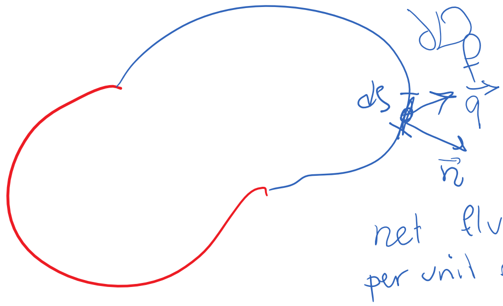
2022/09/29OH

Thursday, September 29, 2022 12:43 PM



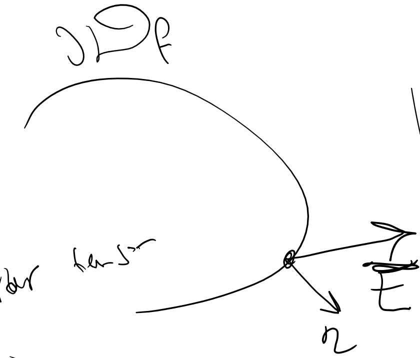
net flux of energy per unit power

scalar $\leftarrow \underbrace{\vec{q}\cdot\vec{n}}_{\text{over } ds}\, ds$

there for natural BC net flux is specified

$\vec{q}\cdot\vec{n} = \bar{q}$

vector, scalar



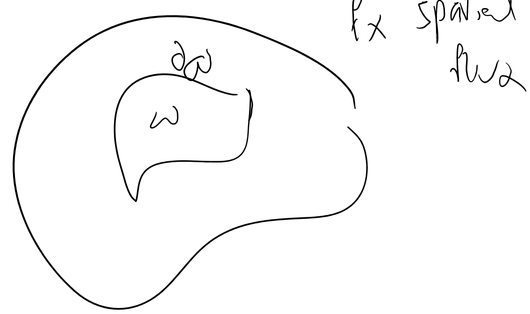
2nd order tensor

$\sigma \cdot n = \bar{t}$

spatial flux, vector

$$-\int_{\partial\omega} f_x \cdot n \, ds + \int_{\omega} r \, dV = 0$$

? ?

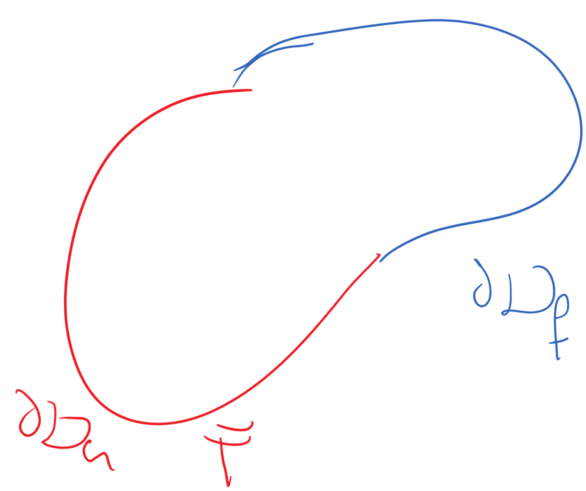


$R_f = \bar{q} - q \cdot n$

$R_u = \bar{T} - T$ we'd strongly satisfy this

$T(x) = \phi_p(x) + \sum \phi_i(x) a_i$

$R_i = \nabla \cdot q - Q$



WRS

$$\int w R_i \, dV + \int w R_f \, ds = 0$$

$$\int_{\mathcal{D}} \omega R_i \, dV + \int_{\partial\mathcal{D}_f} w R_f \, ds = 0$$

$$\int_{\mathcal{D}} \omega (\nabla \cdot q - Q) \, dV + \int_{\partial\mathcal{D}_f} \omega (\bar{q} - q \cdot n) \, ds = 0$$

$q \to -k\nabla T$ Fourier heat model

C^0 no der. — T needs 2 derivatives C^2 → we want to balance it.

WRS

no BC for ω

WRS → weak

$$\int \omega \, \nabla \cdot q \, dV \quad \rightsquigarrow \quad \int \nabla W \, q \quad + \ldots$$

2D $\quad \omega \nabla \cdot \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \omega(\underbrace{q_{1,1}}_{\frac{\partial q_1}{\partial x_1}} + q_{2,2})$

$$\forall i \in \{1,2\}: \int_{\mathcal{D}} \frac{\partial f}{\partial x_i} \, dV = \int_{\partial\mathcal{D}} f n_i \, dS \qquad (\star)$$

$$\omega q_{1,1} = (\omega q_1)_{,1} - \omega_{,1} q_1$$

$$\omega q_{2,2} = (\omega q_2)_{,2} - \omega_{,2} q_2$$

(i) $\displaystyle\int_{\mathcal{D}} \omega q_{1,1} \, dV = \int_{\mathcal{D}} (\omega q_1)_{,1} \, dV - \int_{\mathcal{D}} \omega_{,1} q_1 \, dV = \int_{\partial\mathcal{D}} \omega q_1 n_1 \, ds$

$(\star)$ $i=1$

$$- \int_{\mathcal{D}} \omega_{,1} q_1 \, dV$$

similarly (ii) $\displaystyle\int \omega q_{2,2} \, dV = \int_{\partial\mathcal{D}} \omega q_2 n_2 \, ds - \int_{\mathcal{D}} \omega_{,2} q_{,2} \, dV$

i, ii $\rightarrow$ $$\int w(\underbrace{q_{1,1}+q_{2,2}}_{\nabla\cdot q})\,dV = \int_{\partial\Omega} w\underbrace{(q_1 n_1 + q_2 n_2)}_{q\cdot n}\,ds - \int_{\Omega}\underbrace{(w_{,1} q_1 + w_{,2} q_2)}_{\nabla w\cdot q}\,dV \quad , \quad \nabla w = (w_{,1}, w_{,2})$$

I showed

eq (◇)
$$\boxed{\int_{\Omega} w\,\nabla\cdot q\,dV = \int_{\partial\Omega} w\,q\cdot n\,ds - \int_{\Omega}\nabla w\, q\,dV}$$

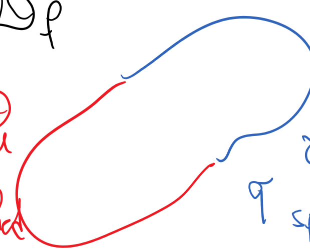
plug this into WRS

$$\int_{\Omega}(\underbrace{w\nabla\cdot q}_{wR_i} - wQ)\,dV + \int_{\partial\Omega_f} w(\underbrace{\bar{q} - q\cdot n}_{R_f})\,ds = 0$$

$$\left(\int_{\partial\Omega} w\,q\cdot n\,ds - \int_{\Omega}\nabla w\, q\,dV\right) - \int_{\Omega} wQ\,dV + \int_{\partial\Omega_f} w\bar{q} - \int_{\partial\Omega_f} w\,q\cdot n\,ds = 0$$

$$\partial\Omega = \partial\Omega_u \cup \partial\Omega_f$$

$$\partial\Omega_u \cap \partial\Omega_f = \phi$$



$$\int_{\partial\Omega} w\,q\cdot n\,ds$$

$$\left(\int_{\partial\Omega_u} w\,q\cdot n\,ds + \cancel{\int_{\partial\Omega_f} w\,q\cdot n\,ds} - \int_{\Omega}\nabla w\, q\,dV\right) - \int_{\Omega} wQ\,dV + \int_{\partial\Omega_f} w\bar{q}\;\cancel{- \int_{\partial\Omega_f} w\,q\cdot n\,ds} = 0$$

$= 0$

want to get rid of this

Choose $\omega = 0$ on $\partial\mathcal{D}_u$

this term goes away

$$\int_{\mathcal{D}} -\nabla w \, q \, dV = \underbrace{\int_{\mathcal{D}} w Q \, dV}_{F_r} - \underbrace{\int_{\partial\mathcal{D}_f} w \bar{q} \, ds}_{F_n}$$

$q = -K \nabla T$

$$\int_{\mathcal{D}} \nabla w \, K \, \nabla T \, dV = -$$

$$\int L_m(w) \, D \, L_m(T) \, dV$$

$D = K$, $L_m = \nabla$