

2016/09/27

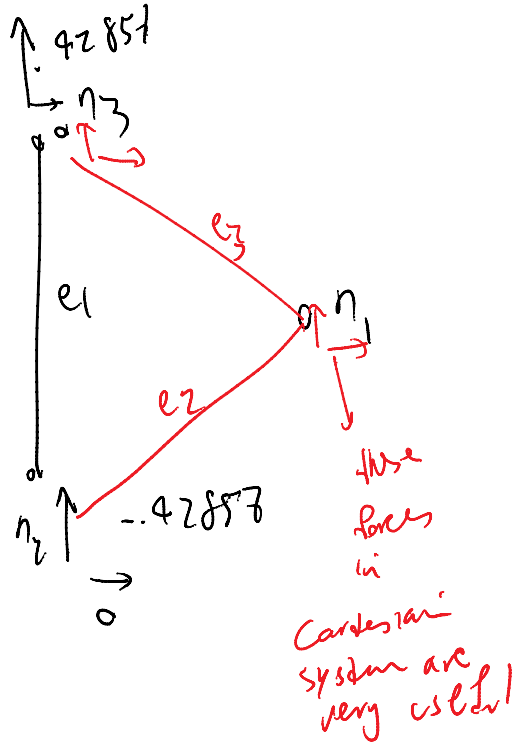
Tuesday, September 27, 2016
10:07 AM

Ansys:

How to list element end point forces

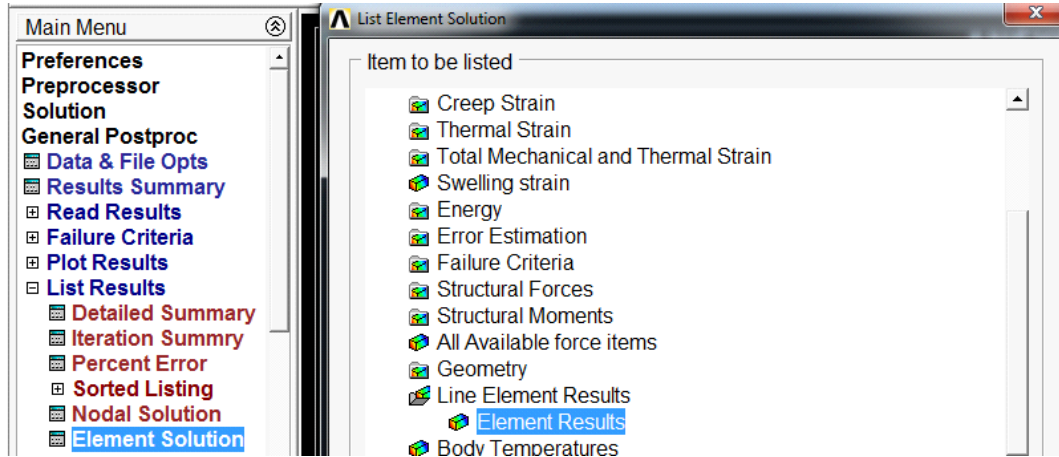
Postprocess -> element solution -> Structural forces

ELEM=	1	FX	FY	FZ
2	0.0000	0.42857	0.0000	
3	0.0000	-0.42857	0.0000	

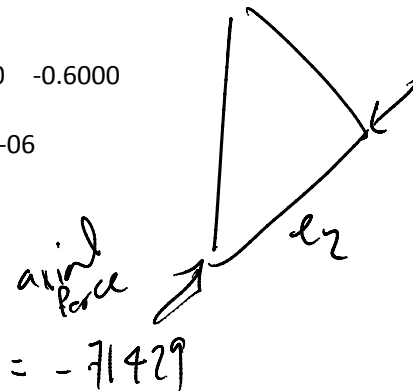


Get axial forces

Axial forces:

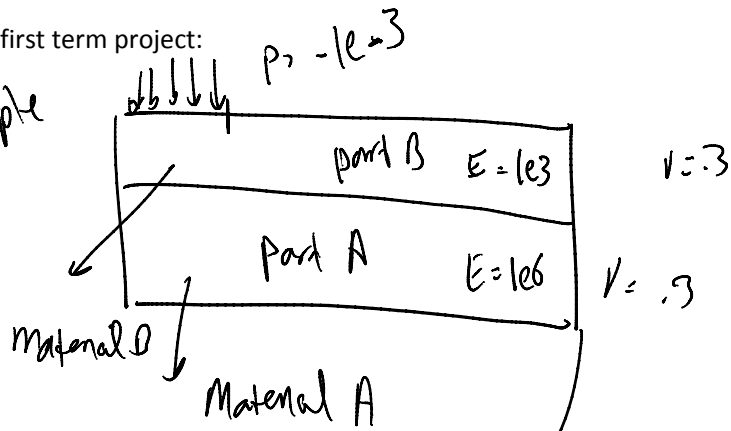


EL= 2 NODES= 2 1 MAT= 1 XC,YC,ZC= 0.8000 -0.6000
 0.000 AREA= 1.0000 LINK180
FORCE=-0.71429 STRESS=-0.71429 EPEL=-0.71429E-06
 TEMP= 0.00 0.00 EPTH= 0.0000

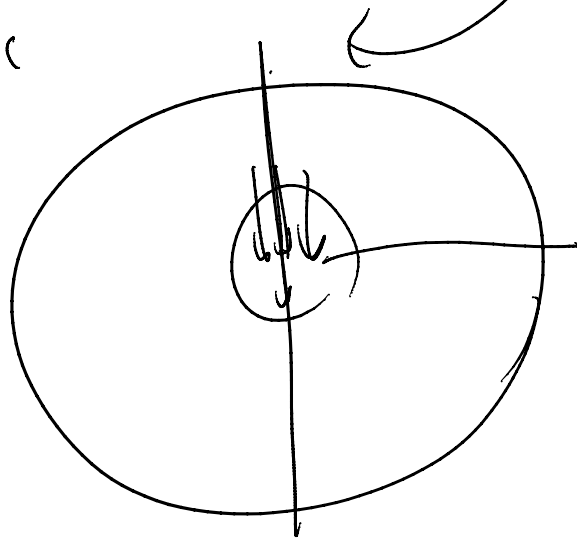


Second example related to the first term project:

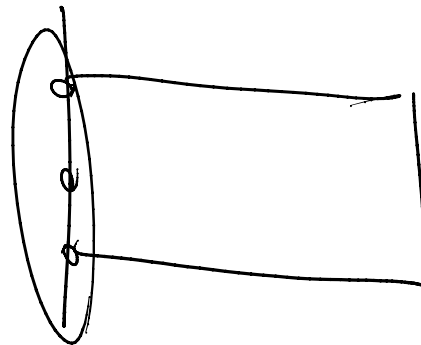
Axisymmetric example



axis
of
rotation

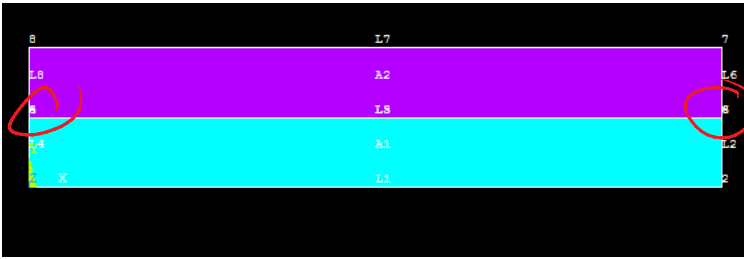


For axisymmetry problem make sure left edge
is on y-axis.



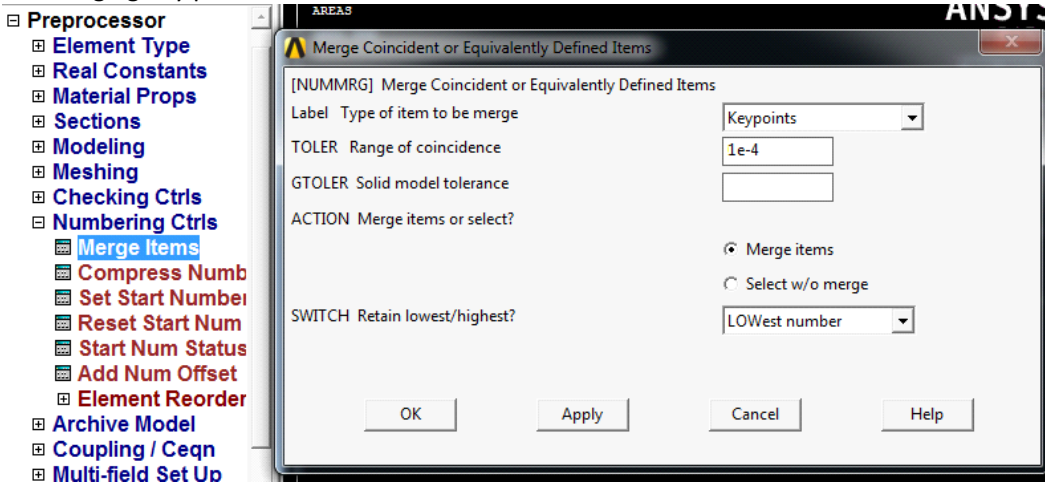
$x=0$

How to merge duplicate keypoints?

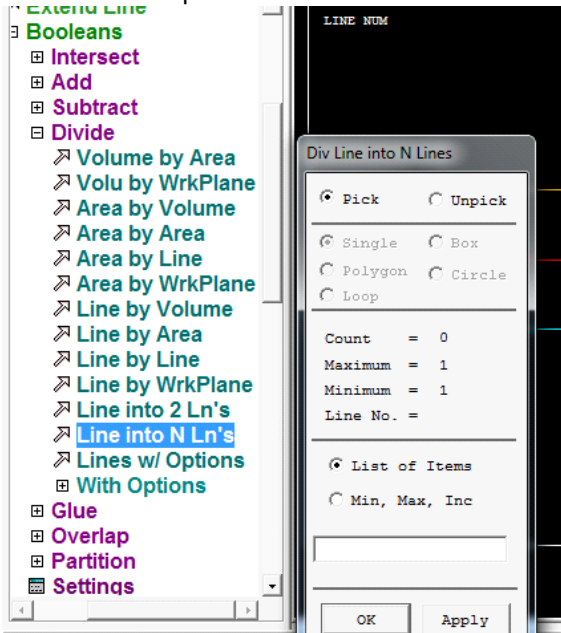


Want to merge them

For merging key points

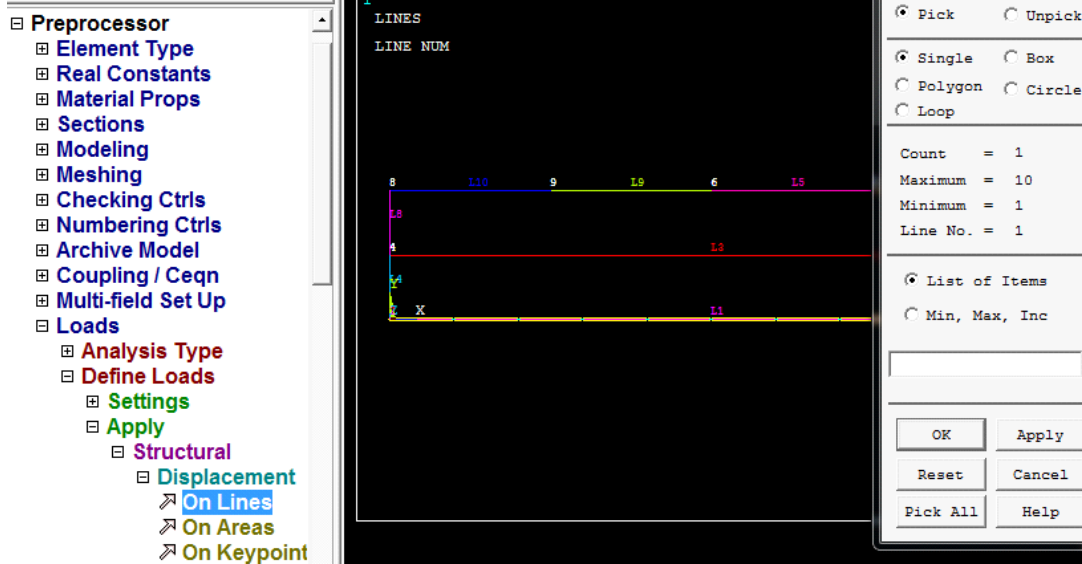


To divide the top line

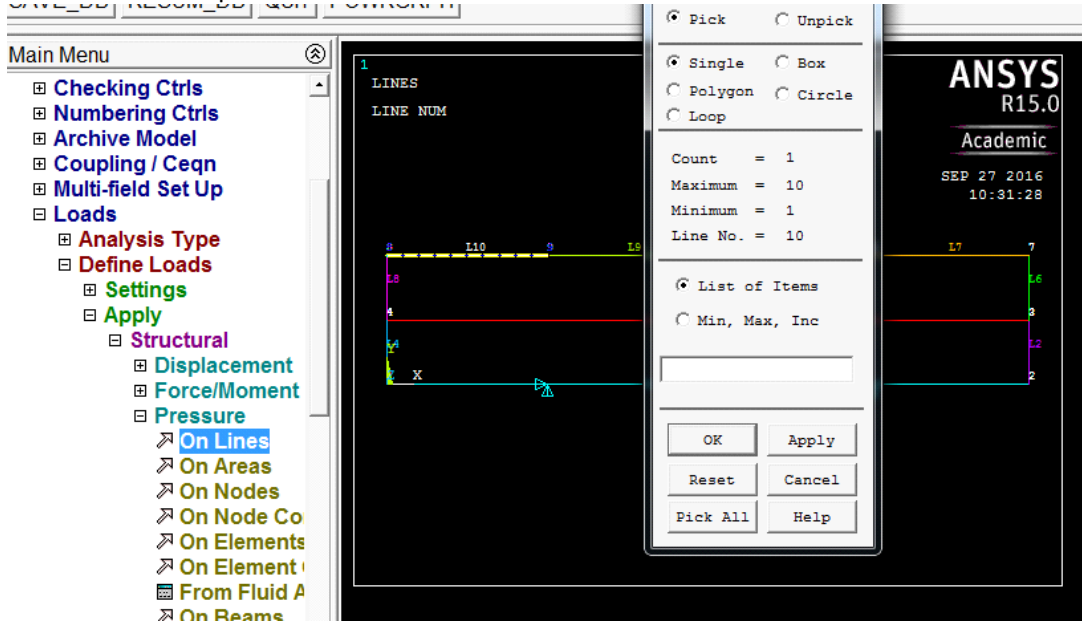


Always apply loads (BCs) to geometry rather than directly applying them to elements and nodes of finite element mesh.

Applying fixed BC on the bottom line

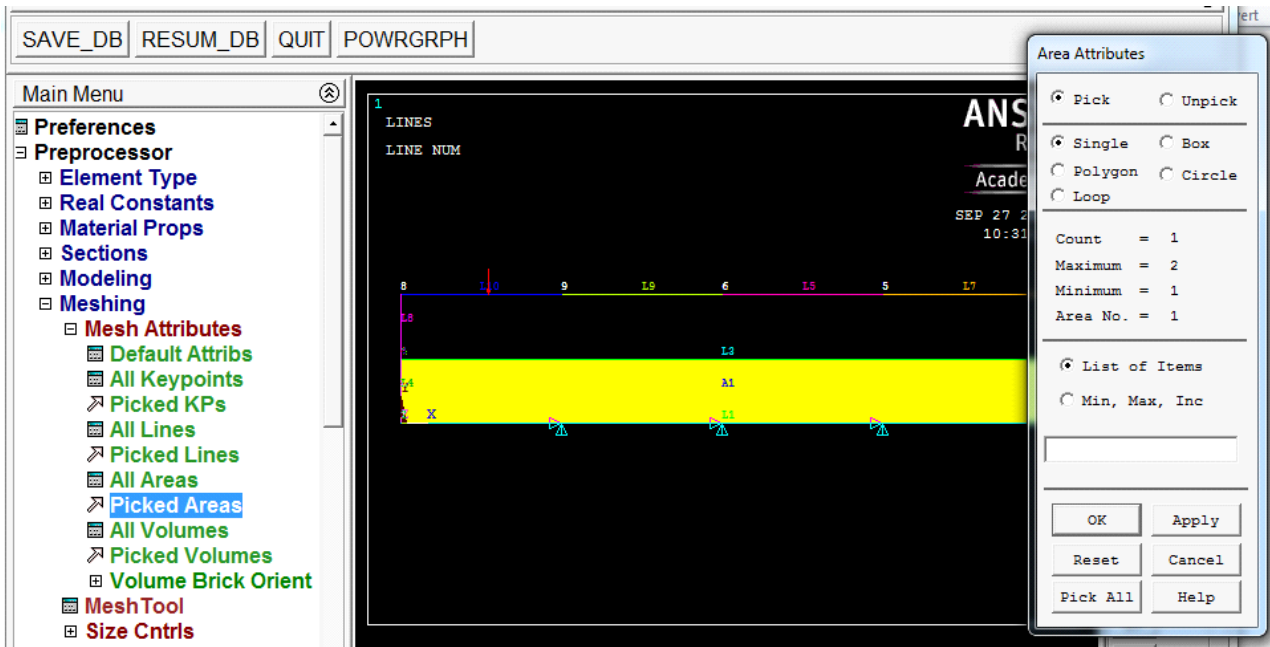


Pressure on the top line

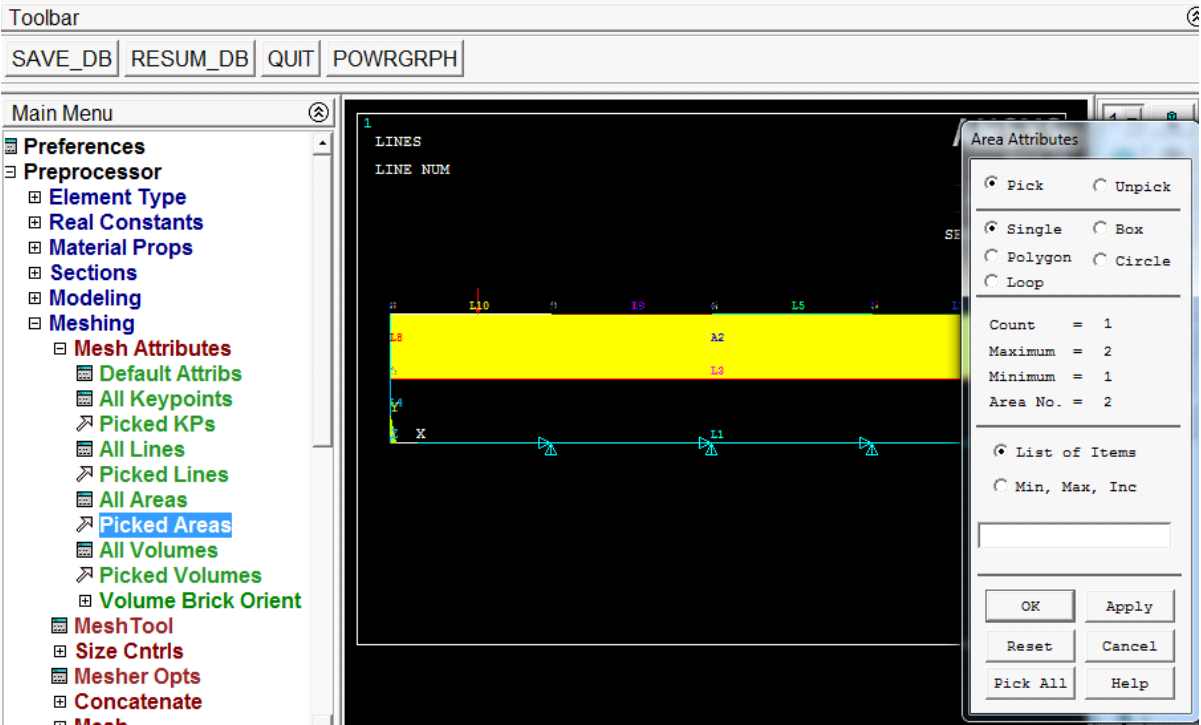


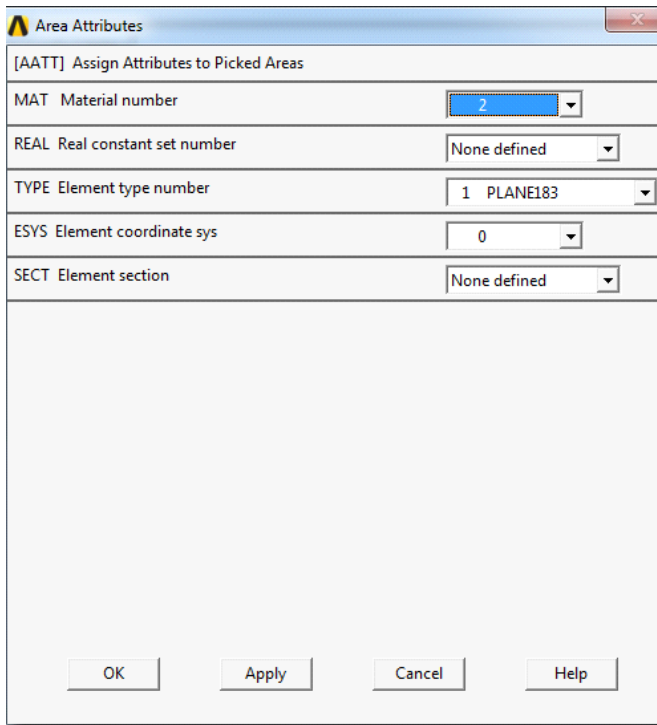
Assigning material numbers to surfaces A and B (bottom and top surfaces)

Choose the bottom area

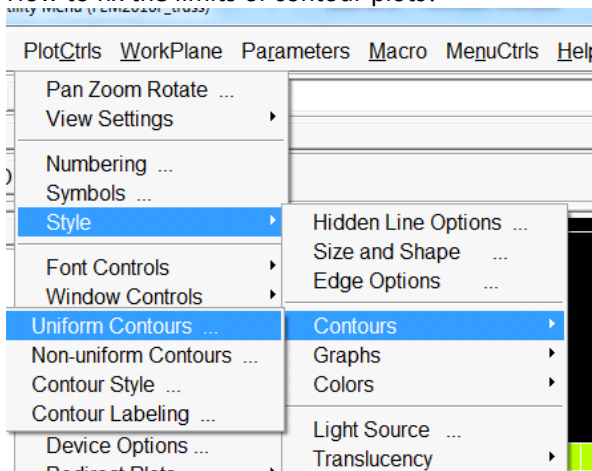


Top one





How to fix the limits of contour plots?



Discretization

1. General idea of discretization and how a system $Ka = F$
 - a. K stiffness matrix
 - b. a unknown vector
 - c. F force vector
 2. Do this derivation for various forms of Weighted residual statement, energy method, least square
 3. Actual numerical examples
-

Discretization means turning a problem with infinite unknowns to one with finite number of unknown

$$y(x) = ?$$

Approximate solution

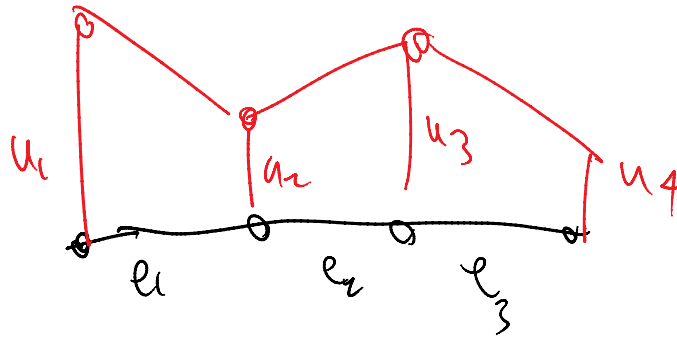
$$y = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

4 unknowns a_1 to a_4

but they don't have geometric interpretation



How about FEM

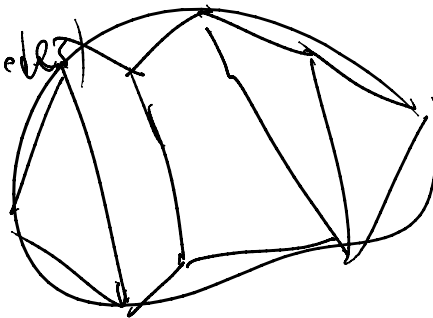


u_1, u_2, u_3, u_4

are the 4 unknowns. They have geometric interpretation

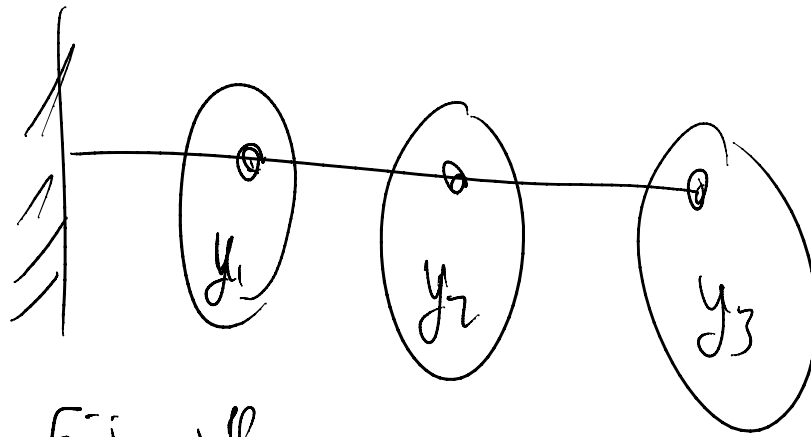
(displacement at element nodes)

2D



How do we discretize the solution?

- Finite difference approach / finite volume
Unknowns are primary fields (e.g. displacements) at grid point locations



Finite difference
 the unknowns are y 's at certain locations

2. Interpolating the solution with functions

essential BC \rightarrow

PDE
 $P_i = \nabla \cdot q = Q$
 $= \nabla \cdot (-k \nabla T) = Q$

Goal: Have approximate expression of solution inside the domain

δD_p
 $q(x) \cdot n = \bar{q}$
 $(-k \nabla T) \cdot n = \bar{q}$
 $R_f = \bar{q} - q \cdot n = \bar{q} + (k \nabla T) \cdot n$

$T(x) = \bar{T}(x)$
 for $x \in \partial D_u$

$R_u = \bar{T}(x) - T(x)$

unknown

$$T(x) = \sum_{i=1}^n \phi_i(\vec{x}) a_i$$

$$T(x) = \sum_{i=1}^n \phi_i(\vec{x}) a_i$$

\downarrow
 $(x_1, x_2) \text{ in } \mathbb{R}^2$

$\phi_i(x)$: trial (test) function

known WE CHOOSE

these functions

a_i : unknowns that we want to solve

n : number of unknowns

Example



$$y(x) = 1 a_1 + x a_2 + x^2 a_3 + x^3 a_4$$

$$\phi_1(x) = 1$$

$$\phi_2(x) = x$$

$$\phi_3(x) = x^2$$

$$\phi_4(x) = x^3$$

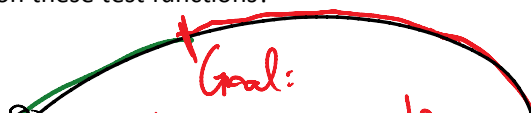
$$\phi_1 \text{ to } \phi_4 \quad \sin x$$

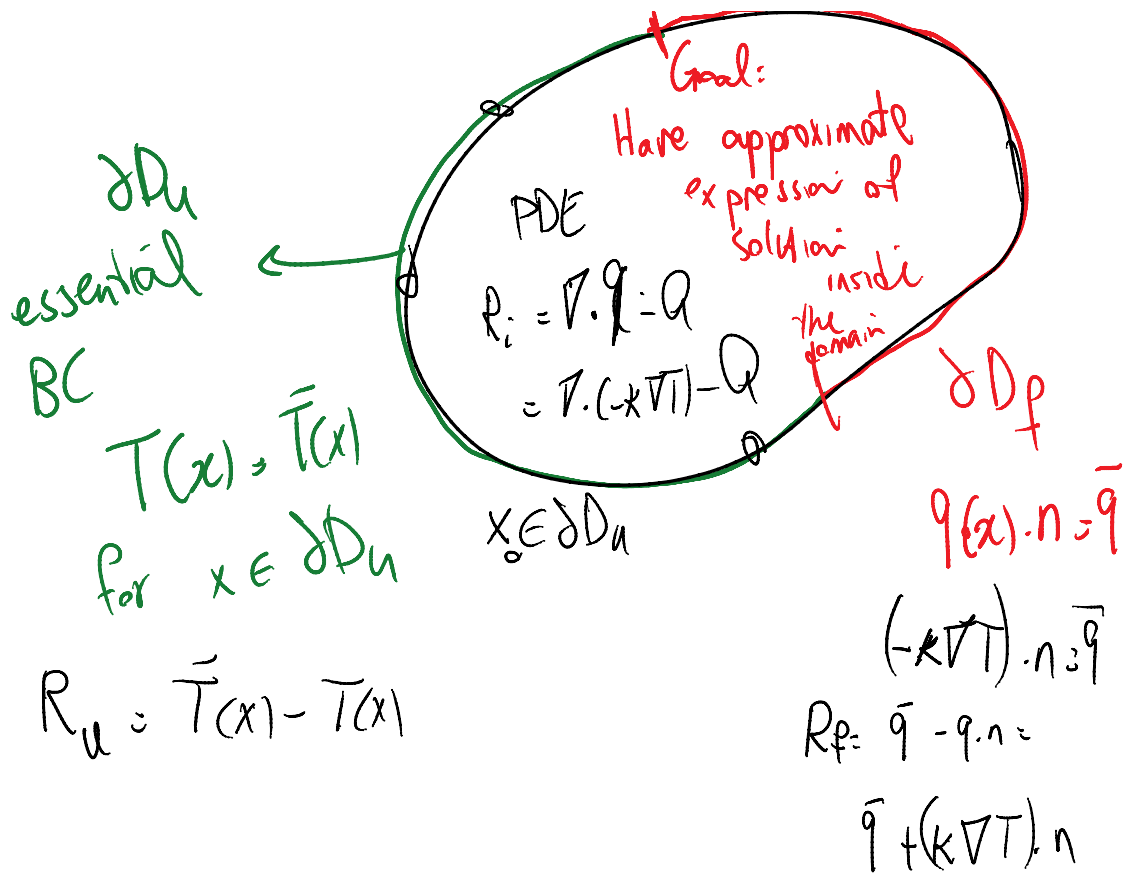
$$\sin 2x$$

$$\sin 3x$$

$$\sin 4x$$

Do we need to stipulate any conditions on these test functions?





We almost always use a method that R_u (essential BC residual) is strongly satisfied.

$$T(x) = \bar{T}(x) \quad \forall x \in \partial D_u$$

want to satisfy this

$$T(x) = a_1 \phi_1(x) + a_2 \phi_2(x) + \dots + a_n \phi_n(x)$$

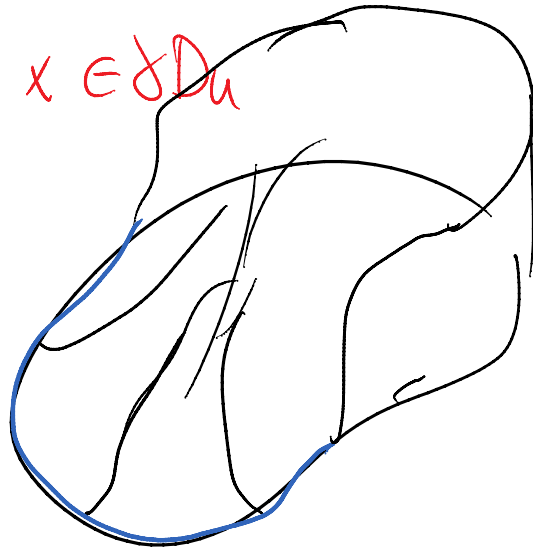
$x_0 \in \partial D_u$

$$T(x_0) = a_1 \phi_1(x_0) + a_2 \phi_2(x_0) + \dots + a_n \phi_n(x_0)$$

$$= \bar{T}(x_0)$$

Let's say that all the trial functions satisfy the homogeneous essential BC (similar to weight functions in weak statement)

$$\phi_i(x) = 0 \quad \forall x \in \partial D_n$$



$$y(x) = \cancel{1} a_1 + x a_2 + x^2 a_3 + \cancel{x^3} a_4$$

$$\phi_1 = \cancel{1} \quad \phi_2 = x \quad \phi_3 = x^2 \quad \phi_4 = x^3$$

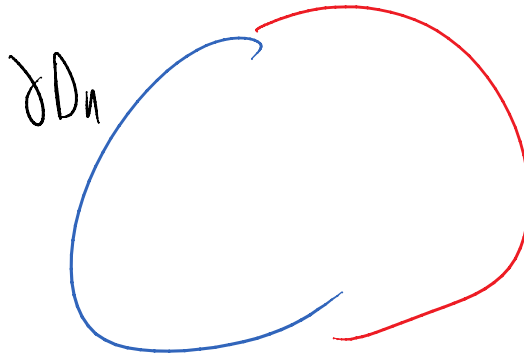
$$\phi_i(x) = 0 \quad \forall x \in \partial D_n$$

in this case $\phi_i(0) = 0$

$$\phi_i(0) = 1 \neq 0$$

How do we actually satisfy essential BC (not the homogeneous version of that)?

$$\sum_{i=1}^n a_i \phi_i$$



$$T(x) = a_i \phi_i(x)$$

$$\forall x \in \partial D_n \quad T(x) = a_i \phi_i(x) = 0$$

not good

Remedy: add a particular function
summation from $i=1 \dots n$

$$T(x) = \overbrace{a_i \phi_i(x)} + \phi_p(x)$$

$$\forall x \in \partial D_n \quad T(x) = \underbrace{a_i \phi_i(x)}_0 + \phi_p(x) = \bar{T}(x)$$

$$\phi_p(x) = \bar{T}(x) \quad \forall x \in \partial D_n$$

Particular function ϕ_p satisfies the essential BC strongly

- 2 Solution is represented by a finite number of functions:

$$u^h(x) = \phi_p(x) + \sum_{i=1}^n a_i \phi_i(x)$$

n unknowns

where u^h is the symbol for discrete solution and $\phi_i(x)$ are trial or test functions. $\phi_p(x)$ is set to satisfy essential boundary conditions and will be discussed later.

This approach is used by (discrete) weighted residual method, weak form, least square, and Ritz energy method.

Different approaches that we could get an exact solution ->
How do we discretize these methods

Any expression in the form

For all

->

For n ... (to get n equations rather than infinite equations) so that we get n unknown n equation system

Approach	Equation	Figure	Discretization	Discretization method
Balance Law (20)	$\forall \Omega \subset \mathcal{D} : \int_{\partial\Omega} (f, n) ds - \int_{\Omega} r dv = 0$		Change $\forall \Omega$ to $\{\Omega_1, \Omega_2, \dots, \Omega_n\}$	Similar to subdomain method in WRM
Strong Form (23)	$\forall x \in \mathcal{D} : \nabla \cdot f - r = 0$		Change $\forall x$ to $\{x_1, x_2, \dots, x_n\}$	Collocation method in WRM. Also FD & FV.
Energy Method (80)	$\forall \tilde{y} \in \mathcal{V} : \Pi(y) \leq \Pi(\tilde{y})$		$\forall \{\tilde{a}_1, \dots, \tilde{a}_n\} : \Pi(a_1, \dots, a_n) \leq \Pi(\tilde{a}_1, \dots, \tilde{a}_n) \Rightarrow \frac{\partial \Pi}{\partial a_1} = \dots = \frac{\partial \Pi}{\partial a_n} = 0$	Ritz Energy Method. Also yields Weak Form.

$$\nabla \Pi = 0$$

Approach	Equation	Figure	Discretization	Discretization method
Weighted Residual Method (45)	$\forall w \in \mathcal{W} : \int_{\mathcal{D}} w \cdot \mathcal{R}_i \, dv + \int_{\partial \mathcal{D}_f} w^f \cdot \mathcal{R}_f \, ds = 0$		Change $\forall w$ to $\{w_1, w_2, \dots, w_n\}$	Weighted Residual Method (WRM)
Least Square (51)	$R^2 = \int_{\mathcal{D}} \mathcal{R}_i^2 \, dv + \int_{\partial \mathcal{D}_f} \mathcal{R}_f^2 \, ds = 0$		Change $R^2 = 0$ to $\forall \{\tilde{a}_1, \dots, \tilde{a}_n\} : R^2(a_1, \dots, a_n) \leq R^2(\tilde{a}_1, \dots, \tilde{a}_n) \Rightarrow \frac{\partial R^2}{\partial a_1} = \dots = \frac{\partial R^2}{\partial a_n} = 0$	Least Square method, a WRM for linear L_M (& L_f).
Weak Form (74)	$\forall w \in \mathcal{W} : \int_{\mathcal{D}} L_m^w(w) L_m(u) \, dv = \int_{\mathcal{D}} w \cdot r \, dv + \int_{\partial \mathcal{D}_f} w \cdot \bar{f} \, ds$		Change $\forall w$ to $\{w_1, w_2, \dots, w_n\}$	Weak Formulation

$\forall R^2 = 0$