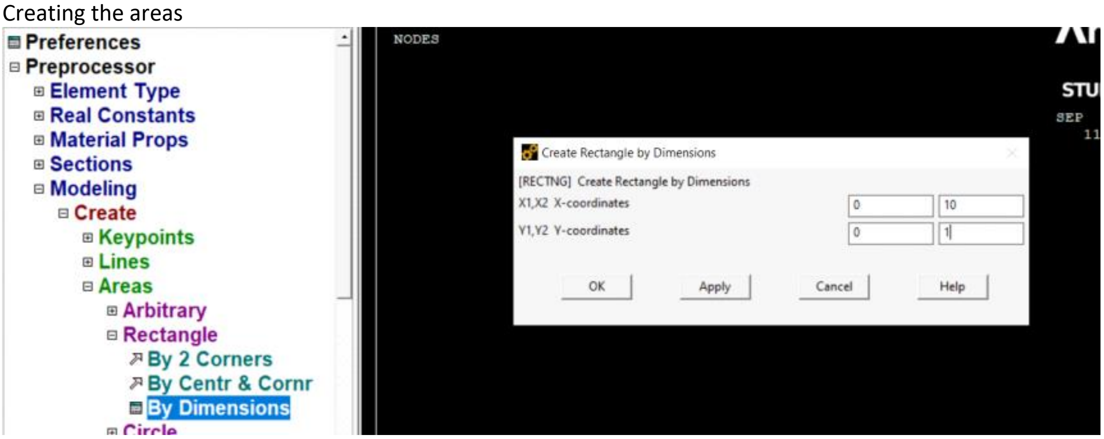
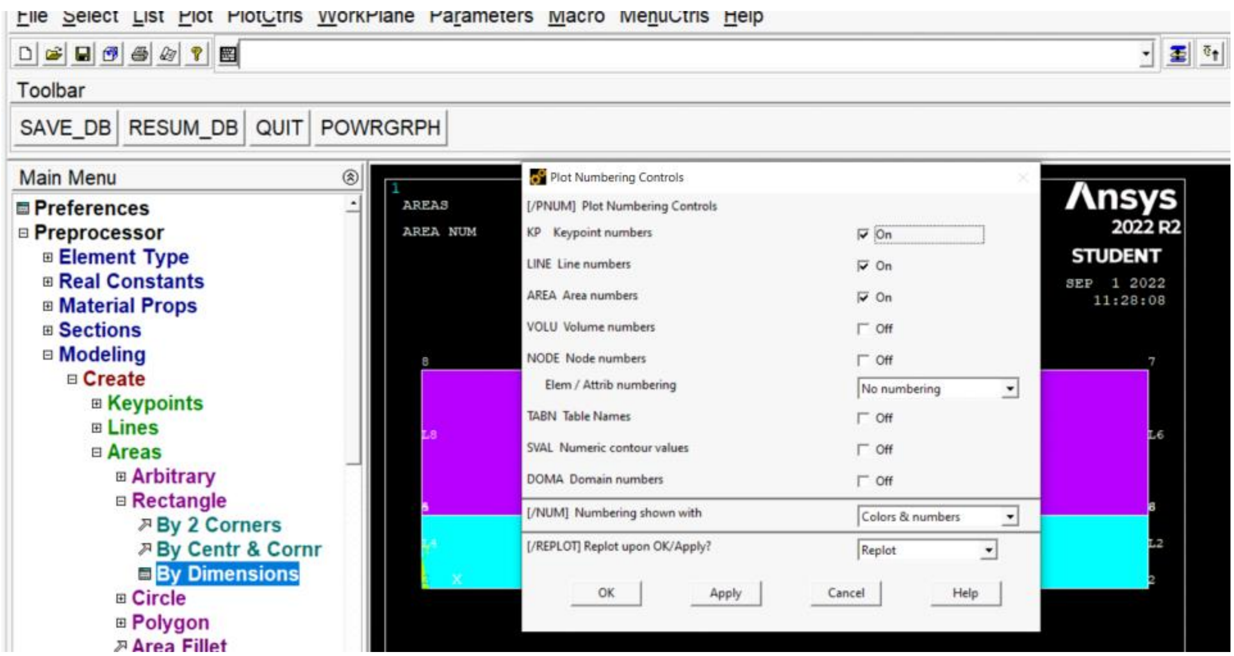
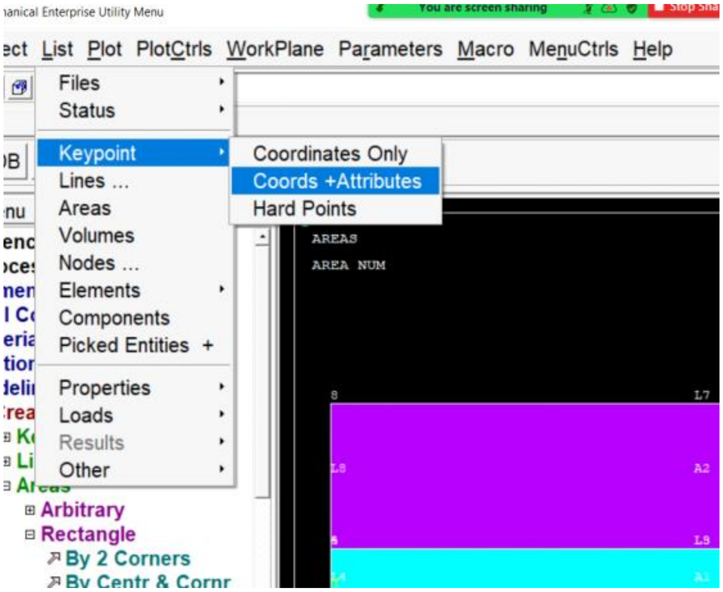
# 2022/09/01

Thursday, September 1, 2022 11:25 AM

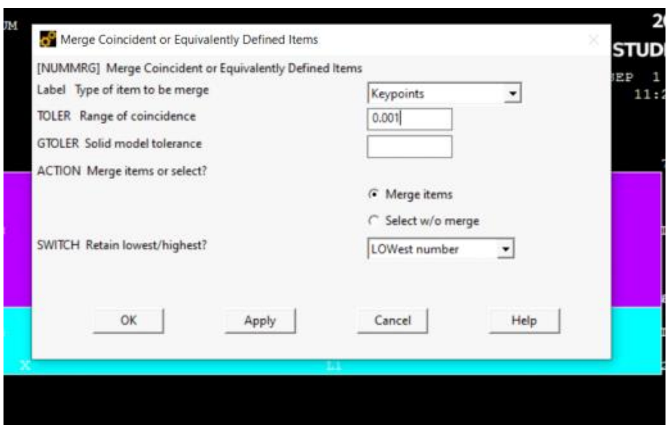
Creating the areas

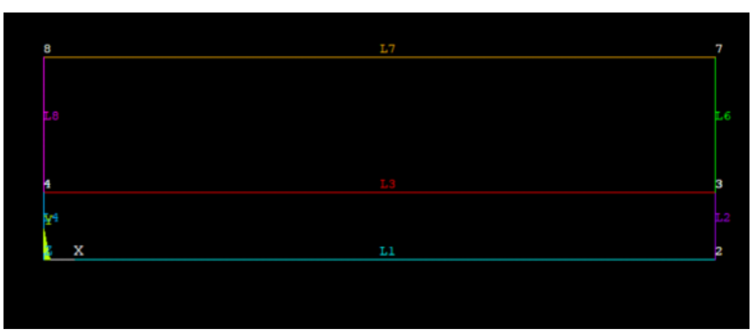





We need to merge the keypoionts and after that the connecting lines will also merge

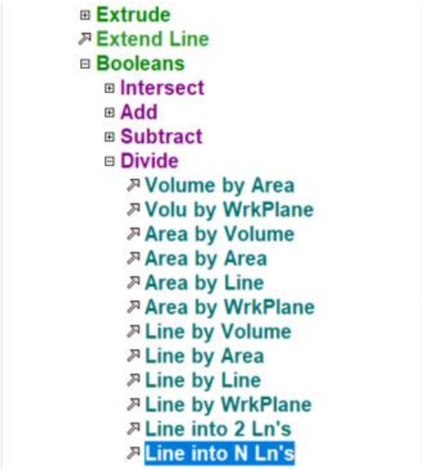


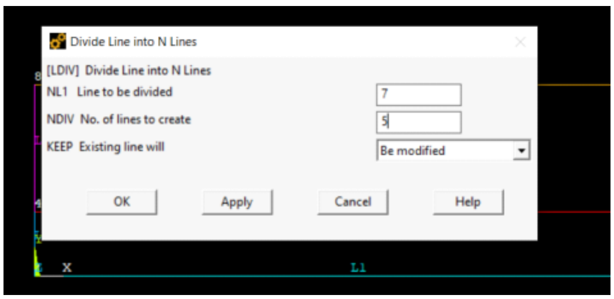



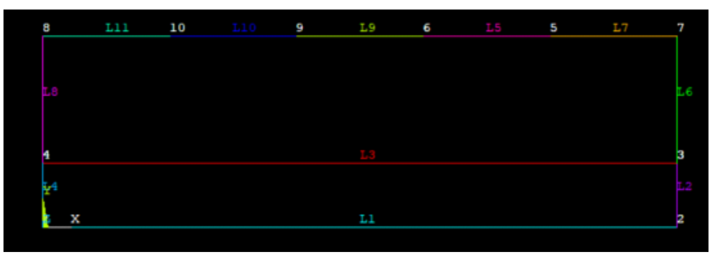

LIST ALL SELECTED KEYPOINTS. DSYS= 0

| NO. | X,Y,Z LOCATION | | | KESIZE | NODE | ELEM | MAT | REAL | TYP | ESYS |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 10.0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 10.0 | 1.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0.00 | 1.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 10.0 | 3.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0.00 | 3.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 |

Dividing the top line to 5 segments so we can apply the load on the first created segment from the left



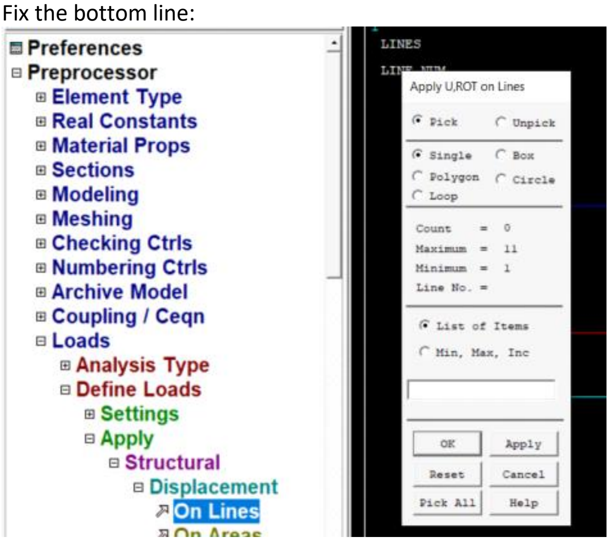



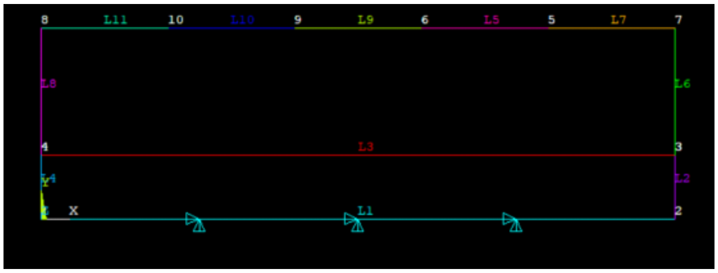




Boundary conditions

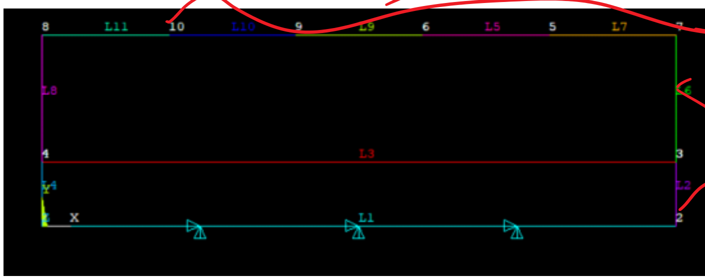
- We don't need to specify that the left edge is the axis of symmetry

- Fix the bottom line:

- 




After fixing all dofs on the bottom surface

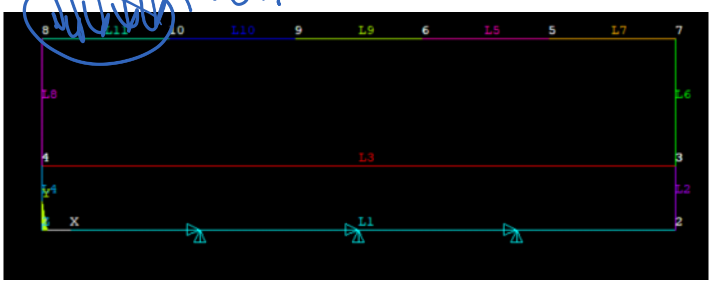



traction free

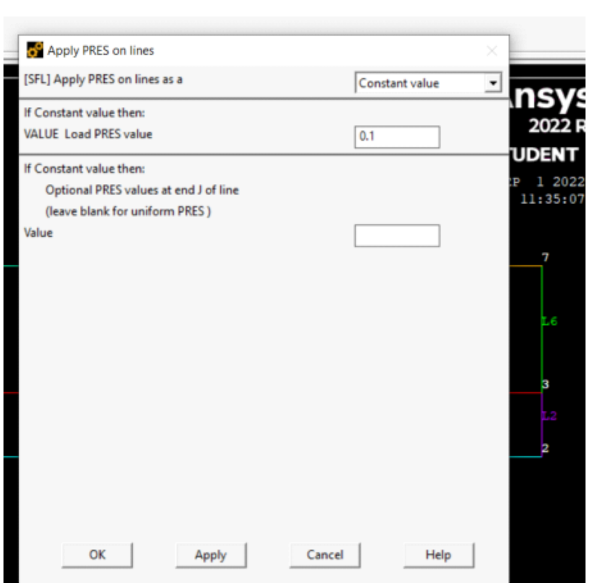
no need to define BC here as this is FEM's default

Apply the pressure on the top surface - first segment

P=0.1







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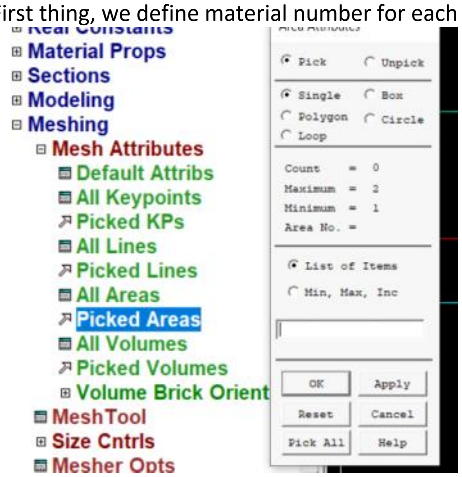
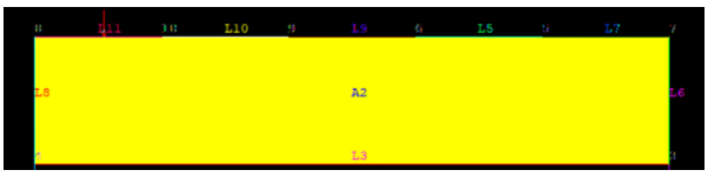
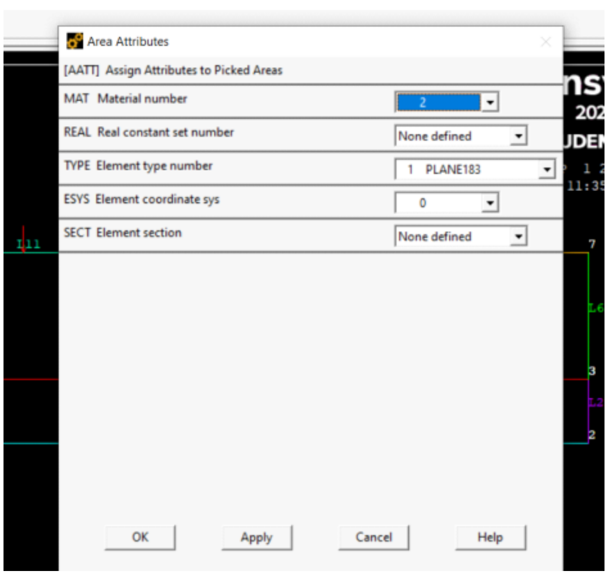
Meshing

First thing, we define material number for each area

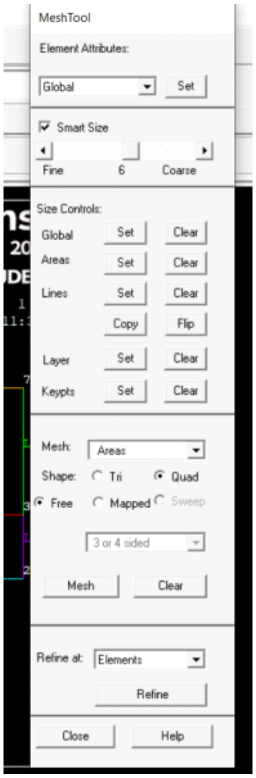


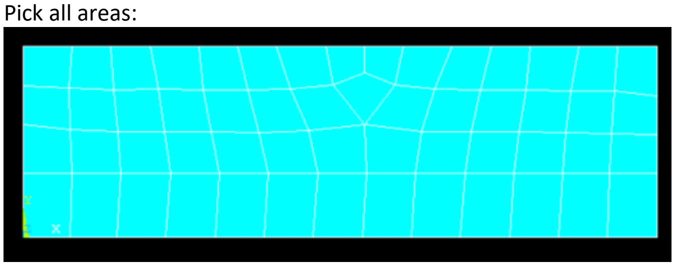




List the areas

```
LIST ALL SELECTED AREAS.
NUMBER LOOP  LINES              AREA    ELEM SIZE  #NODES  #ELEM   MAT  REAL  TYP  ESYS  SECN
   1  1    1    2    3    4  N/A    0.000      0    0    1    0    1    0    0
   2  1    3    6    7    5  N/A    0.000      0    0    2    0    1    0    0
           9   10   11    8
```




Pick all areas:



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Solution:


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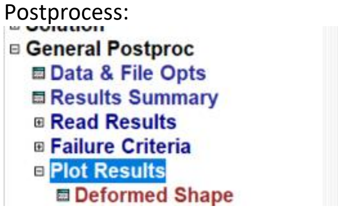
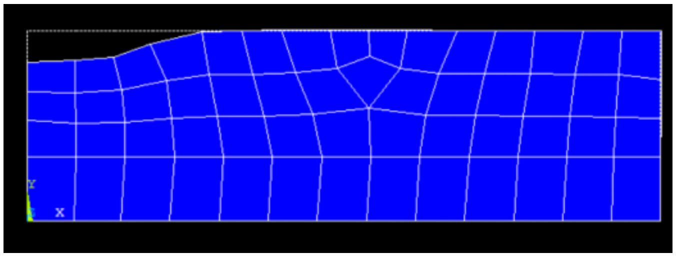
Postprocess:





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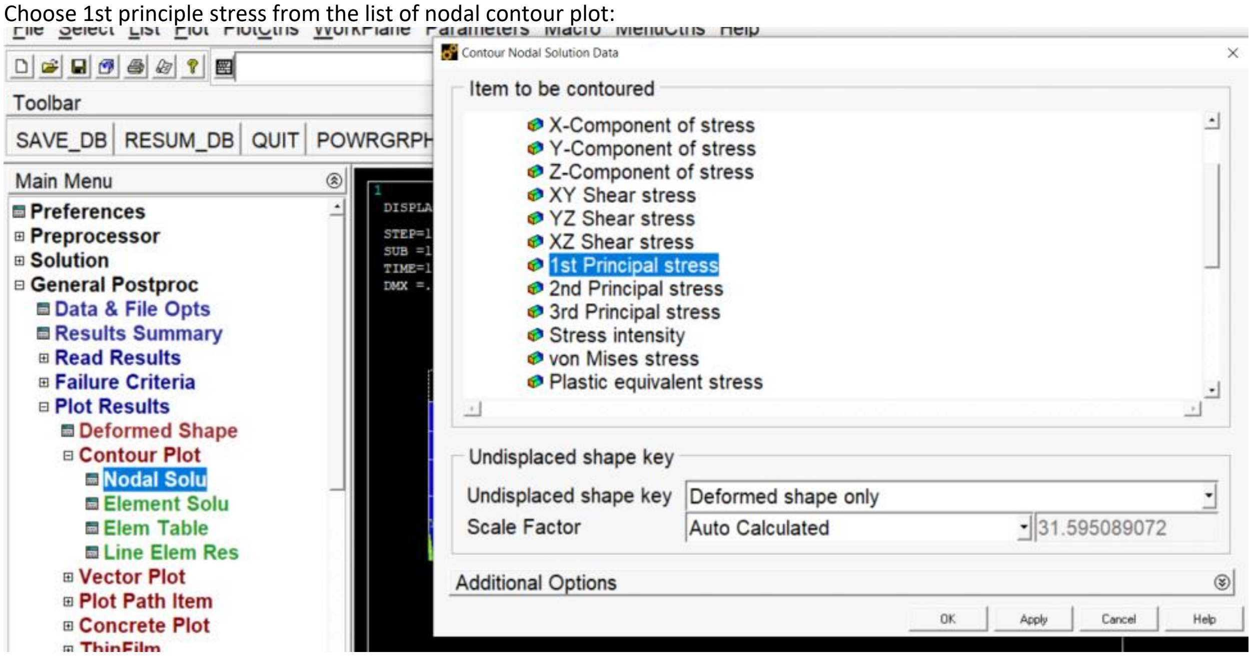
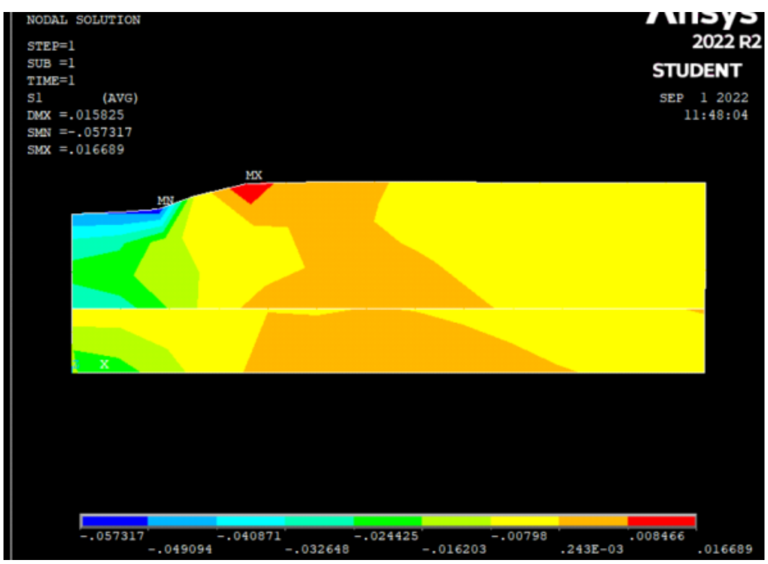
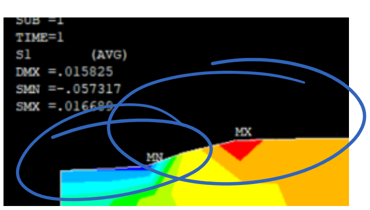
Contour plots

Choose 1st principle stress from the list of nodal contour plot:

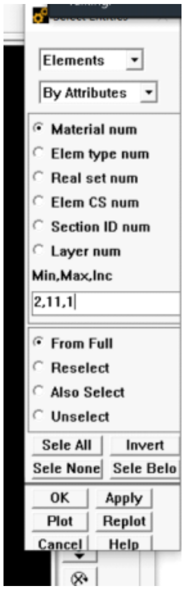


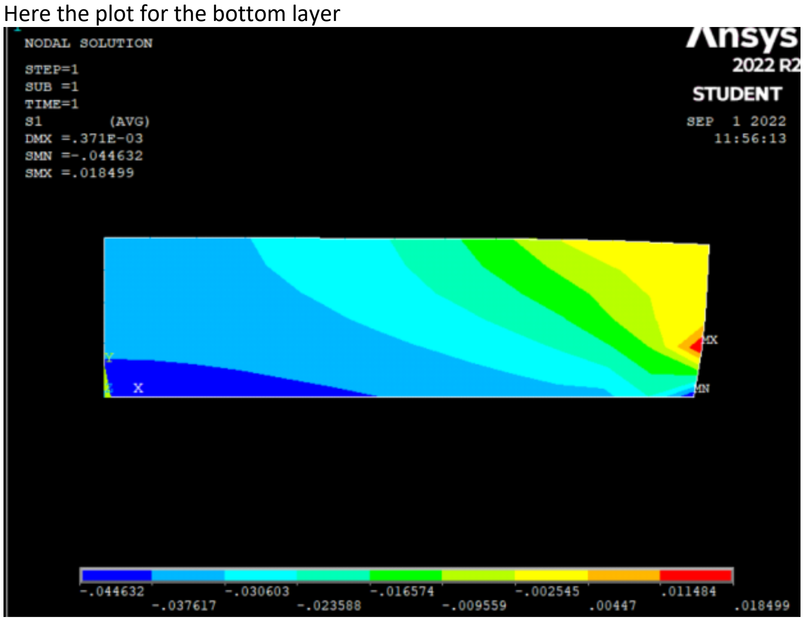




Min and max sigma_1 for the whole domain

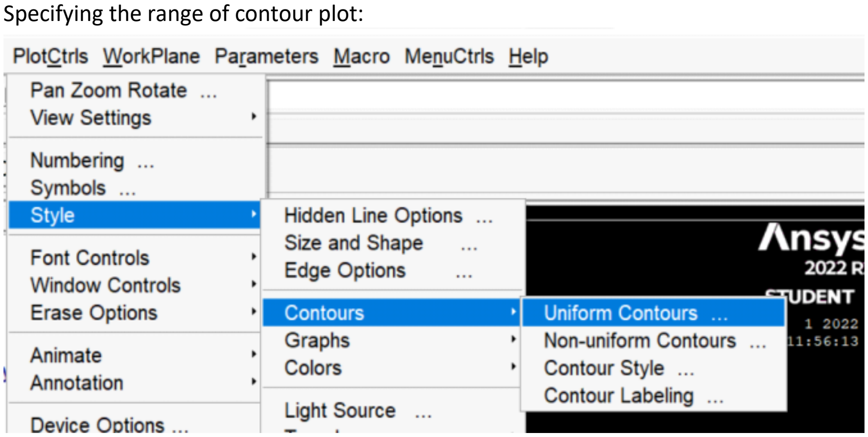
Plotting the results for certain number of layers
Select -> entities ->

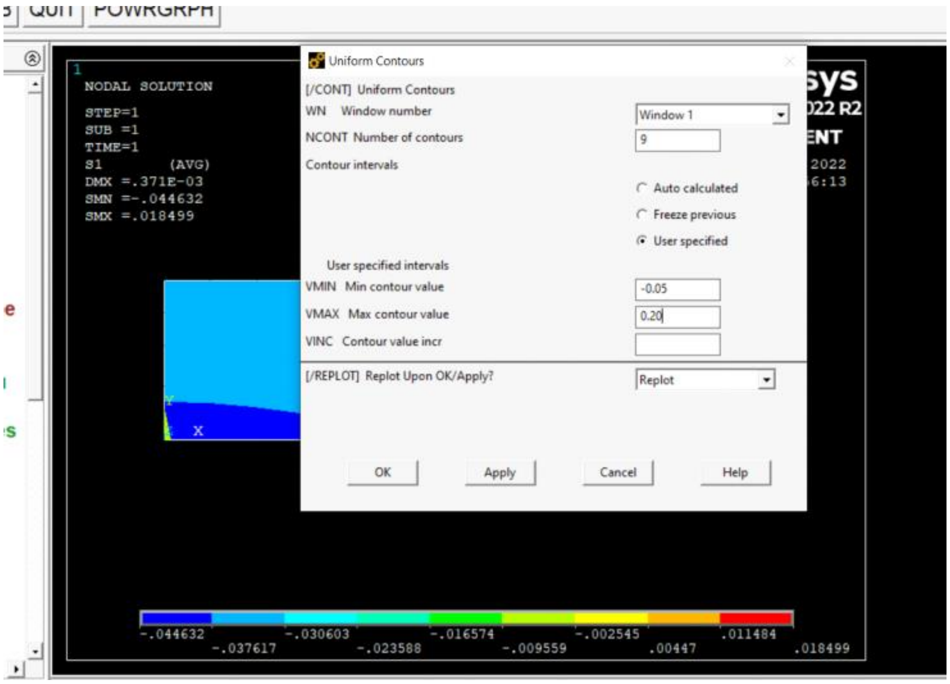


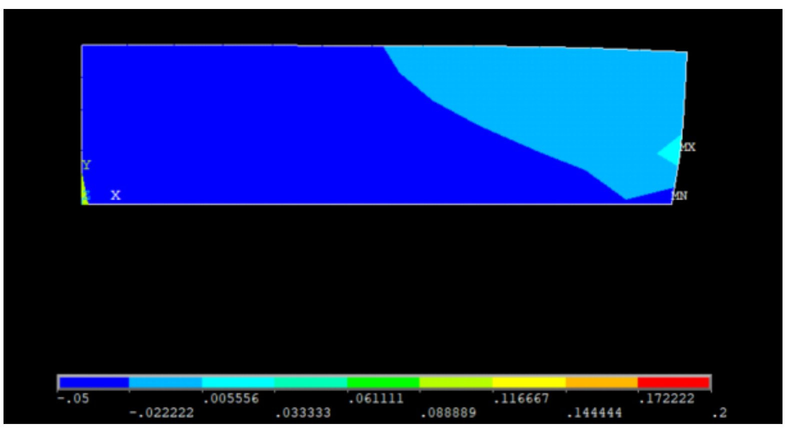

Here the plot for the bottom layer

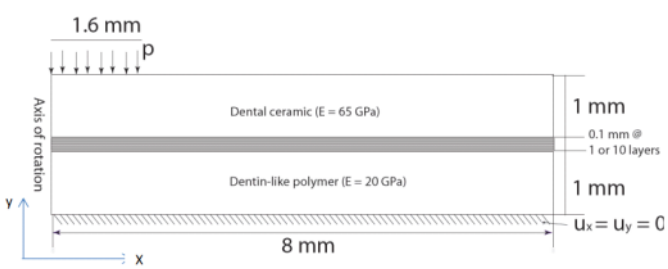



Specifying the range of contour plot:






I ⟶ make sure total length is 0.1 mm

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FEM Formulation

1. Balance laws

$$\sum F_{\text{on } \omega} = 0$$

$$\underline{\quad} \; t_{\text{on } \omega} = 0 \qquad \underline{\qquad\qquad\qquad\qquad} \; \Omega$$

subdomain $\leftarrow \omega \subseteq \Omega \rightarrow$ domain

$$\equiv \int_{\partial\omega} \sigma \cdot n \, ds + \int_{\omega} \rho b = 0$$

$\partial\omega$ → boundary of $\omega$

$\sigma$ → stress tensor

$\rho b$ → body force

$\omega$ → arbitrary

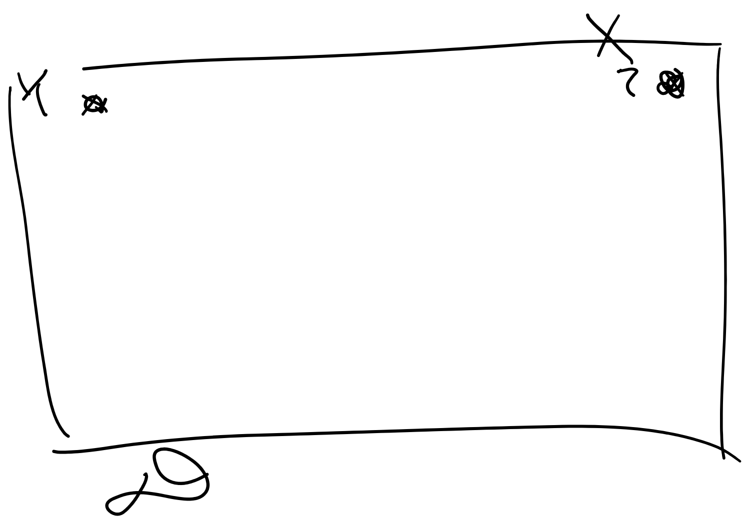
(2) ⟶ we will derive

strong form = (Partial) Differential Equation (PDE)

$$\nabla \cdot \sigma + \rho b = 0 \quad \forall x \in \Omega$$

$\nabla \cdot$ → divergence theorem

(3)




$$R = \nabla \cdot \sigma + \rho b$$

$R$ → residual "error"

Weighted Residual Statement (WRS)

$$\forall w \quad \int_{\Omega} w \underbrace{(\nabla \cdot \sigma + \rho b)}_{R} = 0 \qquad (\star)$$

$w$ → weight function → arbitrary

fixed

$\forall w$ fixed

weight functions

domain

R = Residual

(4) $u = \sum_{i=1}^{N} a_i \, \phi_i(x)$

$a_i$ → unknowns

$\phi_i$ → shape functions, known → we choose this

$\phi_1 = x$

$\phi_2 = x^2$

$\phi_3 = x^3$

Discretization

$\infty$ unknowns → $N$ unknowns $(a_1, \dots, a_n)$

→ $N$ equations

we'll satisfy (A) for $w_1, w_2, \dots, w_N$

we'll choose these functions

$\forall i \in \{1, N\}$ $\int_\Omega w_i \, (\nabla \cdot \sigma + \rho b) = 0$

1D

$w_1 = x$

$w_2 = x^2$ example

$w_3 = x^3$

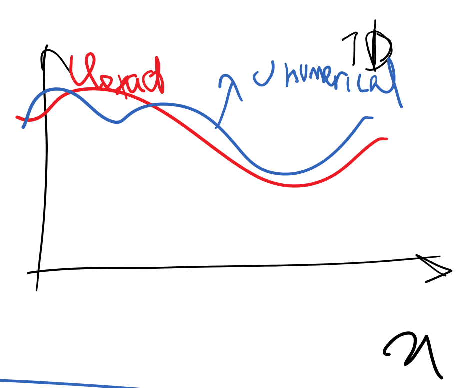
⋮

Find $a_i$'s

$u = \sum a_i \, \phi_i(x)$

$u$ → fully known

$a_i$ → is obtained




$u \to \varepsilon \to \sigma, \dots$

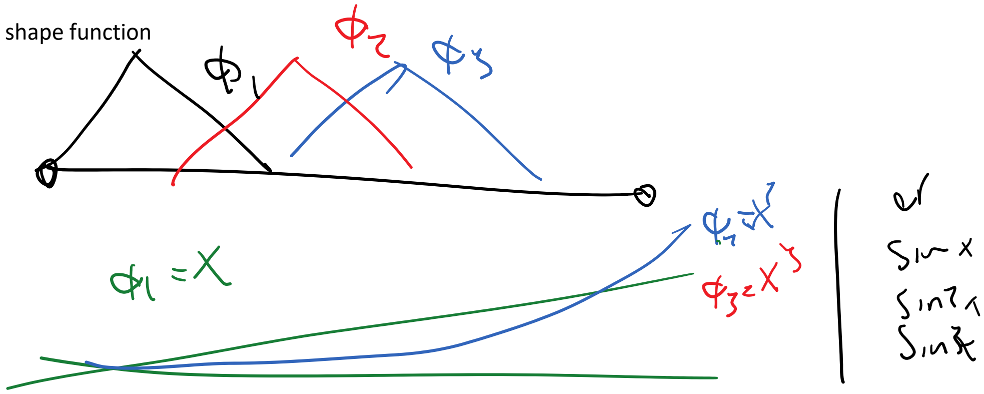
Different methods have different forms of shape function

$\phi_2$ $\phi_3$

Different methods have different forms of shape function

FEM

Spectral method




Discontinuous Galerkin -> Different basis functions

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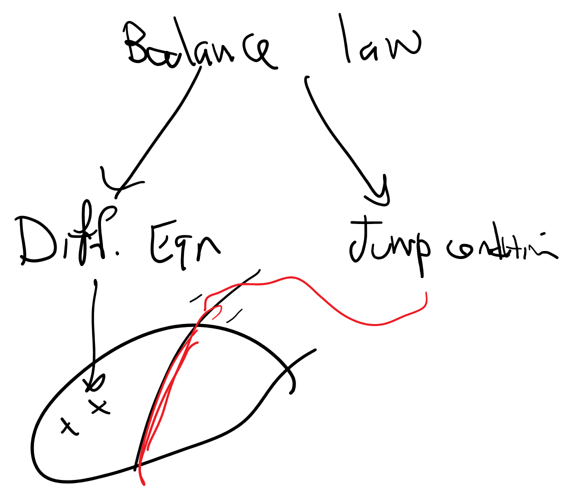
FEM formulation in detail

1. Balance law

- Why start with a balance law?
  - They are the actual physics laws.
  - They contain more *information* than their corresponding PDEs.
  - Larger solution space than the PDEs.
- Can we directly start the FE formulation from a PDE?
  - Yes, FE formulation starts from a differential equation.
  - A PDE may not be derived from a balance law.




Balance of mass, force (linear momentum), energy, …
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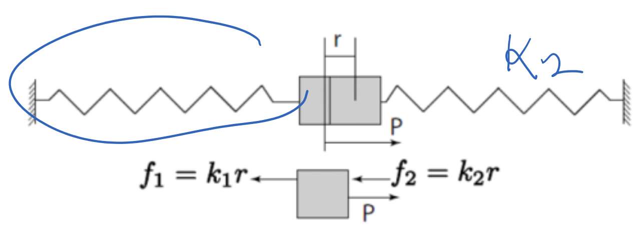
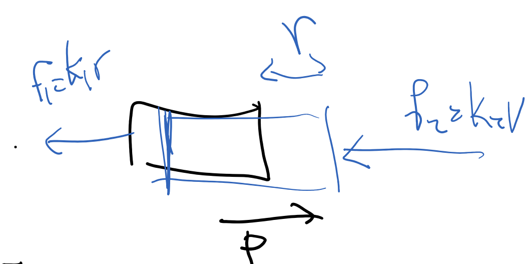
Example of balance of force in discrete setting:

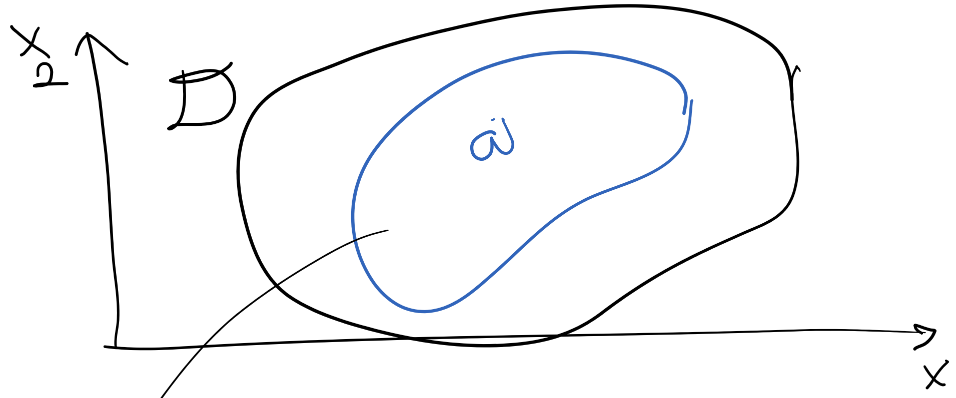



$$-f_1 - f_2 + P = 0$$

$$\to (k_1 + k_2) r = P \to r = \frac{P}{k_1 + k_2}$$

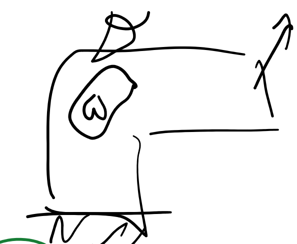
Continuum:

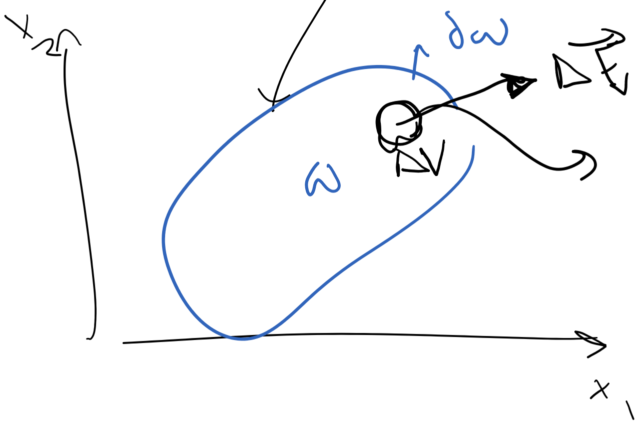



Balance of forces

$$\sum F = 0$$

subdomain $\omega \subseteq D$ domain




$$\frac{\Delta \vec{F}_v}{\Delta V} = \frac{\Delta \vec{F}_v}{\Delta m} \cdot \frac{\Delta m}{\Delta V} \rightarrow \rho \text{ - density}$$

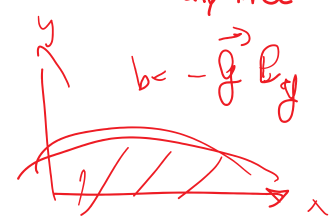
force per volume

incremental mass

$$\lim_{\Delta m \to 0} \frac{\Delta \vec{F}_v}{\Delta m} = \frac{d\vec{F}_v}{dm} = \vec{b}$$

body force

Example $\vec{b} = -g\,\vec{e}_y$




Types of forces: 1. Volumetric force

$$\vec{F}_v = \sum \Delta\vec{F}_v = \sum \frac{\Delta\vec{F}_v}{\Delta V}\Delta V$$

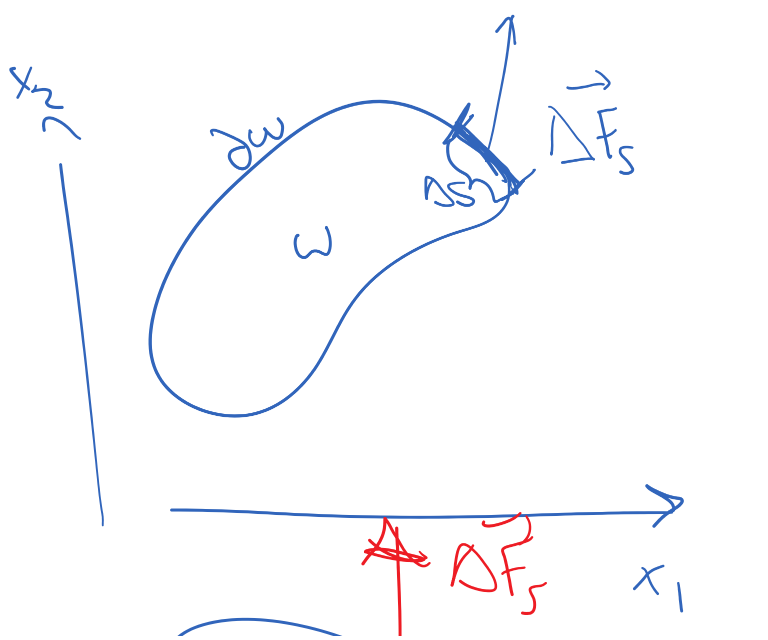
$$= \int_\omega (\rho b)\, dV$$

Surface force

$$\vec{F}_s = \sum \Delta\vec{F}_s$$

$$= \sum \frac{\Delta\vec{F}_s}{\Delta S}\Delta S$$

let $\Delta S \to 0$



let $\Delta S \to 0$

$$= \int_{\partial \omega} \left[ \lim_{\Delta S \to 0} \left( \frac{\Delta F_s}{\Delta S} \right) \right] dS$$


intensity of force

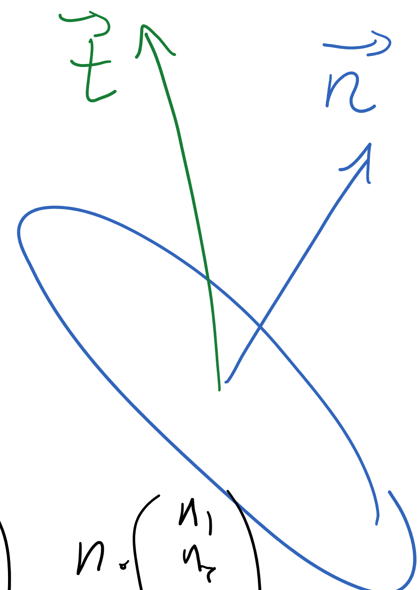
"force per area" $\lim_{\Delta S \to 0} \frac{\Delta F_s}{\Delta S} = \vec{t}$

Pa

MPa

psi

$$F_s = \int_{\partial \omega} \vec{t} \, dS$$




$$\vec{t} = \sigma \cdot \vec{n}$$

$\sigma$ → stress tensor

$$t = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} \quad n = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$$

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & — & \\ & & \sigma_{33} \end{pmatrix}$$

stress tensor