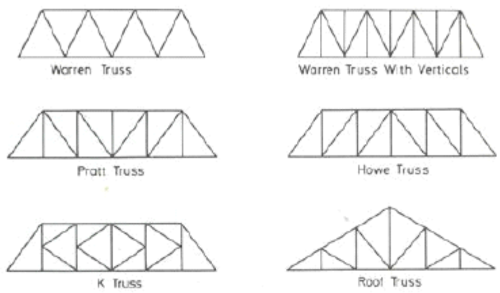
2016/11/01A

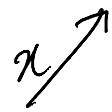
Tuesday, November 01, 2016
10:08 AM

Trusses

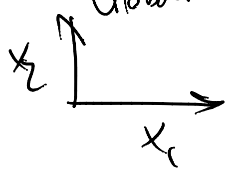


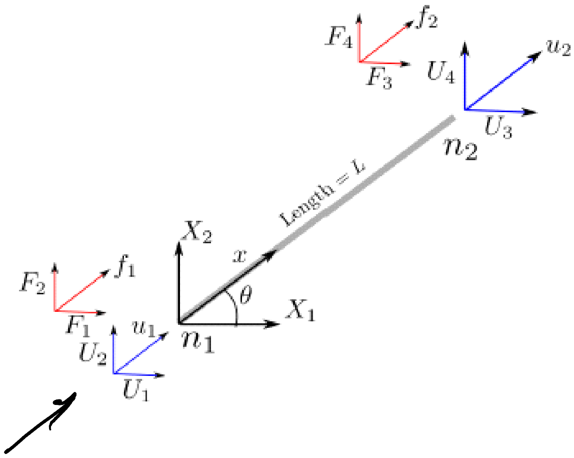
Types of simple Plane truss

local



Global coordinate





$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \begin{matrix} \} \text{ node 1} \\ \\ \} \text{ node 2} \\ \end{matrix} \qquad U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

From bar theory

$$f^e = k^e u^e \qquad f^e = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \qquad k^e = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$u^e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$[\qquad k^e \qquad ||$$

$$\underset{4\times 1}{F} = \underbrace{K^e_{4\times 4}}_{?} \, U_{4\times 1}$$

$$F_{4\times 1} = (T_{Ff})_{4\times 2} \, f_{2\times 1} \longrightarrow$$

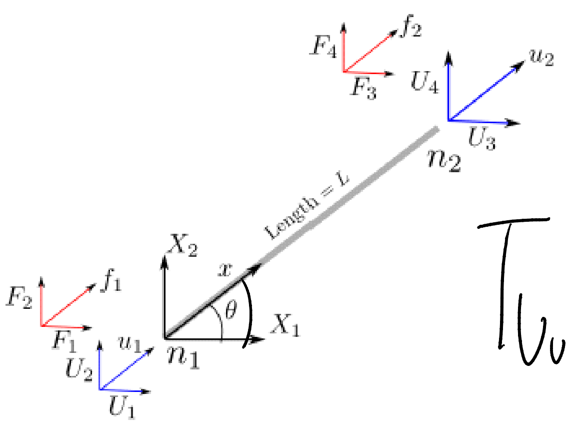
$$\left\{ \begin{array}{l} u_{2\times 1} = (T_{Uu})_{2\times 4} \, U_{4\times 1} \\ f_{2\times 1} = k^e_{2\times 2} \, u^e_{2\times 1} \end{array} \right\} \text{①} \Rightarrow \boxed{f_{2\times 1} = k^e u^e = (k^e T_{Uu})_{2\times 4} U_{4\times 1}} \Rightarrow$$

$$F_{4\times 1} = T_{Ff} \, f_{2\times 1} \Rightarrow (T_{Ff} \, k^e \, T_{Uu}) \, U_{4\times 1}$$

$$\left. \begin{array}{l} F_{4\times 1} = (T_{Ff_{4\times 2}} \, k^e_{2\times 2} \, T_{Uu_{2\times 4}}) \, U_{4\times 1} \\ F_{4\times 1} = K^e_{4\times 4} \, U_{4\times 1} \end{array} \right\}$$

$$\boxed{K^e = T_{Ff}\, k^e\, T_{Uu}}$$

Next step: Find T_{Ff} & T_{Uu}



$$\boxed{\begin{aligned} C &= \cos\theta \\ S &= \sin\theta \end{aligned}}$$

T_{Uu}

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underbrace{\begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix}}_{T_{uU}} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

$$u_1 = cU_1 + sU_2 + 0U_3 + 0U_4$$

$$u_2 = 0U_1 + 0U_2 + cU_3 + sU_4$$

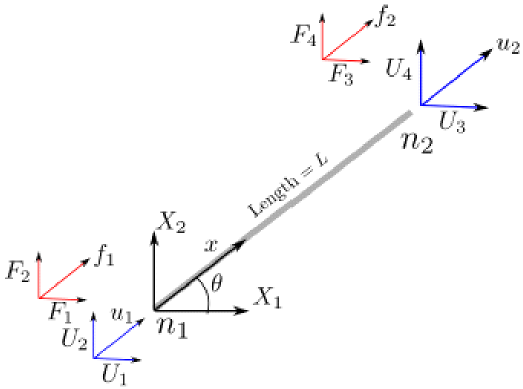
$\underbrace{T_{uU}}$ $\Downarrow$ call this T

2nd transfer

T_{Ff}

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} C & 0 \\ S & 0 \\ 0 & C \\ 0 & S \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{Bmatrix} \vdots \\ F_4 \end{Bmatrix} \quad \begin{bmatrix} c & | & s \end{bmatrix} \rightarrow T_{Ff} = (T_{uU})^T$$



$$F_1 = c\, f_1$$

$$F_2 = s\, f_1$$

$$F_3 = c\, f_2$$

$$F_4 = s\, f_2$$

$$T = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \qquad \begin{aligned} T_{uU} &= T \\ T_{Ff} &= T^t \end{aligned}$$

$$K = T_{Ff}\, k^e\, T_{uU} \Rightarrow$$

$$\boxed{K^e = T^T k^e T, \quad k^e = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} c & s & 0 & 0 \\ 0 & c & c & s \end{bmatrix}}$$

$\mathbf{K} = \mathbf{T}^{\mathrm{T}}\mathbf{k}\mathbf{T} \;\Rightarrow \mathbf{K} = \dfrac{AE}{L}\left[\begin{array}{c|c} \mathbf{k}_b & -\mathbf{k}_b \\ \hline -\mathbf{k}_b & \mathbf{k}_b \end{array}\right]$, where $\mathbf{k}_b = \begin{bmatrix} c^2 & cs \\ cs & s^2 \end{bmatrix}$ that is

$$\mathbf{K} = \frac{AE}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$\begin{Bmatrix} f_1 \\ \vdots \end{Bmatrix} = T \begin{Bmatrix} U_1 \\ U_2 \\ \vdots \end{Bmatrix}$$

$$\mathbf{k} = \frac{AE}{L}\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Another useful relation is

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = T^{T} k \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix}$$

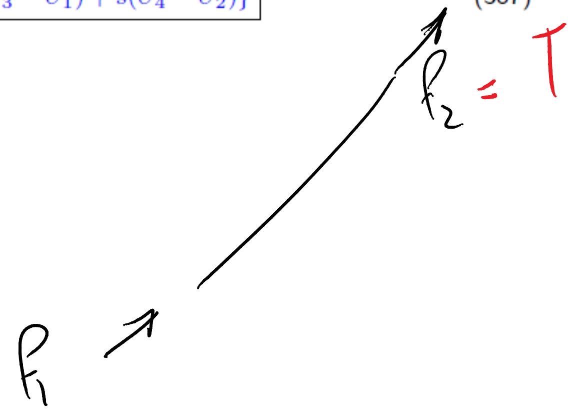
$\Longrightarrow$

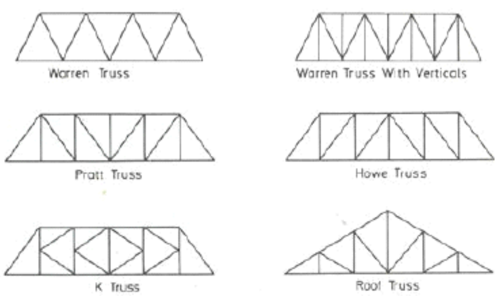
$$\mathbf{T_{Ff}k} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \Rightarrow \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} c & s & -c & -s \\ -c & -s & c & s \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \tag{386}$$

- Noting that f_2 corresponds to tensile axial force in the bar, which we denote by T we have,

$$\boxed{T = AE\left\{c(U_3 - U_1) + s(U_4 - U_2)\right\}} \tag{387}$$

tensile stress





Types of simple Plane truss

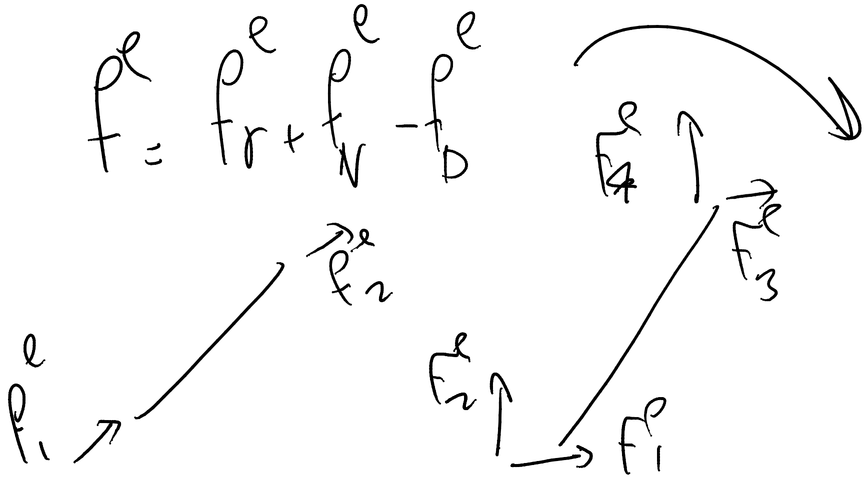
We need to have common coordinate system to match element displacements and add their forces (i.e. the assembly routine). That's why we go to **global coordinate system element stiffness matrix**

How about forces:

$k^e =$ $\quad K^e = T^t k^e T$

$$k^e \rightarrow K^e = T^T k^e T$$

$$f^e = f_r^e + f_N^e - f_D^e$$



$$\underset{4\times1}{F^e} = \underset{4\times2}{T^T} \underset{2\times1}{f^e}$$

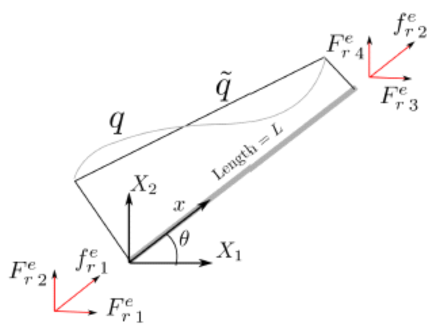
After we evaluate Ke and Fe then we can assemble them into global system stiffness matrix and force vector

Example on how to go from local coordinate to global coordinate element forces

Body force in rotated coordinate system

- While there is no body force exerted on truss members, it is interesting know how the body force we computed in local coordinate system transfers to global coordinate system.
- From (378) we have,

$$\mathbf{f}_r^e = \int_e \mathbf{N}^{e\,\mathrm{T}}.q \,\mathrm{d}x \approx \mathbf{r}^e \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad (391)$$



- Similar to $\mathbf{F} = \mathbf{T}^\mathrm{T}\mathbf{U}$ we have,

$$\mathbf{F}_r^e = \mathbf{T}^\mathrm{T}\mathbf{f}_r^e$$

- Thus,

$$\mathbf{F}_r^e = \mathbf{T}^\mathrm{T}\int_e \mathbf{N}^{e\,\mathrm{T}}.q\,\mathrm{d}x \approx \mathbf{T}^\mathrm{T}\mathbf{r}^e \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad \text{that is}$$

$$\begin{bmatrix} F_{r\,1}^e \\ F_{r\,2}^e \\ F_{r\,3}^e \\ F_{r\,4}^e \end{bmatrix} = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \int_e \begin{bmatrix} N_1^e \\ N_2^e \end{bmatrix} q\,\mathrm{d}x \approx \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \frac{L^e}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad \text{exact for linear } q \qquad (392)$$

- The transformation from local to global forces is general and applies to other forces, *e.g.*, $\mathbf{f}_N^e$, and other element types, *e.g.*, beams.

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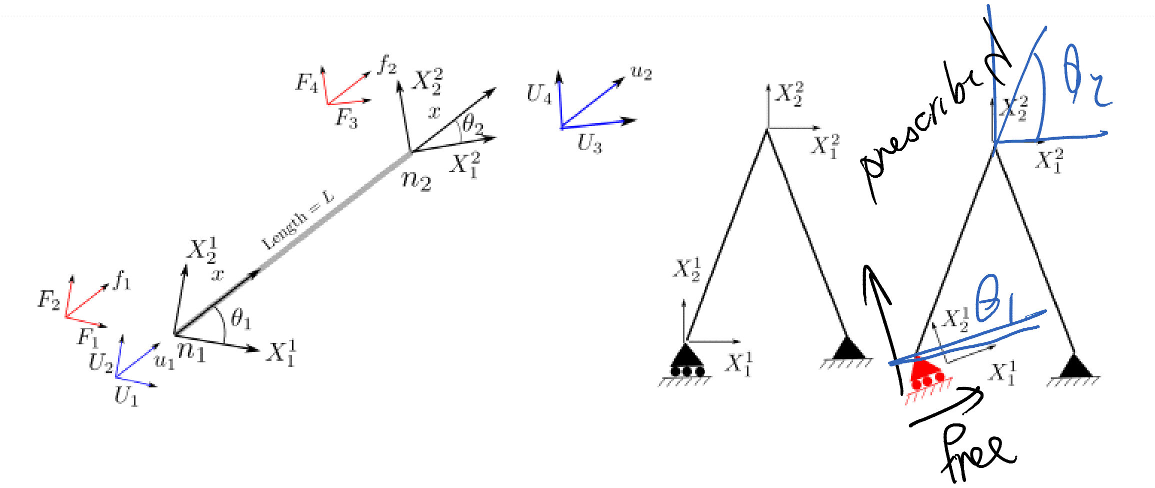
Relevant for HW4

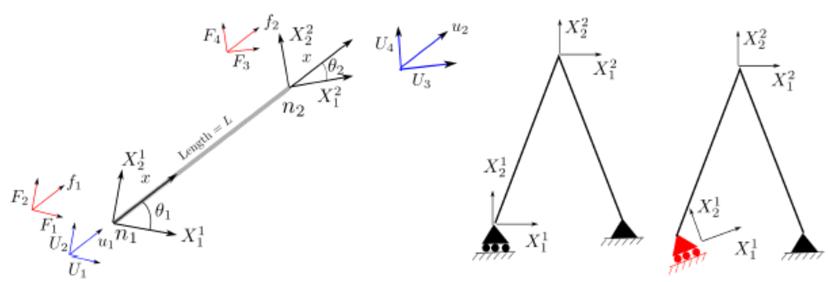
Cases that we need to have two different coordinate systems for the ends of a truss element



- In some instances we need to employ two different coordinate systems at the end points of a bar or in general coordinate system(s) that are not aligned with global coordinate system.
- For example the support highlighted in red in the right figure, do decouple displacement at the support and set the normal displacement to zero (Dirichlet BC) and tangential one free (Neumann BC) we need to employ the rotated coordinate system X_1^1, X_2^1.
- We have two different angles, θ_1 and θ_2. We define,

$$c_1 = \cos(\theta_1) \qquad s_1 = \sin(\theta_1)$$
$$c_2 = \cos(\theta_2) \qquad s_2 = \sin(\theta_2)$$

Truss element /two different coordinate systems



- As before $\mathbf{T} := \mathbf{T_{uU}} = \mathbf{T_{Ff}}$ and in this case is given by,

$$\mathbf{T} = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & c_2 & s_2 \end{bmatrix} \tag{393}$$

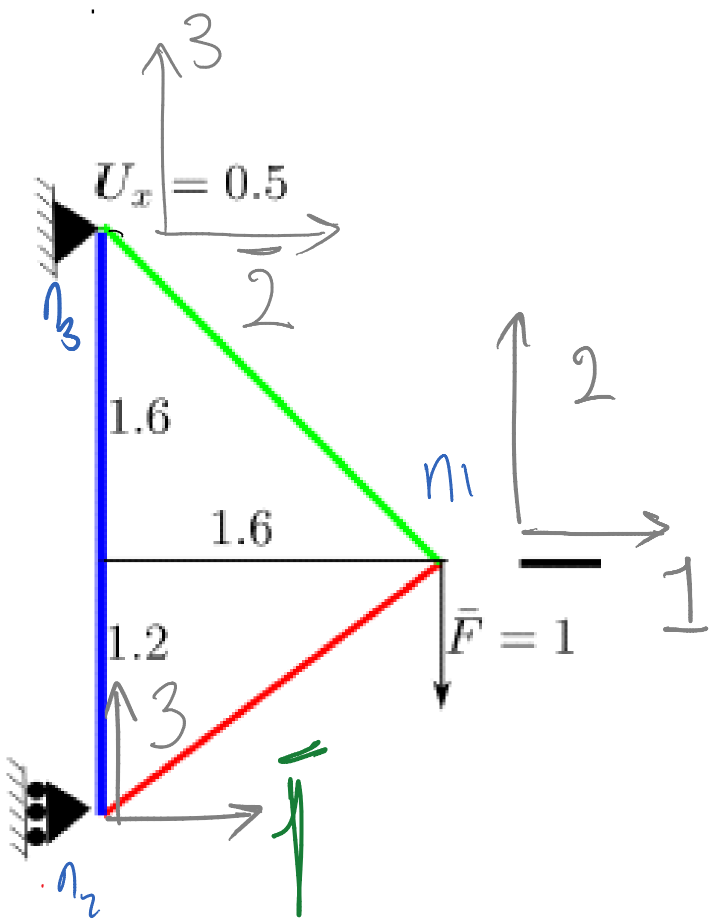
- Accordingly, from $\mathbf{K} = \mathbf{T}^{\mathrm{T}}\mathbf{k}\mathbf{T}$ we obtain,

$$\mathbf{K} = \begin{bmatrix} c_1^2 & c_1 s_1 & -c_1 c_2 & -c_1 s_2 \\ c_1 s_1 & s_1^2 & -c_2 s_1 & -s_1 s_2 \\ -c_1 c_2 & -c_2 s_1 & c_2^2 & c_2 s_2 \\ -c_1 s_2 & -s_1 s_2 & c_2 s_2 & s_2^2 \end{bmatrix} \tag{394}$$

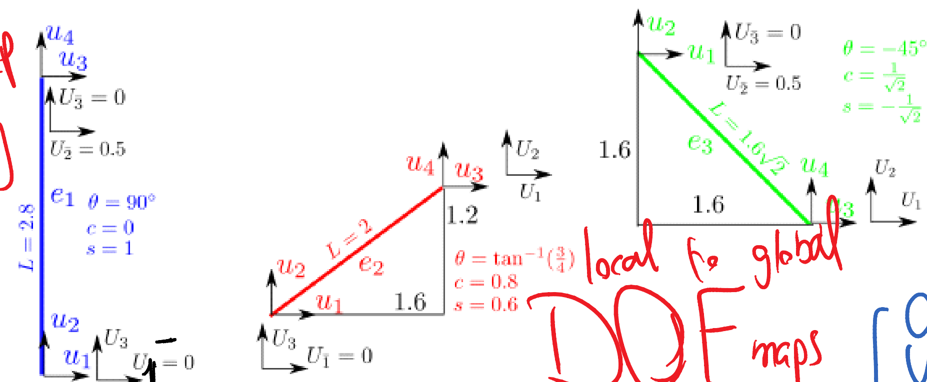
- Finally the axial tensile force in the bar, which is the second line of $\mathbf{kT_{uU}} = \mathbf{kT}$ is (compare to one global coordinate in (387)):

$$\boxed{T = -c_1 U_1 - s_1 U_2 + c_2 U_3 + s_2 U_4} \tag{395}$$

Truss example



Nodal map $M_n^e = [2\ 3]$



local to global DOF maps

- Table below summarizes parameters for each element:

e	L^e	θ	c	s	M_t^e
e_1	2.8	90°	0	1	$[\bar{1}\ 3\ \bar{2}\ \bar{3}]$
e_2	2	$\tan^{-1}(\frac{3}{4})$	0.8	0.6	$[\bar{1}\ 3\ 1\ 2]$
e_3	$1.6\sqrt{2}$	$-45°$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$[\bar{2}\ 3\ 1\ 2]$

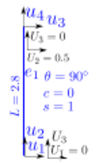
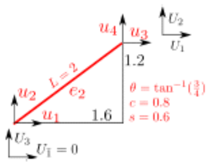
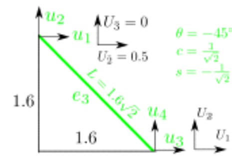
$$a^{e_1} = \begin{bmatrix} 0 \\ U_3 \\ .5 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ .5 \\ 0 \end{bmatrix}$$

$$K^e = \frac{AE}{L}\left[\begin{array}{cc|c} c^2 & cs & -(\ ) \\ cs & s^2 & \\ \hline -(\ ) & & +(\ ) \end{array}\right]$$

$$f_D^e = \underline{k^e a^e}$$

encircled 0's

encircled 0's are free dofs

e	e1	e2	e3
e			
$\mathbf{k}^e$	$\mathbf{k}^{e1} = \frac{(1)(1)}{2.8}\begin{bmatrix} 0&0&0&0\\ 0&1&0&-1\\ 0&0&0&0\\ 0&-1&0&1 \end{bmatrix} = \begin{array}{c c} & \begin{matrix} \bar{1} & 3 & \bar{2} & \bar{3} \end{matrix} \\ \begin{matrix} \bar{1}\\ 3\\ \bar{2}\\ \bar{3} \end{matrix} & \begin{bmatrix} 0&0&0&0\\ 0&0.3571&0&-0.3571\\ 0&0&0&0\\ 0&-0.3571&0&0.3571 \end{bmatrix} \end{array}$	$\mathbf{k}^{e2} = \frac{(1)(1)}{2}\begin{bmatrix} 0.64&0.48&-0.64&-0.48\\ 0.48&0.36&-0.48&-0.36\\ -0.64&-0.48&0.64&0.48\\ -0.48&-0.36&0.48&0.36 \end{bmatrix} = \begin{array}{c c} & \begin{matrix} \bar{1} & 3 & 1 & 2 \end{matrix} \\ \begin{matrix} \bar{1}\\ 3\\ 1\\ 2 \end{matrix} & \begin{bmatrix} 0.32&0.24&-0.32&-0.24\\ 0.24&0.18&-0.24&-0.18\\ -0.32&-0.24&0.32&0.24\\ -0.24&-0.18&0.24&0.18 \end{bmatrix} \end{array}$	$\mathbf{k}^{e3} = \frac{(1)(1)}{1.6\sqrt{2}}\begin{bmatrix} 0.5&-0.5&-0.5&0.5\\ -0.5&0.5&0.5&-0.5\\ -0.5&0.5&0.5&-0.5\\ 0.5&-0.5&-0.5&0.5 \end{bmatrix} = \begin{array}{c c} & \begin{matrix} \bar{2} & \bar{3} & 1 & 2 \end{matrix} \\ \begin{matrix} \bar{2}\\ \bar{3}\\ 1\\ 2 \end{matrix} & \begin{bmatrix} 0.221&-0.221&-0.221&0.221\\ -0.221&0.221&0.221&-0.221\\ -0.221&0.221&0.221&-0.221\\ 0.221&-0.221&-0.221&0.221 \end{bmatrix} \end{array}$

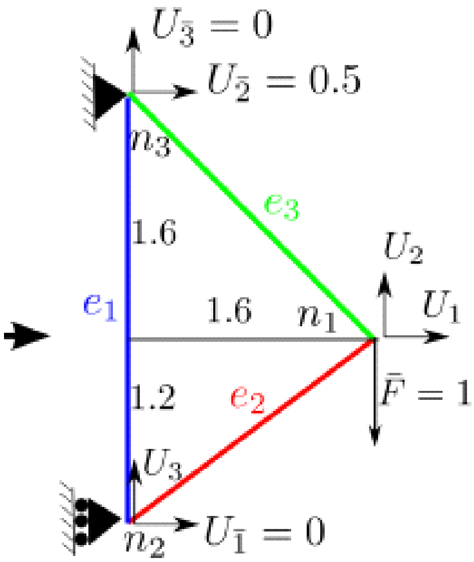
$$\mathbf{K} = \begin{bmatrix} 0.32+0.221 & 0.24-0.221 & -0.24 \\ 0.24-0.221 & 0.18+0.221 & -0.18 \\ -0.24 & -0.18 & 0.3571+0.18 \end{bmatrix} = \begin{bmatrix} 0.5410 & 0.019 & -0.24 \\ 0.019 & 0.401 & -0.18 \\ -0.24 & -0.18 & 0.5371 \end{bmatrix}$$

$\mathbf{f}_D^e$	$\mathbf{k}^{e1}\mathbf{a}_1^e = \begin{bmatrix} 0&0&0&0\\ 0&0.3571&0&-0.3571\\ 0&0&0&0\\ 0&-0.3571&0&0.3571 \end{bmatrix}\begin{bmatrix} 0\\ 0\\ 0.5\\ 0 \end{bmatrix} = \begin{array}{c c} \begin{matrix} \bar{1}\\ 3\\ \bar{2}\\ \bar{3} \end{matrix} & \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} \end{array}$	$\mathbf{k}^{e2}\mathbf{a}_2^e = \begin{bmatrix} 0.32&0.24&-0.32&-0.24\\ 0.24&0.18&-0.24&-0.18\\ -0.32&-0.24&0.32&0.24\\ -0.24&-0.18&0.24&0.18 \end{bmatrix}\begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} = \begin{array}{c c} \begin{matrix} \bar{1}\\ 3\\ 1\\ 2 \end{matrix} & \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} \end{array}$	$\mathbf{k}^{e3}\mathbf{a}_3^e = \begin{bmatrix} 0.221&-0.221&-0.221&0.221\\ -0.221&0.221&0.221&-0.221\\ -0.221&0.221&0.221&-0.221\\ 0.221&-0.221&-0.221&0.221 \end{bmatrix}\begin{bmatrix} 0.5\\ 0\\ 0\\ 0 \end{bmatrix} = \begin{array}{c c} \begin{matrix} \bar{2}\\ \bar{3}\\ 1\\ 2 \end{matrix} & \begin{bmatrix} 0.1105\\ -0.1105\\ -0.1105\\ 0.1105 \end{bmatrix} \end{array}$
$\mathbf{f}_e^e$	$\mathbf{f}_e^{e1} = \mathbf{f}_r^{e1} + \mathbf{f}_N^{e1} - \mathbf{f}_D^{e1} = \begin{array}{c c} \begin{matrix} \bar{1}\\ 3\\ \bar{2}\\ \bar{3} \end{matrix} & \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} \end{array}$	$\mathbf{f}_e^{e2} = \mathbf{f}_r^{e2} + \mathbf{f}_N^{e2} - \mathbf{f}_D^{e2} = \begin{array}{c c} \begin{matrix} \bar{1}\\ 3\\ 1\\ 2 \end{matrix} & \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} \end{array}$	$\mathbf{f}_e^{e3} = \mathbf{f}_r^{e3} + \mathbf{f}_N^{e3} - \mathbf{f}_D^{e3} = \begin{array}{c c} \begin{matrix} \bar{2}\\ \bar{3}\\ 1\\ 2 \end{matrix} & \begin{bmatrix} -0.1105\\ 0.1105\\ 0.1105\\ -0.1105 \end{bmatrix} \end{array}$

Assembled forces from elements is

$$F^e = \begin{bmatrix} 0.1105 \\ -0.1105 \\ 0 \end{bmatrix}$$

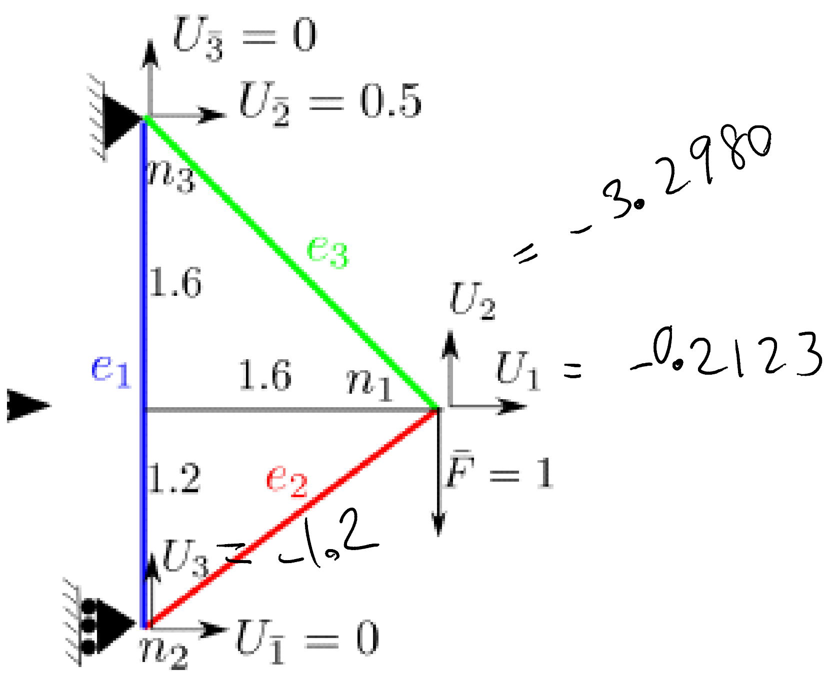
$$F = \underbrace{F^e}_{\text{element contributions}} + \underbrace{F^n}_{\text{nodal dof contributions}}$$



$$F^{n} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 0.32+0.221 & 0.24-0.221 & -0.24 \\ 0.24-0.221 & 0.18+0.221 & -0.18 \\ -0.24 & -0.18 & 0.3571+0.18 \end{bmatrix} = \begin{bmatrix} 0.5410 & 0.019 & -0.24 \\ 0.019 & 0.401 & -0.18 \\ -0.24 & -0.18 & 0.5371 \end{bmatrix}$$

$$\mathbf{F} = \mathbf{F}_N + \mathbf{F}_e = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.1105 \\ -0.1105 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.1105 \\ -1.1105 \\ 0 \end{bmatrix} \Rightarrow \boxed{\mathbf{U} = \mathbf{K}^{-1}\mathbf{F} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} -0.2123 \\ -3.2980 \\ -1.200 \end{bmatrix}}$$

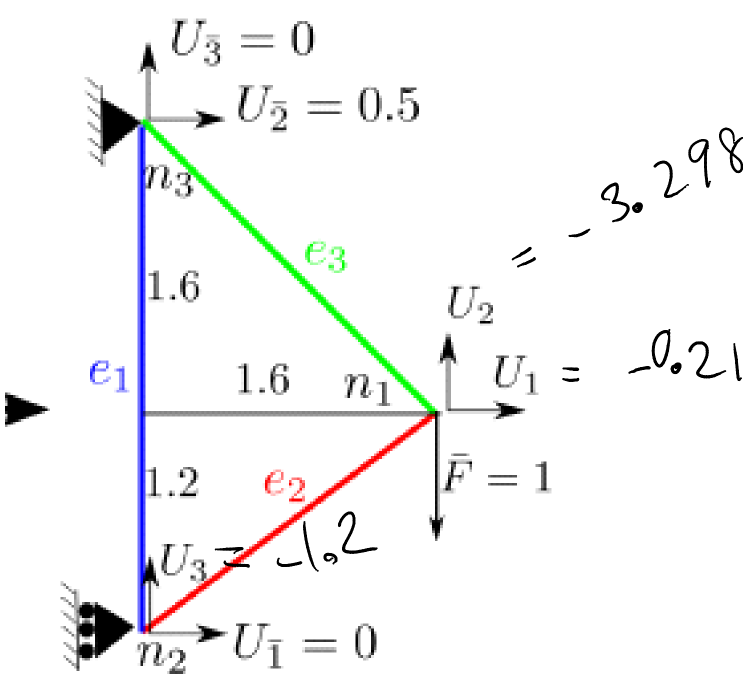


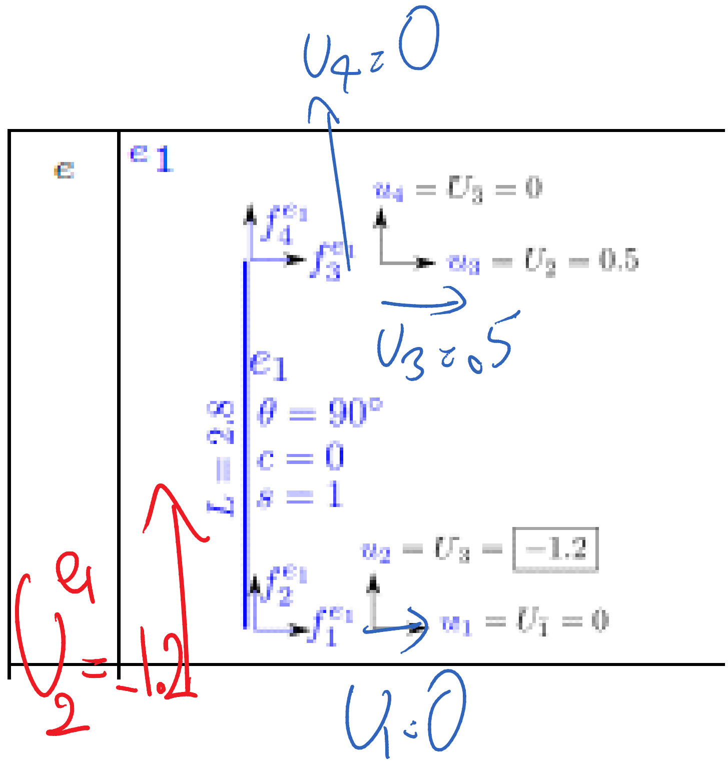
Things that we want to obtain:

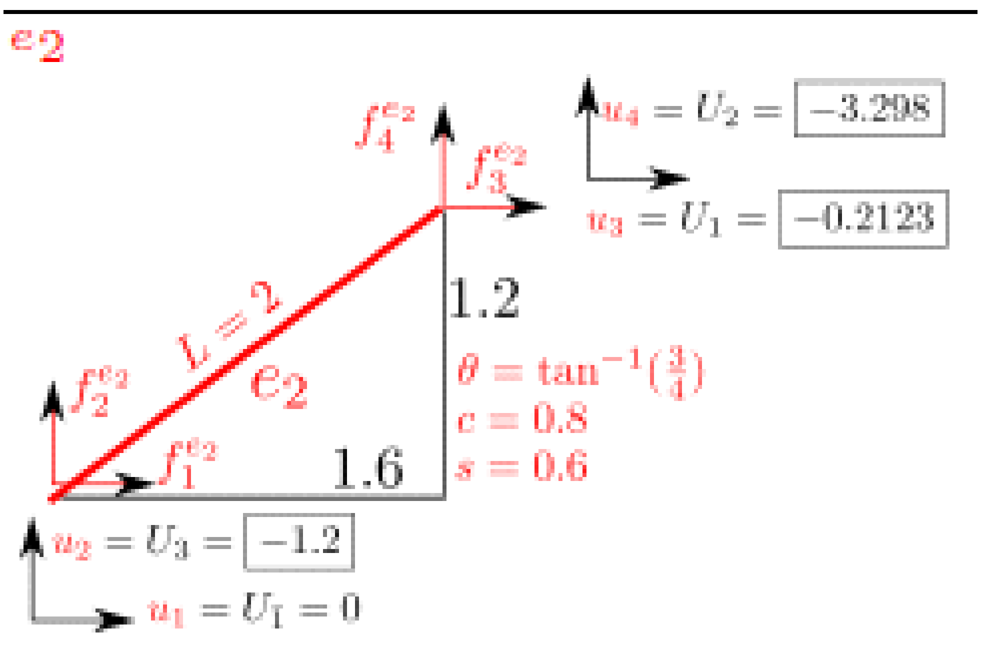
1. Element axial force (the force in its LOCAL coordinate system)
2. Support forces

To calculate element axial forces we need to do two things

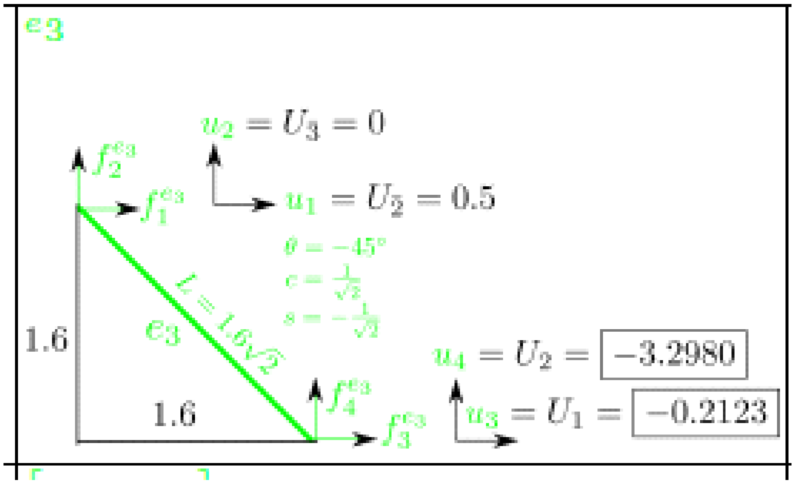
A. Transfer U1, U2, U3 (free dofs) and $U_{\bar{1}}, U_{\bar{2}}, U_{\bar{3}}$ (prescribed dofs) to element local displacements







Element 3



$\mathbf{u}^e$	$\begin{bmatrix} 0 \\ -1.2 \\ 0.5 \\ 0 \end{bmatrix}$ $c=0$ $s=1$	$\begin{bmatrix} 0 \\ -1.2 \\ -0.2123 \\ -3.2980 \end{bmatrix}$ $c=.8$ $s=.6$	$\begin{bmatrix} 0.5 \\ 0 \\ -0.2123 \\ -3.2980 \end{bmatrix}$

Element displacements in the global coordinate system

To compute element axial forces we use

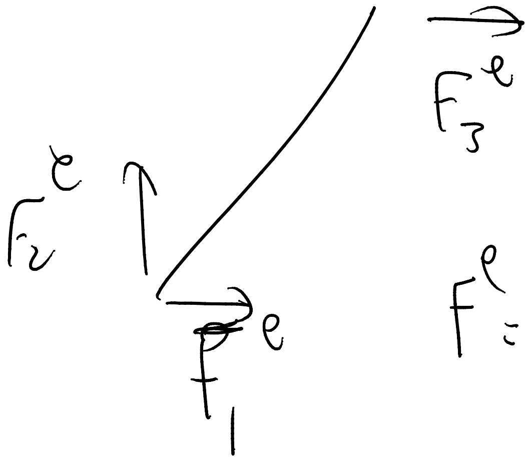
$$\boxed{T = \frac{AE}{L}\{c(U_3 - U_1) + s(U_4 - U_2)\}}$$

T^e	$T^{e1} = \frac{1\times 1}{2.8}\{0 \times (0.5 - 0) +1 \times (0 + 1.2)\} = 0.4286$	$T^{e2} = \frac{1\times 1}{2}\{0.8 \times (-0.2123 - 0) +0.6 \times (-3.2928 + 1.2)\} = -0.7128$	$T^{e3} = \frac{1\times 1}{1.6\sqrt{2}}\{\frac{1}{\sqrt{2}} \times (-0.2123 - 0.5) -\frac{1}{\sqrt{2}} \times (-3.2928 - 0)\} = 0.8064$

To compute support forces we evaluate the element forces all expressed in the global coordinate system.

For each element have 4 end point forces

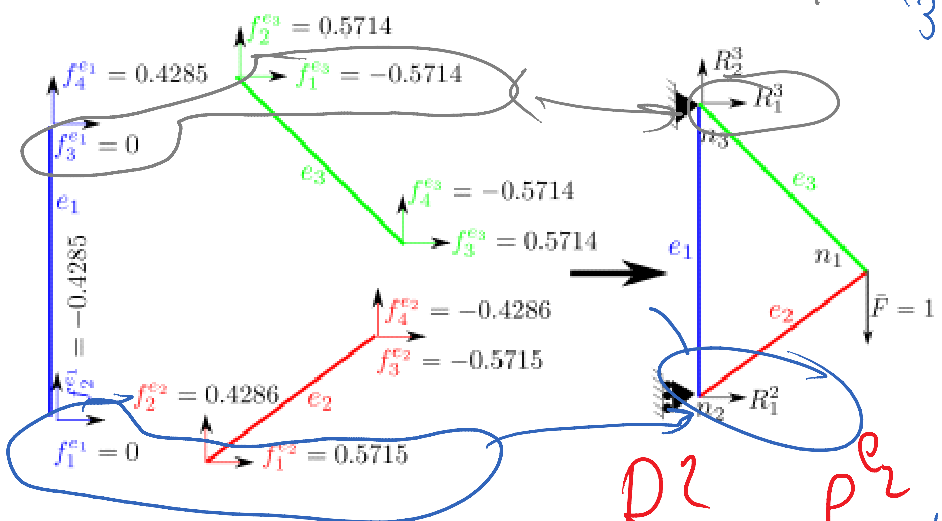
$F_a^e \uparrow$



$$F^e = k^e \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

$-f^e$	$\mathbf{k}^{e1}\mathbf{a}_1^e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.357 & 0 & -0.357 \\ 0 & 0 & 0 & 0 \\ 0 & -0.357 & 0 & 0.357 \end{bmatrix} \begin{bmatrix} 0 \\ -1.2 \\ 0.5 \\ 0 \end{bmatrix} = \begin{array}{c} \bar{1} \\ 3 \\ \bar{2} \\ \bar{3} \end{array} \begin{bmatrix} 0 \\ -0.4285 \\ 0 \\ 0.4285 \end{bmatrix}$	$\mathbf{k}^{e2}\mathbf{a}_2^e = \begin{bmatrix} 0.32 & 0.24 & -0.32 & -0.24 \\ 0.24 & 0.18 & -0.24 & -0.18 \\ -0.32 & -0.24 & 0.32 & 0.24 \\ -0.24 & -0.18 & 0.24 & 0.18 \end{bmatrix} \begin{bmatrix} 0 \\ -1.2 \\ -0.2123 \\ -3.2980 \end{bmatrix} = \begin{array}{c} \bar{1} \\ 3 \\ 1 \\ 2 \end{array} \begin{bmatrix} 0.5715 \\ 0.4286 \\ -0.5715 \\ -0.4286 \end{bmatrix}$	$\mathbf{k}^{e3}\mathbf{a}_3^e = \begin{bmatrix} 0.221 & -0.221 & -0.221 & 0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ 0.221 & -0.221 & -0.221 & 0.221 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0 \\ -0.2123 \\ -3.2980 \end{bmatrix} = \begin{array}{c} 2 \\ \bar{3} \\ 1 \\ 2 \end{array} \begin{bmatrix} -0.5714 \\ 0.5714 \\ 0.5714 \\ -0.5714 \end{bmatrix}$

$$R_1^3 = f_3^{e_4} + f_1^{e_3} = -.5714$$



$$R_1^2 = f_1^{e_2} + f_1^{e_1}$$

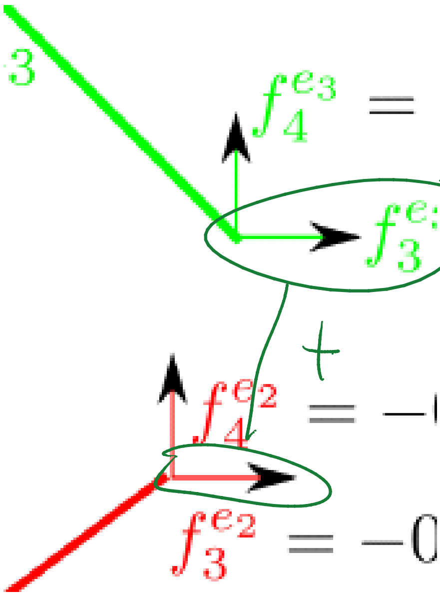
$$= .5715$$

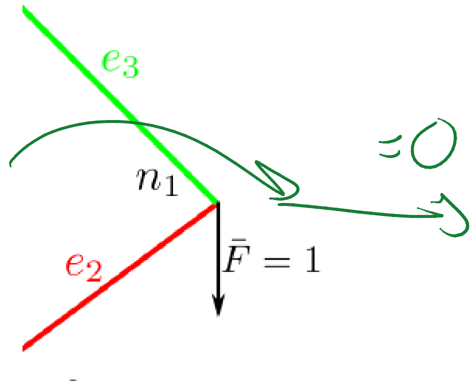
$$R_1^2 = f_1^{e_1} + f_1^{e_2} = 0 + 0.5715 = 0.5715 \quad (397a)$$

$$R_1^3 = f_3^{e_1} + f_1^{e_3} = 0 + -0.5714 = -0.5714 \quad (397b)$$

$$R_2^3 = f_4^{e_1} + f_2^{e_3} = 0.4285 + 0.5714 = 0.9999 \quad (397c)$$

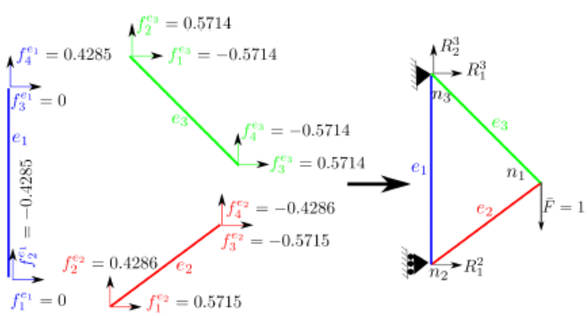
Question





Although we do not need to recalculate forces for free dofs (because they are given) we can check the correctness of our FEM solutions

Truss Example: verification of forces at free dofs



- Also, if we want to double-check our calculations on **free dofs**. This step is not needed and it may be done as a verification for hand calculations:

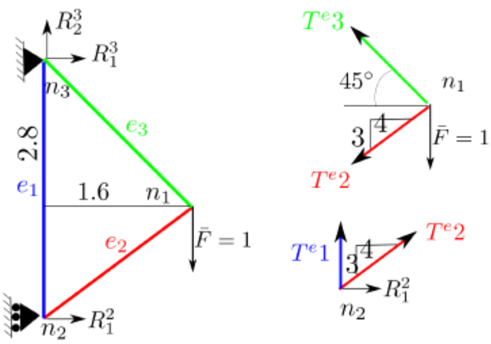
$$F_1^1 = f_3^{e_2} + f_3^{e_3} = -0.5715 + 0.5714 = -0.0001 \qquad (398a)$$

$$F_2^1 = f_4^{e_2} + f_4^{e_3} = -0.4286 + -0.5714 = -1 = \bar{F} \qquad (398b)$$

$$R_2^2 = f_2^{e_1} + f_2^{e_2} = -0.4285 + 0.4286 = 0.0001 \qquad (398c)$$

Exact solution

Truss Example: Direct solution method



- Since this is a statically determinate structure, we can easily solve the forces and verify our FEM forces.

$$\Sigma F_2 = 0 \quad \Rightarrow \quad R_2^3 - F = 0 \quad \Rightarrow \quad R_2^3 = 1 \tag{399a}$$

$$\Sigma M_{n_3} = 0 \quad \Rightarrow \quad 2.8 R_1^2 - 1.6 F = 0 \quad \Rightarrow \quad R_1^2 = \frac{4}{7} = 0.5714 \tag{399b}$$

$$\Sigma F_1 = 0 \quad \Rightarrow \quad R_1^2 + R_1^3 = 0 \quad \Rightarrow \quad R_1^3 = -\frac{4}{7} = -0.5714 \tag{399c}$$

$$\Sigma F_1 = 0(@n_2) \quad \Rightarrow \quad R_1^2 + \frac{4}{5} T^e 2 = 0 \quad \Rightarrow \quad T^e 2 = -\frac{5}{7} = -0.7143 \tag{399d}$$

$$\Sigma F_2 = 0(@n_2) \quad \Rightarrow \quad T^e 1 + \frac{3}{5} T^e 2 = 0 \quad \Rightarrow \quad T^e 1 = \frac{3}{7} = 0.4286 \tag{399e}$$

$$\Sigma F_1 = 0(@n_1) \quad \Rightarrow \quad -\frac{4}{5} T^e 2 - \frac{1}{\sqrt{2}} T^e 3 = 0 \quad \Rightarrow \quad T^e 3 = \frac{4}{7}\sqrt{2} = 0.8081 \tag{399f}$$

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FEM solution captures the exact solution!
Why?

Whenever FEM discrete solution is capable of modeling the exact solution function it actually captures the exact solution.

Slides 329 to 334 are the material we covered before