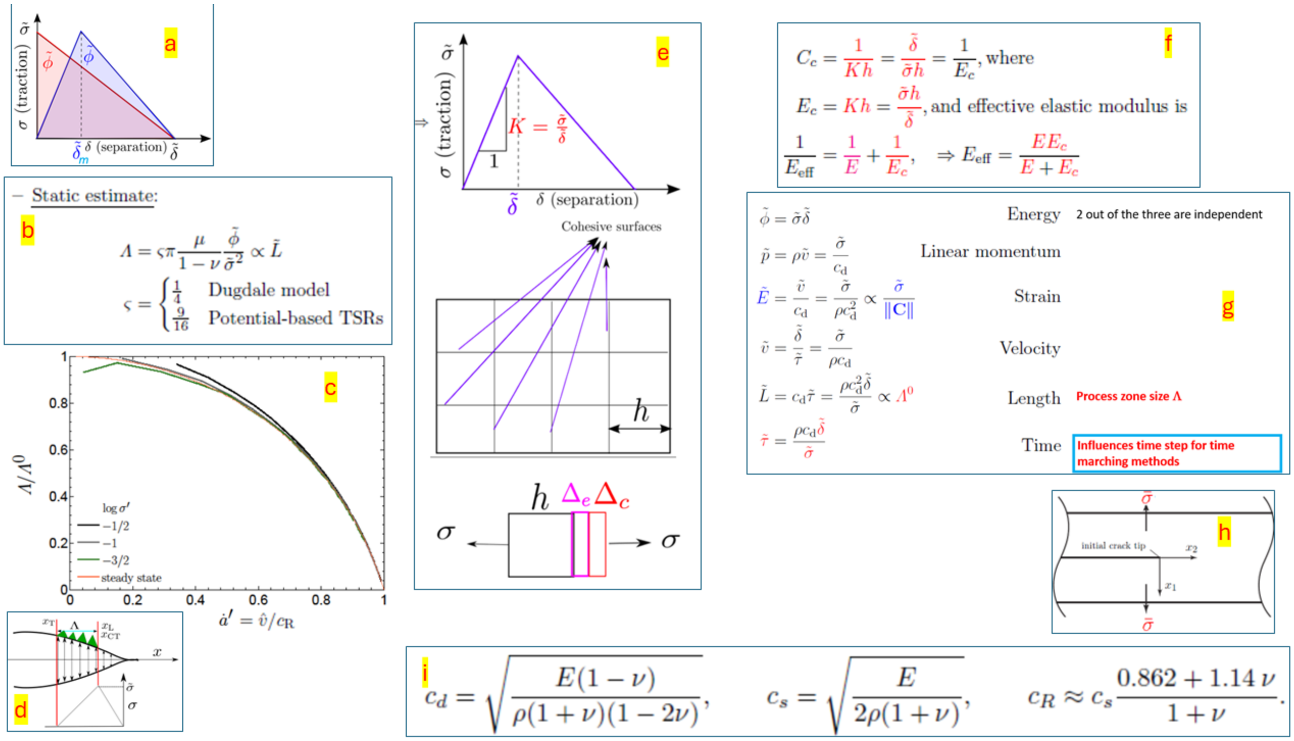

Figure 1: Some figures and equations related to TSR questions.

Figure 1 contains some relevant equations and schematics pertained to the questions below. Consider the following material properties for polymethyl methacrylate (PMMA):

- Young's modulus, $E = 3.24$ GPa
- Poisson's ratio, $\nu = 0.35$
- Mass density $\rho = 1190$ kg/m^3
- Strength $\tilde{\sigma} = 0.025E = 81$ MPa
- Work of separation $\tilde{\phi} = 324$ Pa-m

1. (**50 Points**). Scales:
   (a) For intrinsic or extrinsic TSR in fig. 1a Compute max separation $\tilde{\delta}$ in m (same value for both models).
   (b) Compute longitudinal (dilatational or pressure) wave speed c_d from fig 1.i in m/s.
   (c) Compute strain scale $\tilde{\epsilon} = \tilde{\sigma}/E$ (shown as $\tilde{E}$ in fig 1g) (unit less).
   (d) Compute $\tilde{L} = \tilde{\phi}E/\tilde{\sigma}^2$ (slightly different from the equation in fig 1.g) in m.
   (e) Compute $\tilde{\tau} = \tilde{L}/c_d$ (again slightly different from the expression in fig 1.g) in s (sec).

2. (**40 Points**). Process zone size Λ, $\tilde{L}$, and $\tilde{\delta}$.
   (a) A decent estimate of quasi-static process zone size from fig 1.b for the used TSRs (using $\varsigma = 0.25$) is, $\Lambda = \pi/8/(1-\nu^2)\tilde{L}$. Compute Λ in m.
   (b) If at least 4 elements are needed in the process zone (see fig. 1d) compute the the maximum element size h in m.

(c) (extra credit): Imagine that the crack is propagating at 80% of Rayleigh wave speed. Using the corresponding value of 0.8 on the x-axis of fig. 1c and referring to the orange line, provide a rough estimate on the ratio of this dynamic process zone size to its quasi-static limit ($\mathit{\Lambda}/\mathit{\Lambda}_0$ in this figure)? If we want to have at least 4 elements in the process zone size to the instant where the crack reaches 80% of Rayleigh wave speed, provide the now more stringent element size h limit.

(d) What is the ratio $\tilde{\delta}/\mathit{\Lambda}$ (unit-less)? How does this ratio (or $\tilde{\delta}/\tilde{L}$) compare in terms of its order of magnitude with $\tilde{\epsilon}$? Referring to a similar concept in PFM, how does the ratio of CTOD$/r_p$ compare with $\tilde{\epsilon}$? (Please be brief, answer all in less than 3-4 lines).

3. (**20 Points**). Time scale:

   (a) Provide the time scale at which a point on potential crack line (ahead of the crack in fig 1.h) goes from bonded to fully debonded (hint: provide $\tilde{\tau}$). [1]

   (b) Whether we use an explicit of implicit time advance scheme, we need to ensure the time step is sufficiently smaller than $\tilde{\tau}$ for accuracy considerations. Again, assuming at least 4 time steps are needed in $\tilde{\tau}$, determine the maximum reasonable time step $\mathit{\Delta} t$ for the properties provided (compute $\mathit{\Delta} t = \tilde{\tau}/4$).[2]

4. (**30 Points**). Artificial compliance: Consider a refined finite element mesh with $h = 3.2 \times 10^{-4}$ m, where intrinsic cohesive models are inserted between all elements. Elements can be other than the squares shown in fig. 1e, but the E_c in fig. 1f will have a correction factor. Assuming that $\tilde{\delta}_m = 0.4\tilde{\delta}$ in fig. 1a for the extrinsic model (the TSR in blue), and noting that $K = \tilde{\sigma}/\tilde{\delta}_m$ compute E_{eff} from fig. 1f. What's the ratio E_{eff}/E?

5. (**20 Points**). TSR: From separation to traction value: Consider $\delta = 0.2\tilde{\delta}$. Compute the corresponding traction from

   - Intrinsic model as above ($\tilde{\delta}_m = 0.4\tilde{\delta}$) [Hint: see if the given δ falls on the loading or unloading line.]
   - Extrinsic model.

[1] Note with TSRs, there is not a definite time that a point goes from bonded to debonded, but the real time of fracture is proportional to $\tilde{\tau}$.

[2] Note: Without going to details, for an explicit solver, one can easily see that choosing a small enough element size and having a time step that satisfies the time marching stability condition, results in small enough time step (sufficiently smaller than $\tilde{\tau}$.