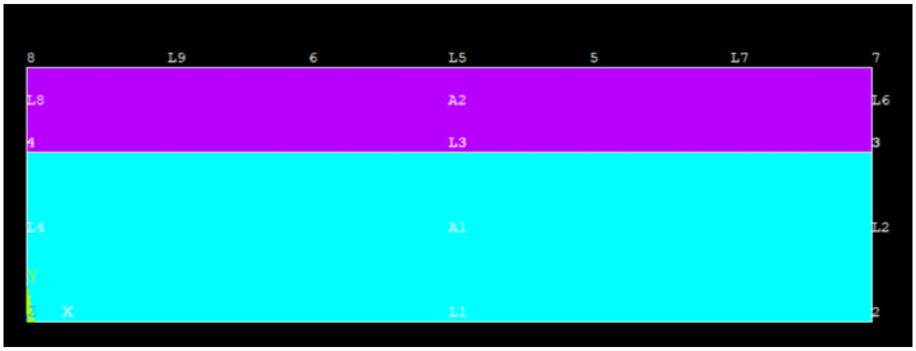
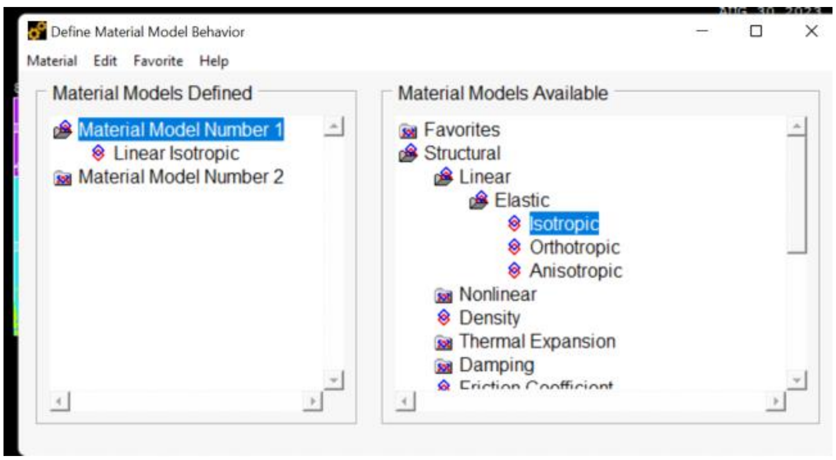
2023/08/30

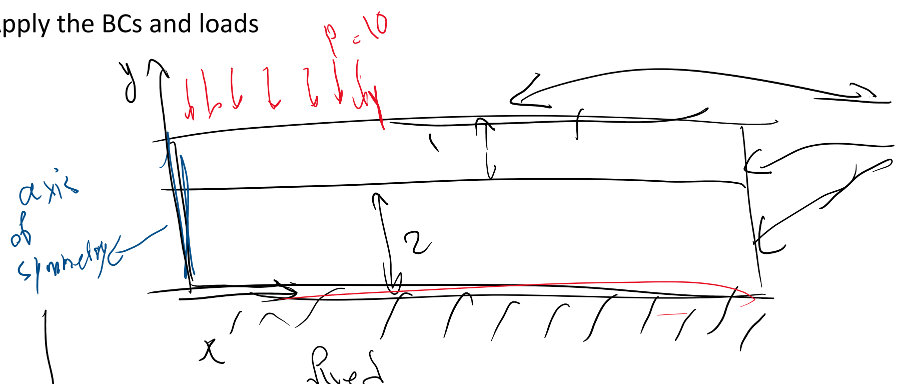
Wednesday, August 30, 2023 11:16 AM



We add material properties similar to the truss problem



Apply the BCs and loads

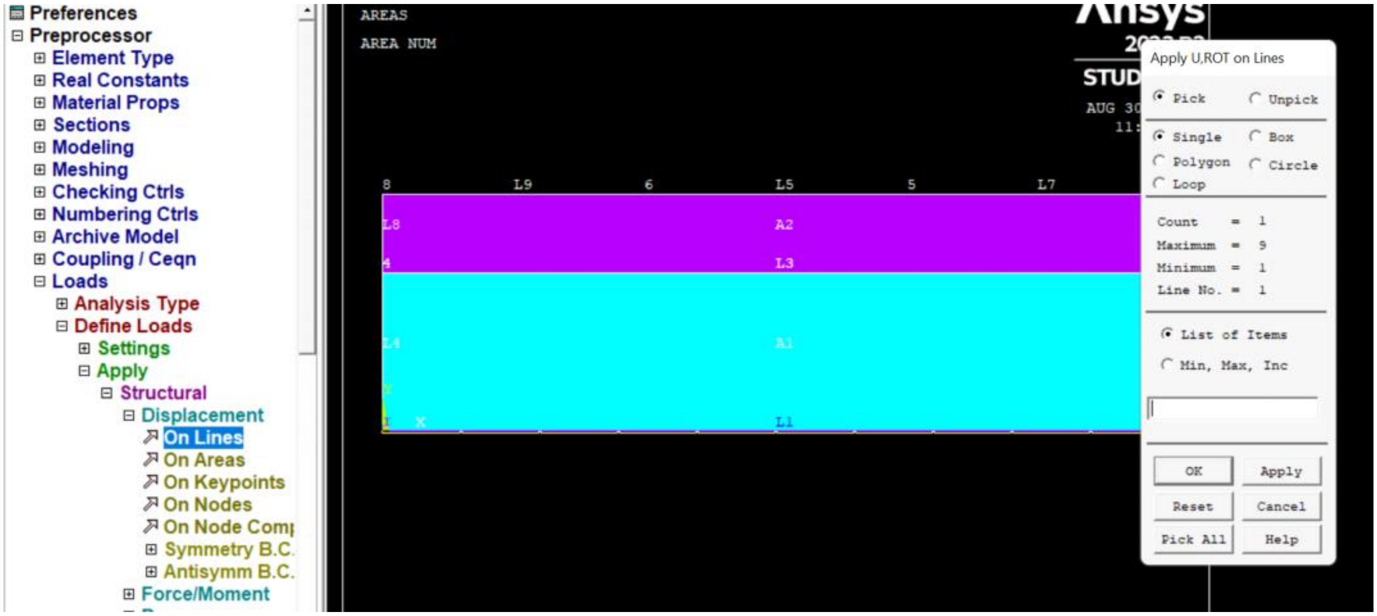


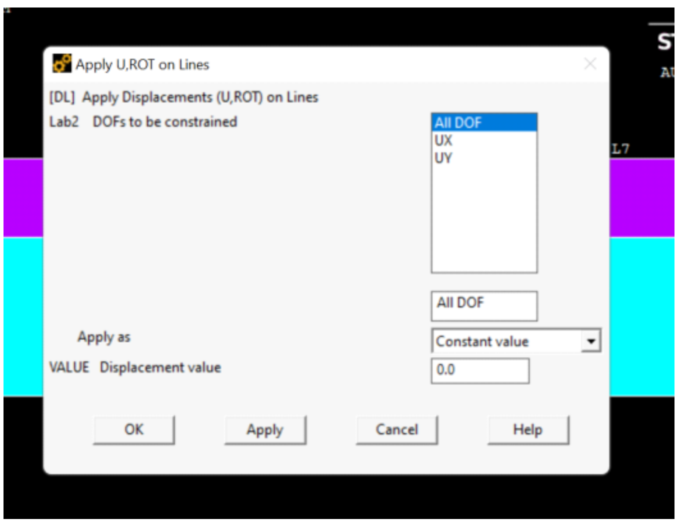
traction free

in FEM all surface by default are traction free

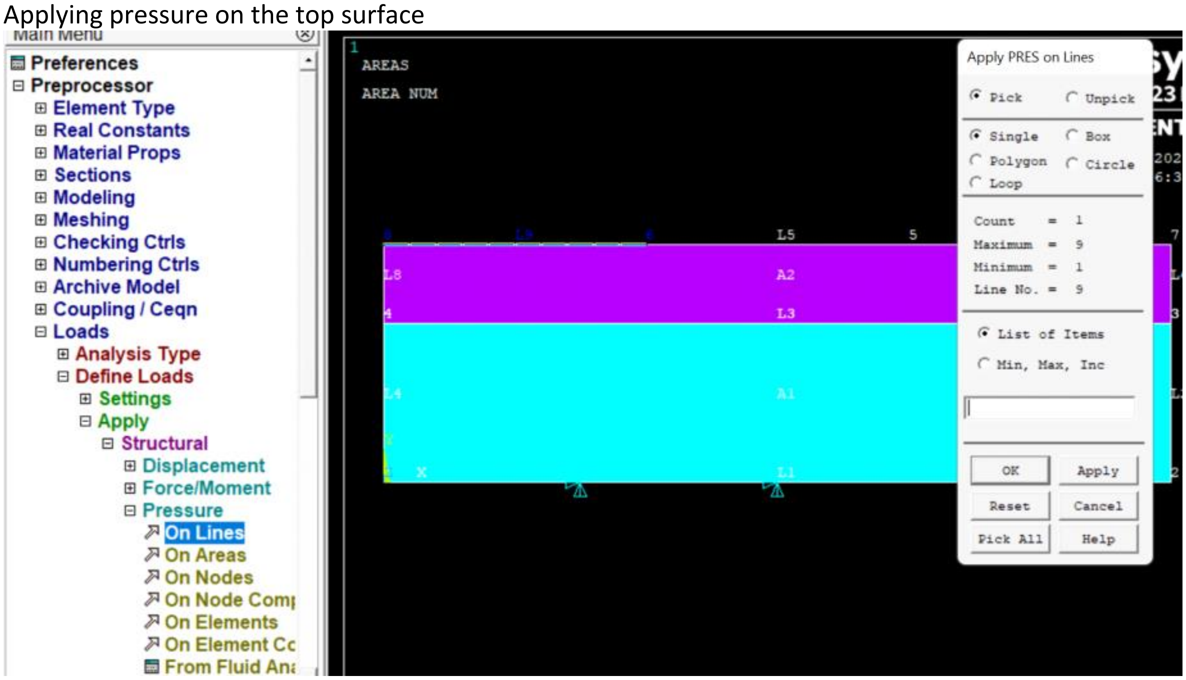
when we defined Q8 element type under options we choose axisymmetry

Fixing the bottom:



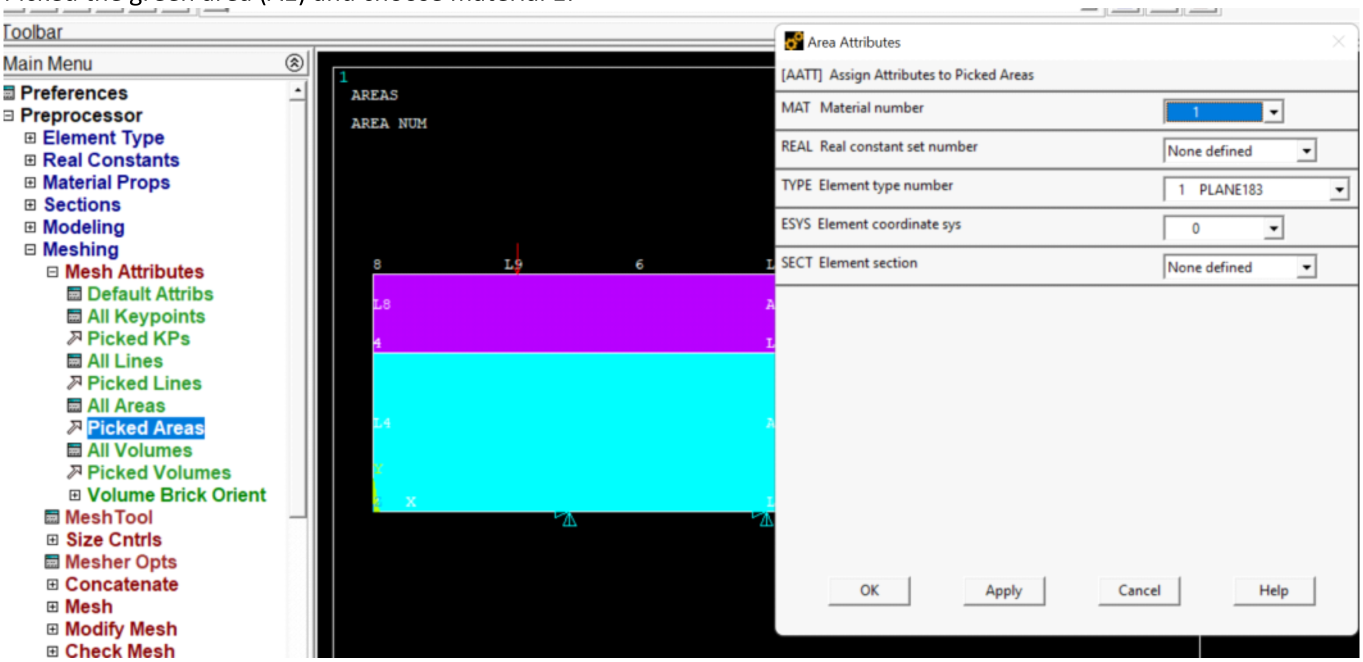


Applying pressure on the top surface

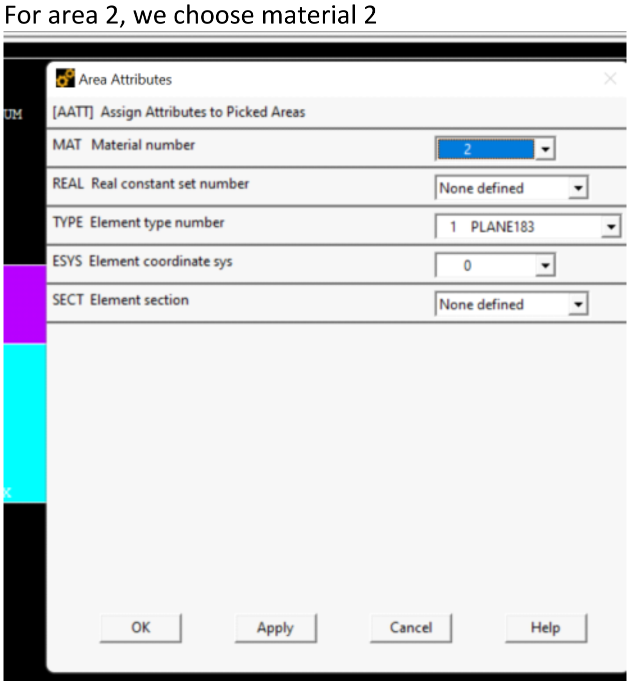


At this point we can mesh the domain (create the elements)

Picked the green area (A1) and choose material 1:

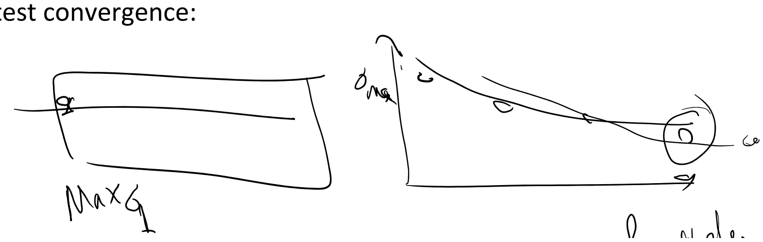


For area 2, we choose material 2



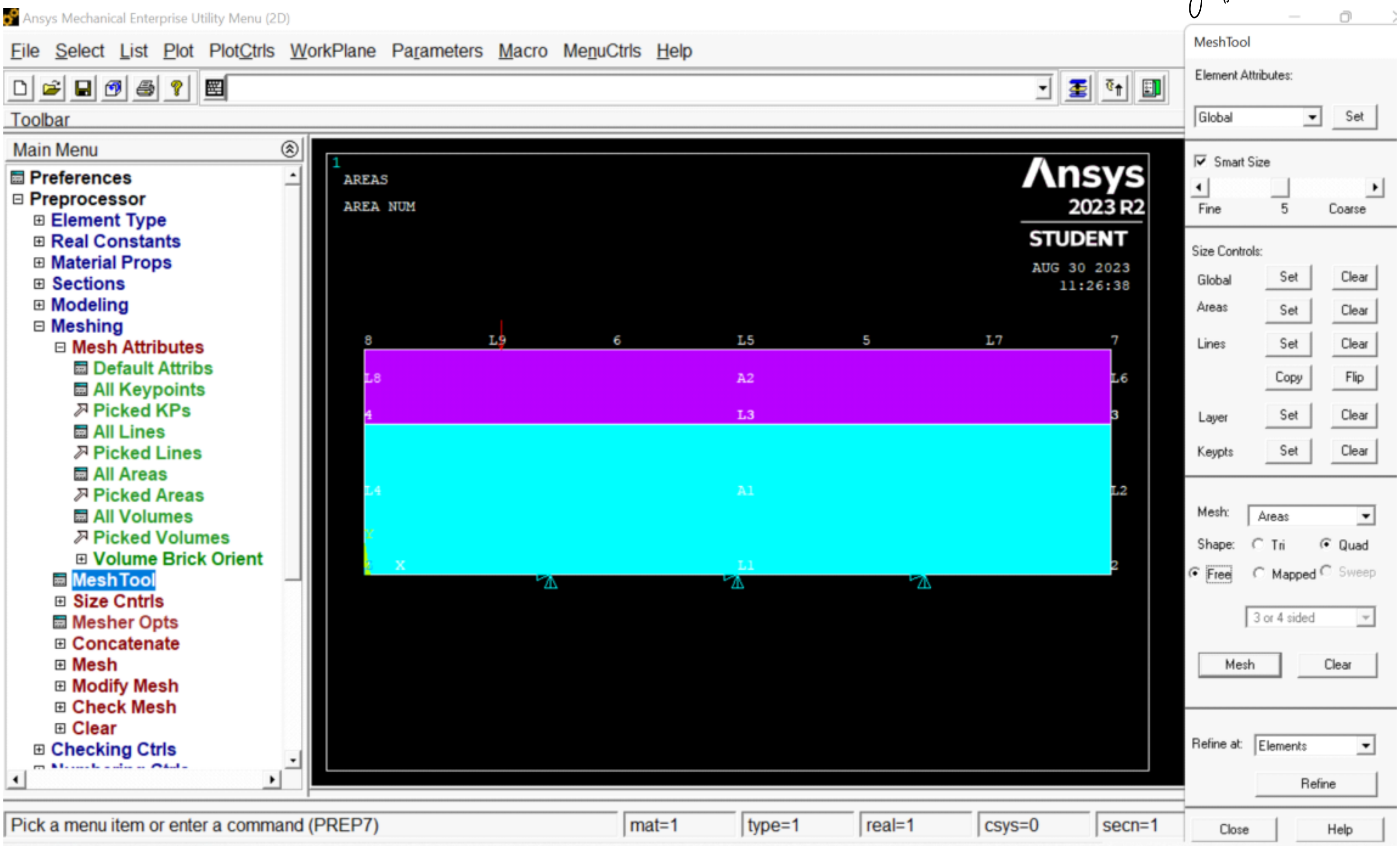
We need to mesh it!

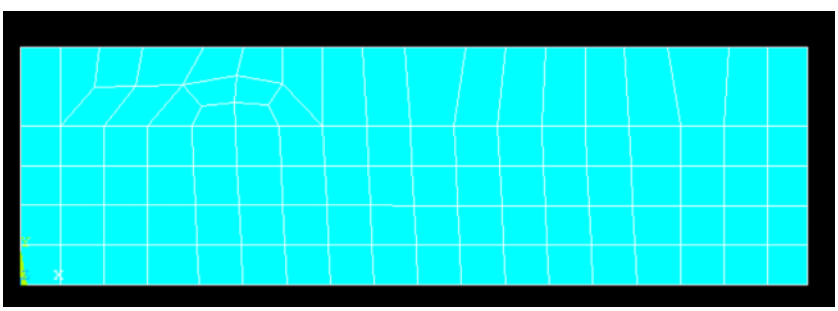
Use mesh tool and smart size. For the term project, convergence study start with the coarsest mesh (e.g. 10) and refine it as much as the academic version allows. You can then test convergence:



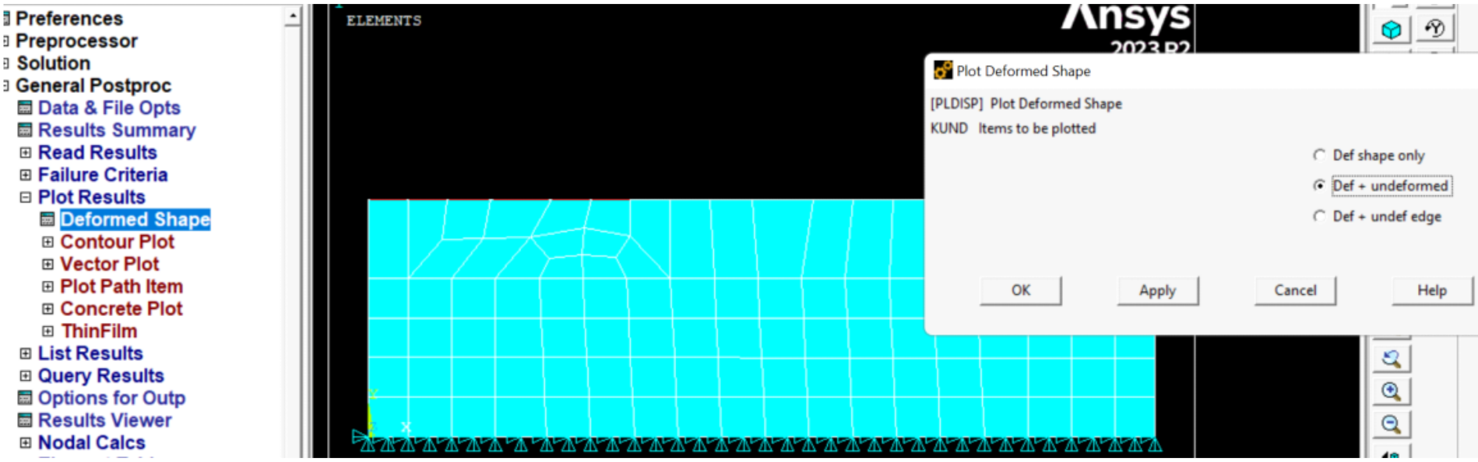
Max Q

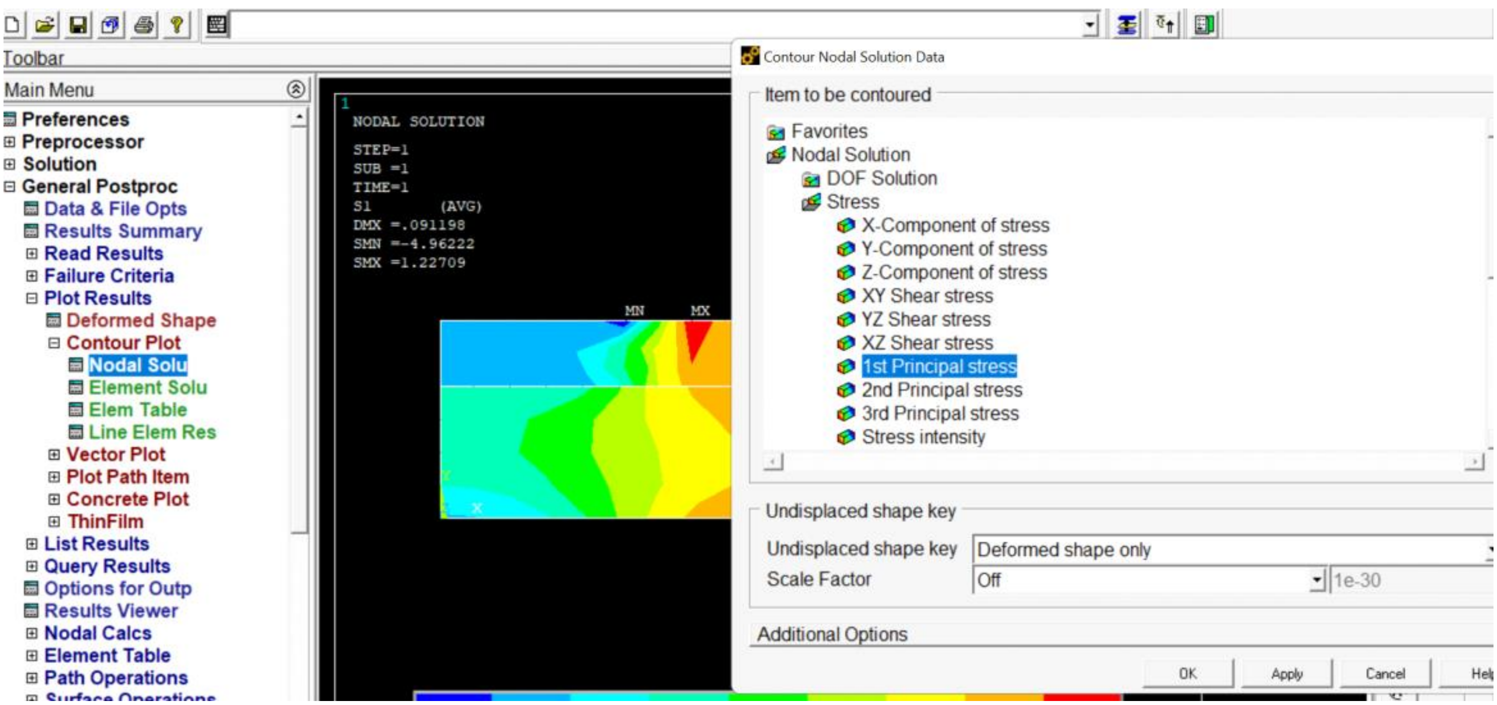
lg #elem

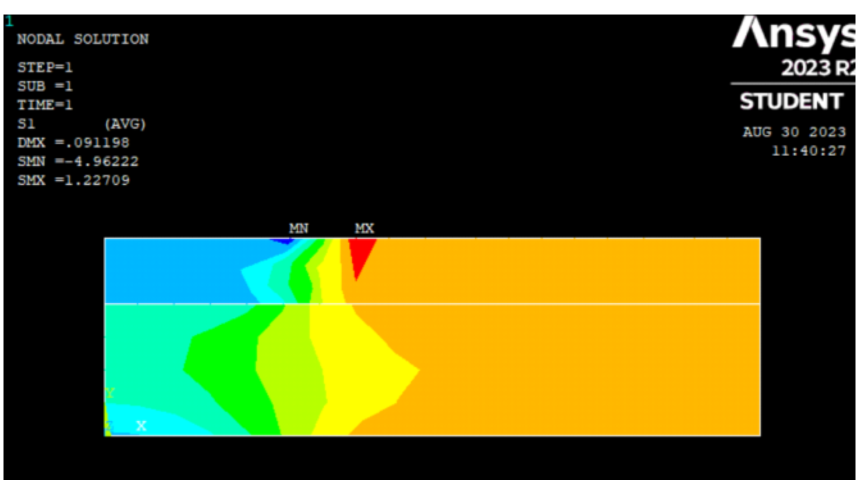




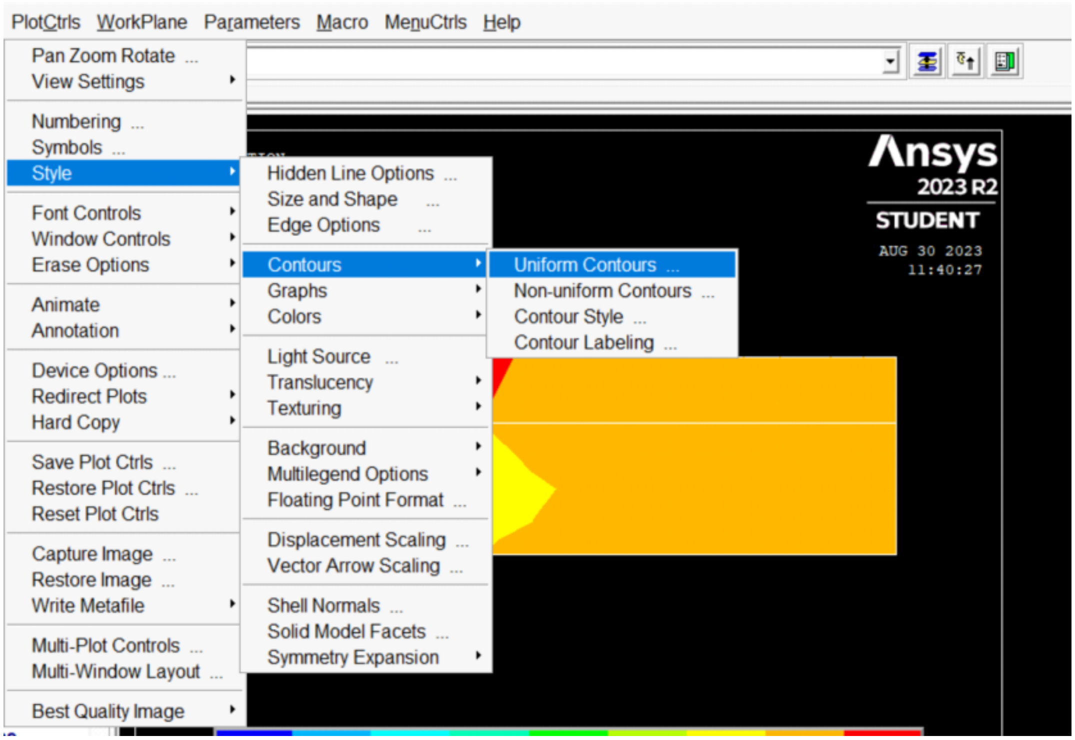
Solve the problem:

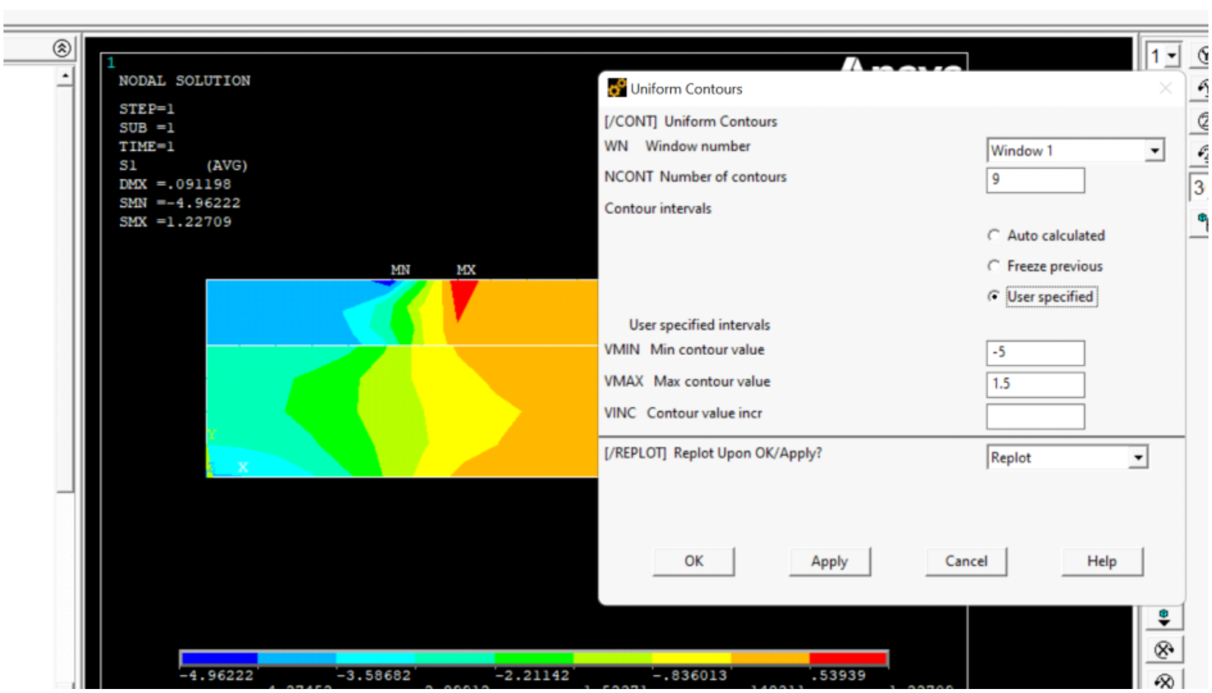


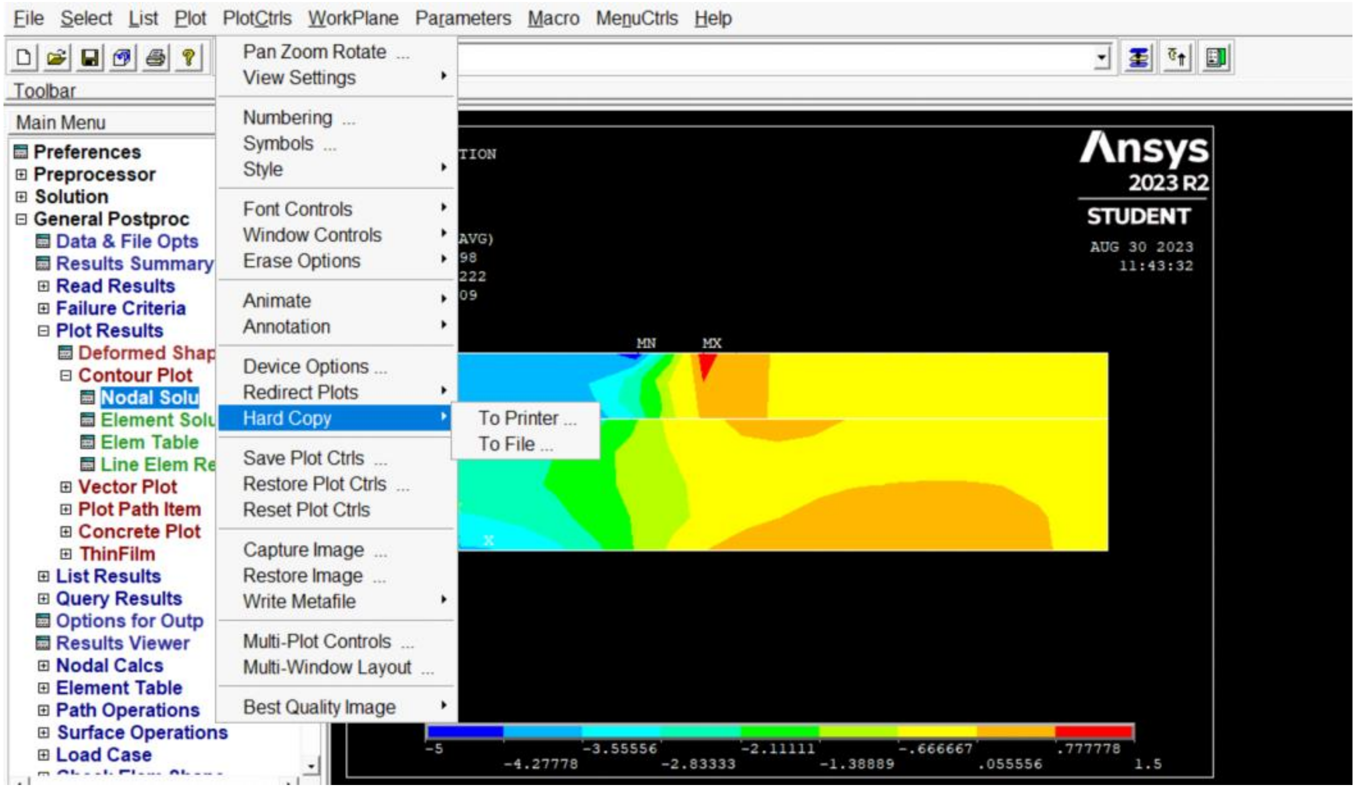




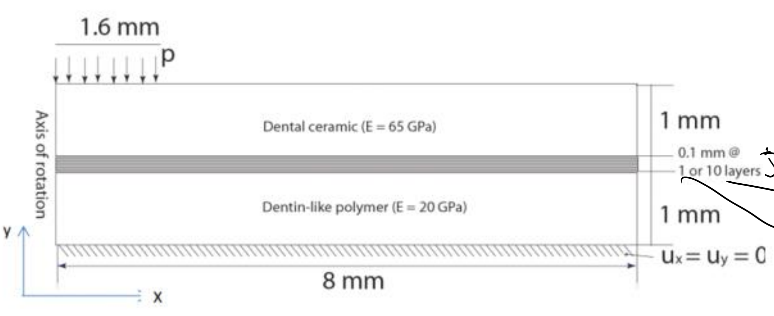
Change the range of values in contour plot


PlotCtrls WorkPlane Parameters Macro MenuCtrls Help
Pan Zoom Rotate ...
View Settings
Numbering ...
Symbols ...
Style
Font Controls
Window Controls
Erase Options
Animate
Annotation
Device Options ...
Redirect Plots
Hard Copy
Save Plot Ctrls ...
Restore Plot Ctrls ...
Reset Plot Ctrls
Capture Image ...
Restore Image ...
Write Metafile
Multi-Plot Controls ...
Multi-Window Layout ...
Best Quality Image
Hidden Line Options ...
Size and Shape ...
Edge Options ...
Contours
Graphs
Colors
Light Source ...
Translucency
Texturing
Background
Multilegend Options
Floating Point Format ...
Displacement Scaling ...
Vector Arrow Scaling ...
Shell Normals ...
Solid Model Facets ...
Symmetry Expansion
Uniform Contours ...
Non-uniform Contours ...
Contour Style ...
Contour Labeling ...
Ansys
2023 R2
STUDENT
AUG 30 2023
11:40:27


1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
S1 (AVG)
DMX =.091198
SMN =-4.96222
SMX =1.22709
MN
MX
Uniform Contours
[/CONT] Uniform Contours
WN Window number
Window 1
NCONT Number of contours
9
Contour intervals
Auto calculated
Freeze previous
User specified
User specified intervals
VMIN Min contour value
-5
VMAX Max contour value
1.5
VINC Contour value incr
[/REPLOT] Replot Upon OK/Apply?
Replot
OK
Apply
Cancel
Help
-4.96222
-3.58682
-2.21142
-.836013
.53939



Term project:



each sublayer is $\frac{0.1}{10}$ mm

0.1

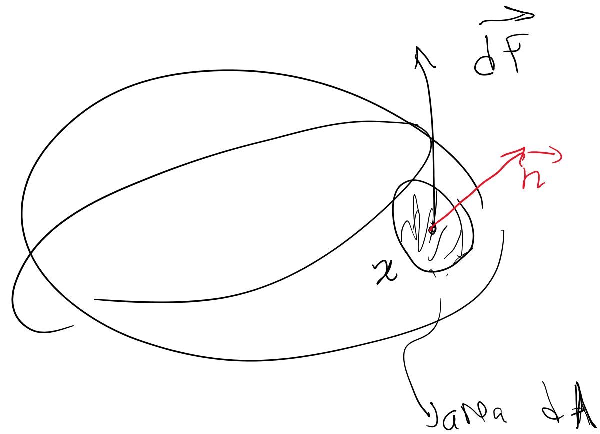
for 10 sublayer

Balance laws:

2D elastostatics

$\vec{dF}$: increment of force acting on dA

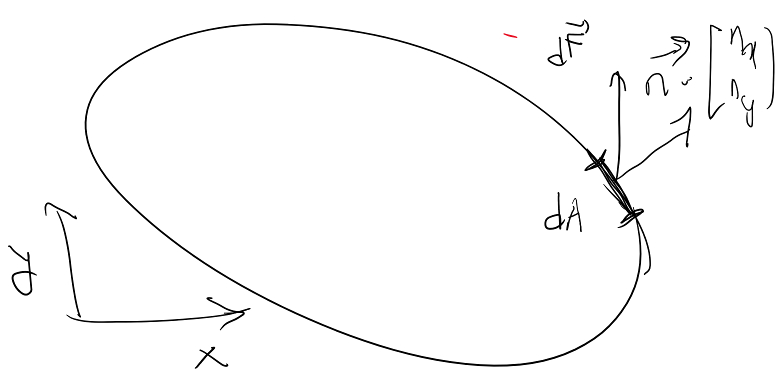
$$\vec{t} = \lim \frac{d\vec{F}}{dA} \quad \text{as } dA \to 0$$



$$\Longleftrightarrow \quad d\vec{F} = \vec{t}\, dA \quad \text{for small } dA$$

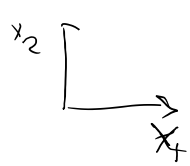
$$\vec{t} = \sigma \vec{n}$$

σ → stress tensor



$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

or written as $\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$



$$\begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix}$$

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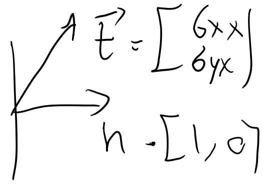
meaning of stress tensor

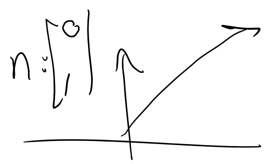
$$\begin{bmatrix} \sigma_{xx} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yx}\end{bmatrix} = \begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{yx} & \sigma_{yy}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$

$\vec{n} = \begin{bmatrix}1\\0\end{bmatrix}$



$\vec{n} = \begin{bmatrix}0\\1\end{bmatrix}$

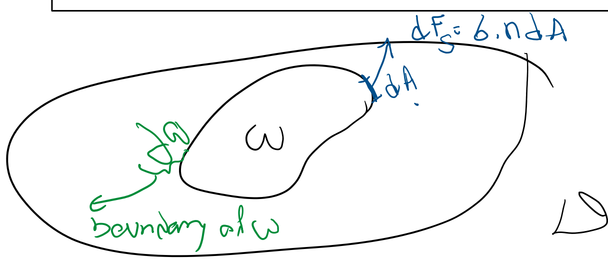


$\vec{t} = \begin{bmatrix}\sigma_{xy}\\ \sigma_{yy}\end{bmatrix}$

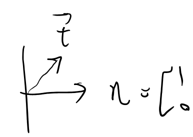
(1)

$$\left.\begin{aligned}\vec{t} &= \sigma\vec{n}\\ d\vec{F} &= \vec{t}\,dA\end{aligned}\right\} \Rightarrow d\vec{F} = \sigma\cdot n\,dA = \sigma\,d\vec{A}$$

$d\vec{A} = (dA)\,\vec{n}$



$\sigma = \begin{bmatrix}5 & 3\\ 3 & 4\end{bmatrix}$



$t = \sigma n = \begin{bmatrix}5\\3\end{bmatrix}$

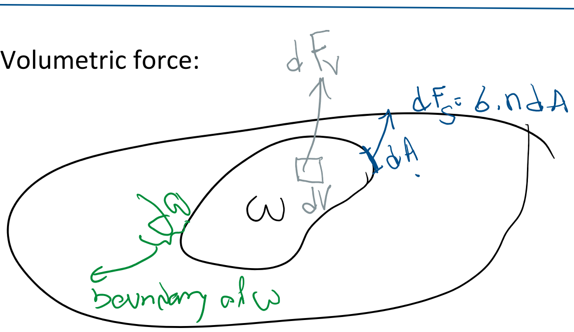
(2)

$$F_s = \int_{\partial\omega} dF_s = \int_{\partial\omega} \sigma.n\,ds$$

F_s: surface on ω

surface contribution

Volumetric force:



$d\vec{F}_V = \vec{b}\,(dm)$

$\vec{b}$: intensity of force per unit mass

dm: mass of dV

$$\left.\begin{aligned}d\vec{F}_V &= \vec{b}\,(dm)\\ dm &= (\rho)\,dV\end{aligned}\right\} \Rightarrow \underline{dF_V = \rho\vec{b}\,dV}$$

$F = mg$

$dm = (\rho)\, dV \} \rightarrow dF_V = \rho b\, dV$

recall $\rho = \frac{dm}{dV}$

(F) mg Vbf

For gravity $\vec{b} = \begin{bmatrix} 0 \\ -g \end{bmatrix}$

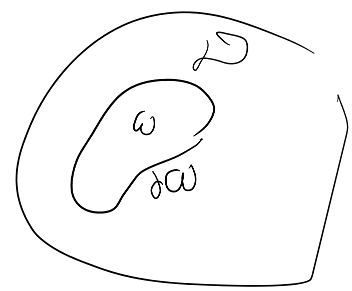
③ volumetric force

$$F_V = \int_{\omega} dF_V = \int_{\omega} \rho b\, dV$$

2 & 3

$$F(\text{on } \omega) = F_S + F_V$$

$$= \int_{\partial\omega} \sigma . n\, ds + \int_{\omega} \rho b\, dV$$



the forces of this course $=$ 0 statics

$\frac{dP}{dt}$ linear momentum $= \frac{d}{dt}\int_{\omega} \vec{v}\, dm$ $dm = \rho dV$

$P = \sum m_i v_i$

$$= \frac{d}{dt}\int_{\omega} \vec{v}\rho\, dV$$

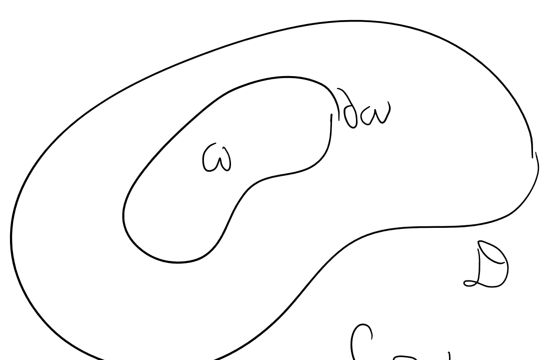
p linear momentum density

$$= \frac{d}{dt}\int_{\omega} p\, dV$$

$p = \rho\vec{v}$

For us (statics)

$$\int_{\partial\omega} \sigma.n\, ds + \int_{\omega} \rho b\, dv = 0$$



In general

$$-\int f_r \vec{n}\, ds + \int S\, dV = 0 \quad = \quad \int S\, dV$$

$$-\int_{\partial\omega} f_x \cdot \vec{n}\, ds + \int_{\omega} S\, dV = 0 \equiv \int_{\partial\omega} f_x \cdot n\, ds = \int_{\omega} S\, dV$$

for many balance laws

f_x : outward spatial flux density

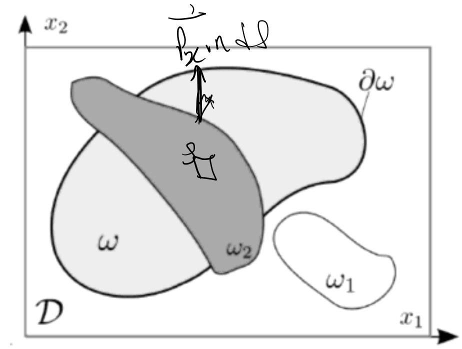
S: source term

Examples

Solid mechanics:	heat conduction
$f_x = -\sigma$	$f_x = \vec{q}$ heat flux
Solid mechanics	
ρb	Q heat source

Summary: Most balance laws have a term on the boundary (spatial flux density goes to it) and a term inside the domain (source term)

$$-\int_{\partial\omega} f_x \cdot n\, ds + \int_{\omega} S\, dV = 0$$



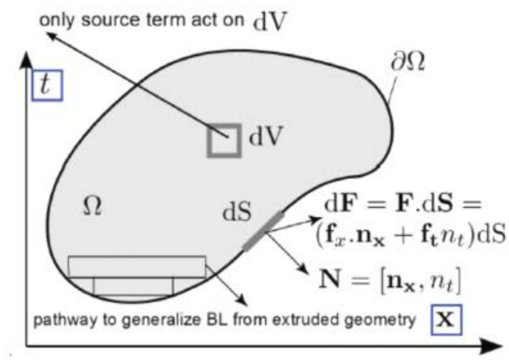
FYI, dynamics balance laws look the same

General form of balance laws using spacetime flux

Using the same definitions from previous page we define the spacetime flux by

$$\mathbf{F} = [\mathbf{f}_x | \mathbf{f}_t] \qquad (15)$$

then the balance law for dynamics reads:



$$\boxed{\forall \Omega \subset \mathcal{D} : \int_{\partial\Omega} \mathbf{F}.\mathrm{dS} - \int_{\Omega} \mathbf{r}\, \mathrm{dV} = \int_{\partial\Omega} (\mathbf{f}_x.\mathbf{n_x} + \mathbf{f_t} n_t)\mathrm{dS} - \int_{\Omega} \mathbf{r}\, \mathrm{dV} = \mathbf{0}} \qquad (16)$$

This can be directly compared to $\int_{\Delta t} \mathbf{F} = \Delta\mathbf{P}$ in previous discrete examples.

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