Library
Beam
Shell
Pretension
Joints
Reinforcing
Pipe
Link
Add
Edit


Add or Edit Link Section
[SECTYPE] Add Link Section 1
Section Name
1
[SECDATA] Section Data
Link area
0.001
[SECCONTROL] Section control
Added Mass (Mass/Length)
0
Tension Key
Tension and Compression
OK
Apply
Cancel
Help