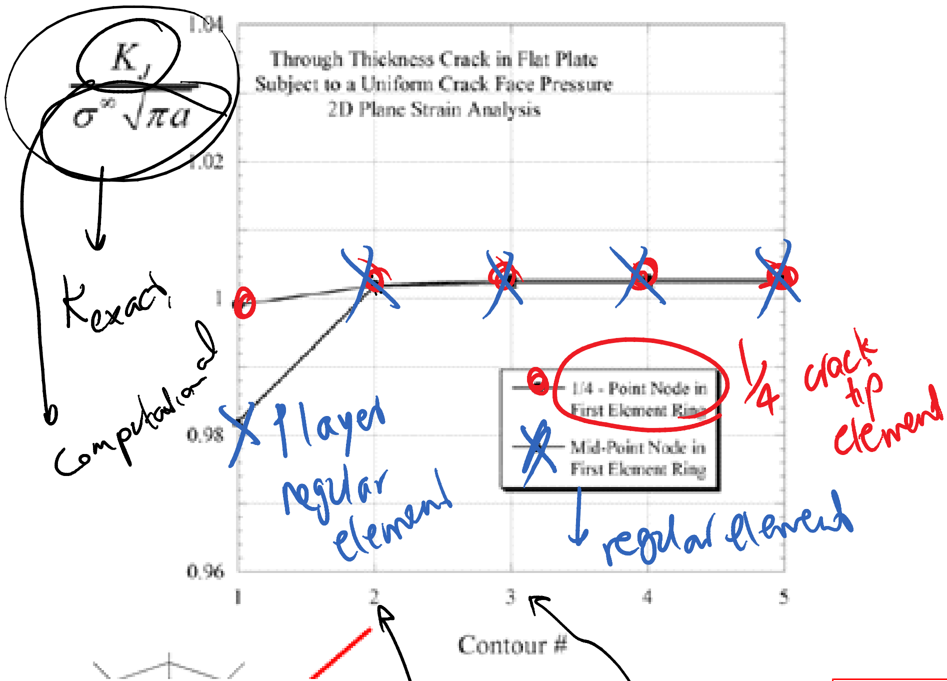
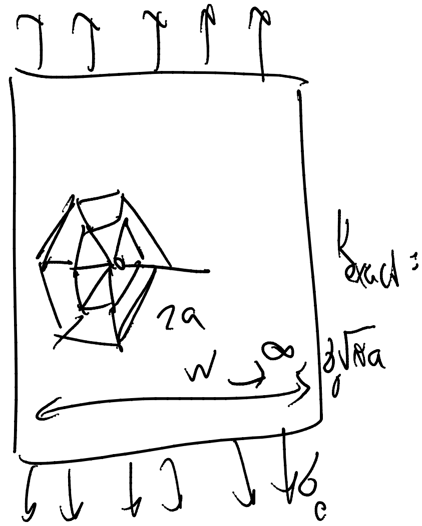
2015/11/05

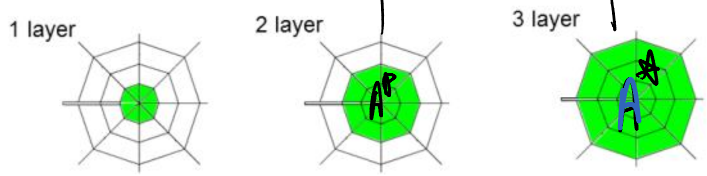
Thursday, November 05, 2015
1:09 PM

**Comparison of the accuracy of different methods proposed for the evaluation of K/G**:



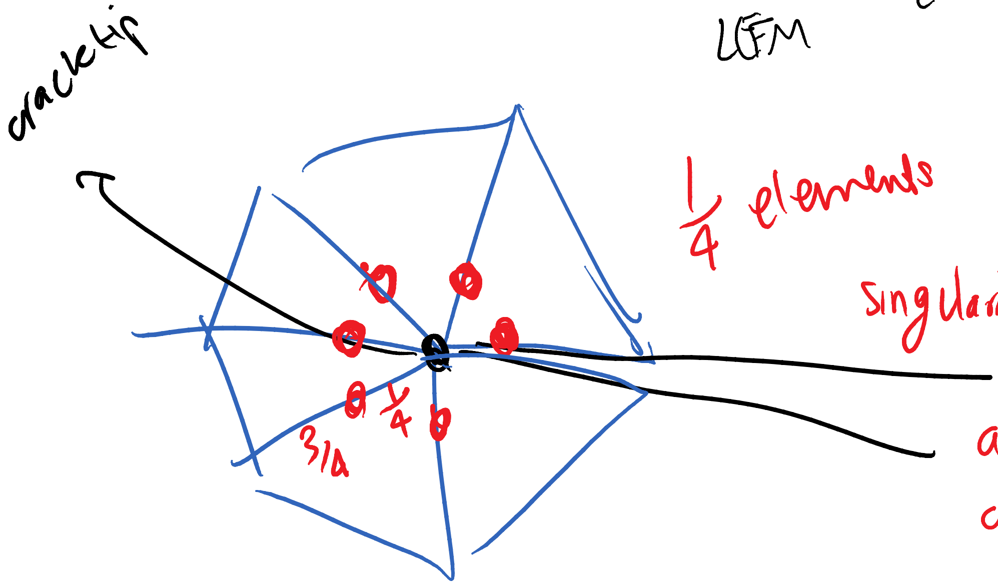

EDI: the most accurate one:



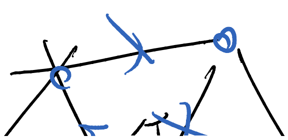

$$J = \int_{A^*} \left[ \sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right] \frac{\partial q}{\partial x_i} dA$$

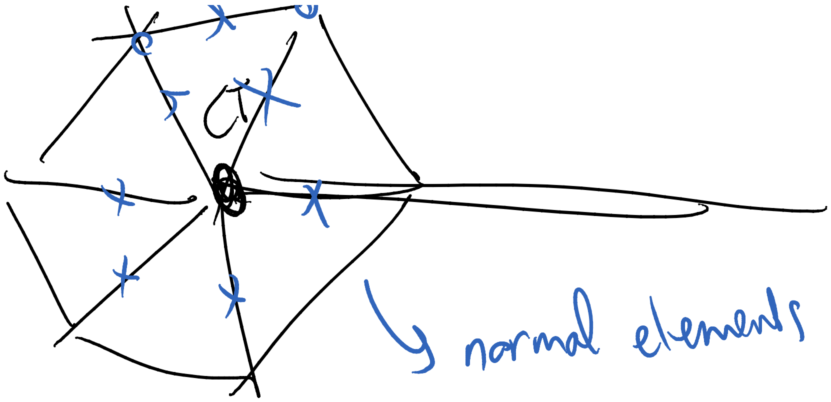
$J = G = \frac{K_I^2}{E'} \Rightarrow K_I = \sqrt{E' J}$

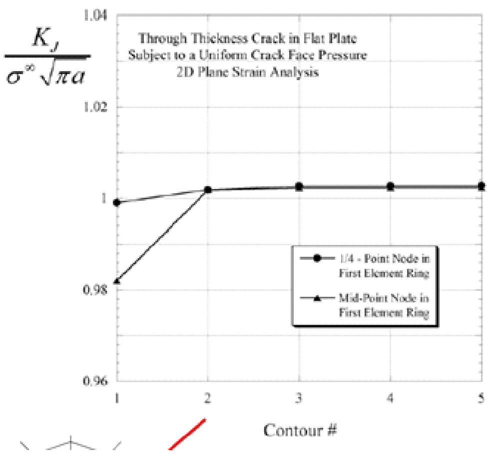
LEFM




$\frac{1}{4}$ elements create $\frac{1}{\sqrt{r}}$ singularity of $\varepsilon$, $\sigma$ around the crack tip

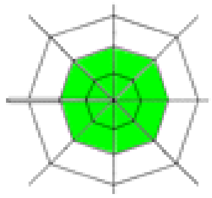





EDI approach:

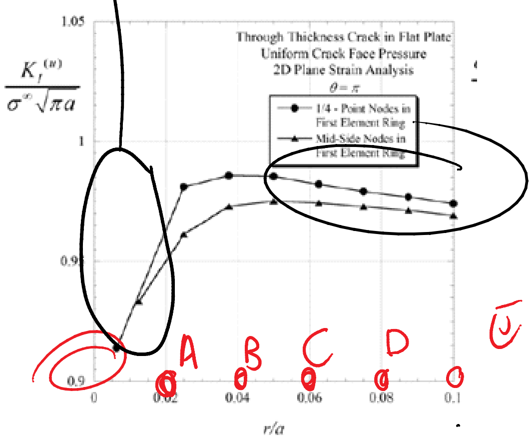
- Results are very accurate
- Singular elements make a noticeable difference only ONE layer of elements is used



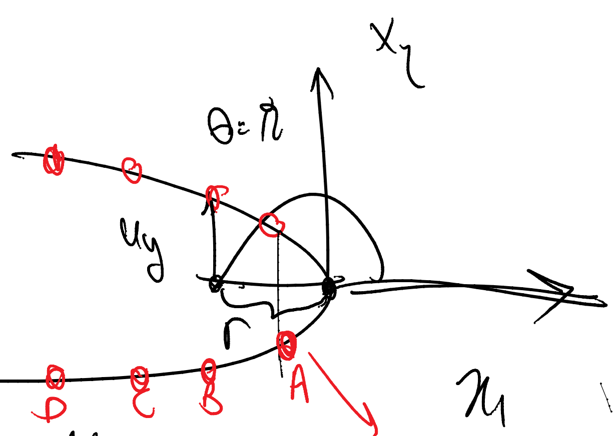
Recommendation: If possible
use more than one layer of elements OR
If using one layer use singular (1/4) elements
In that layer

numerical solutions are not very accurate around the crack tip

Method 2: Displacement-based computation of K




expected to not get very good results because we have in ☆ lim $r \to 0$




K from displacement *u* ☆

$$K_I = \lim_{r\to 0}\left[\frac{E' u_y}{4}\sqrt{\frac{2\pi}{r}}\right] \qquad (\theta = \pi)$$

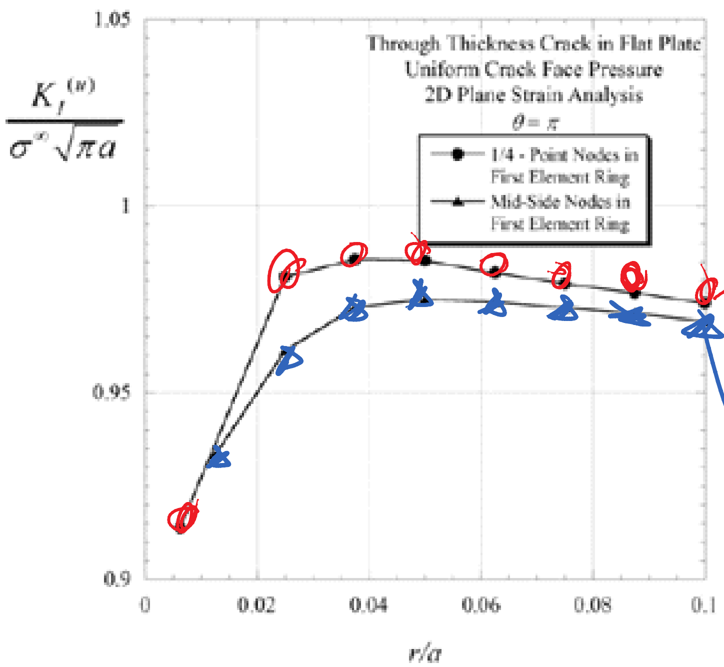
$$\Longleftarrow \quad u_y(\pi) = \frac{4K_I}{E'}\sqrt{\frac{r}{2\pi}} \quad \Longrightarrow \quad r/a = .02$$

$$K_I = \lim_{r \to 0} \left[ \frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$ $\longleftarrow$ $u_y(r) = \frac{4K_I}{E'}\sqrt{\frac{r}{2\pi}}$ $\Rightarrow$ $r/a = .02$

crack length

distance to crack tip

How much does using the singular (1/4) elements improve the results?




∴ 3.4% better solution

$\frac{1}{4}$ node position element

CT

regular element

or

# K from displacement *u*

$$K_I = \lim_{r \to 0} \left[ \frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$

For displacement-based calculation of K:

- Very close to the crack tip K is not very good (numerical solution inaccuracies around the crack tip cause this)
- Away from the crack tip asymptotic property of the equation for K is lost
- Use of singular (1/4) elements has more effect in this case

$\rightarrow$ best distance is somewhere in between

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Final approach:

Stress-based calculation of K

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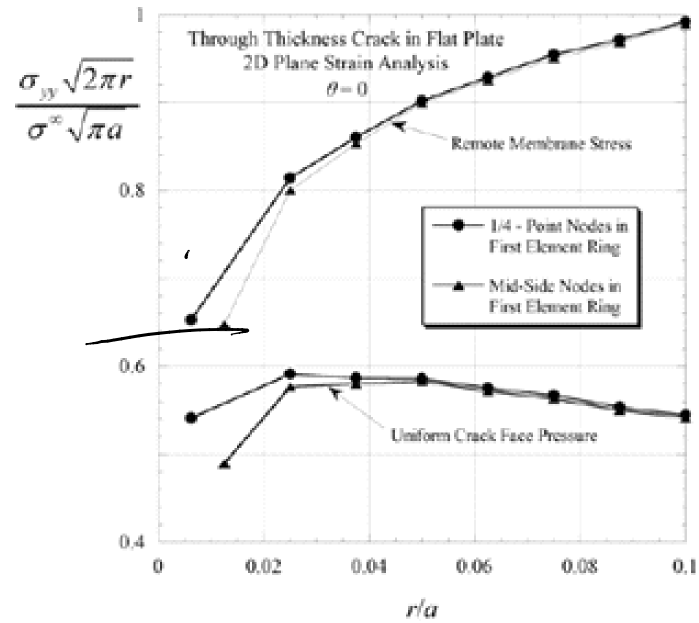
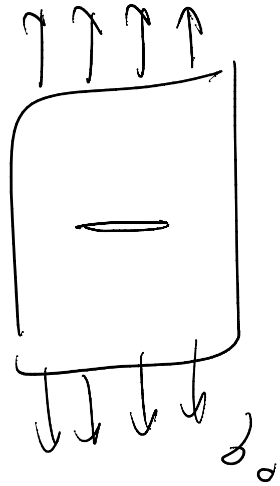
Stress-based calculation of K



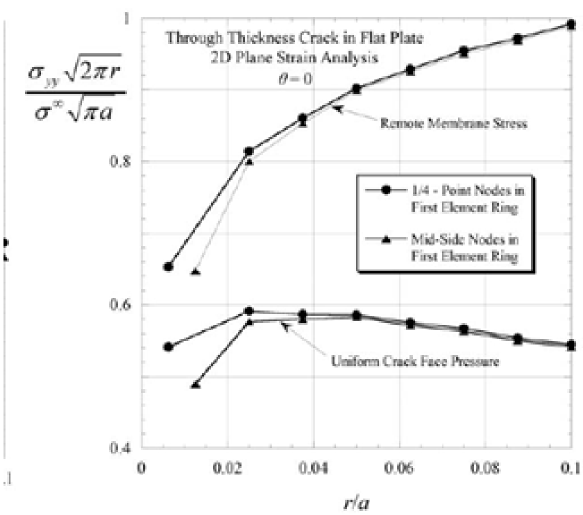



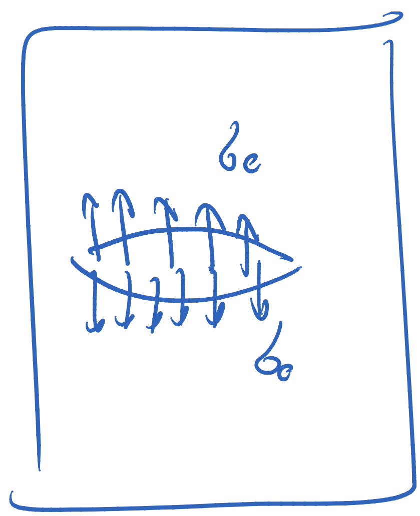

much higher errors
(~ 40% as r→0)

K from stress σ

$$K_I = \lim_{r \longrightarrow 0} \left( \sqrt{2\pi r}\ \sigma_{22}|_{\theta=0} \right)$$

Not only the results are less accurate they are also more sensitive on how the load is applied!




Much worse results when stresses are applied on the crack surfaces

order p-1 order p

- Stress-based method is the worst for these reasons:

Much worse results when stresses are applied on the crack surfaces

- Stress-based method is the worst for these reasons:
  1 ○ Stresses are singular around the crack tip -> most difficult to get a good numerical solution for stresses
  2 ○ Stress is interpolated with lower order polynomial than displacement
  3 ○ Results are very sensitive to how the load is applied

$$\overset{\text{order } p-1}{\sigma} = C \nabla \overset{\text{order } p}{u}$$

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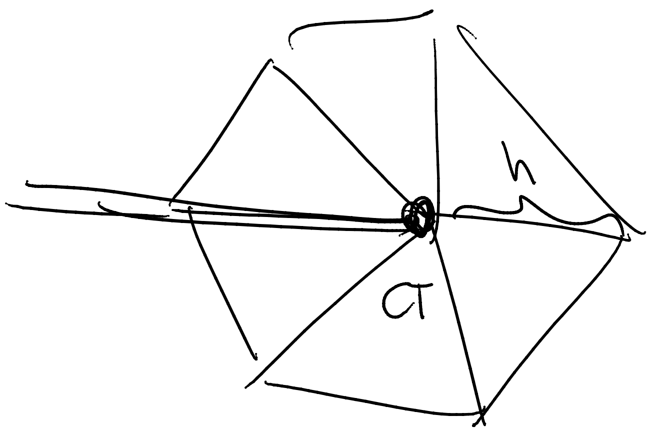
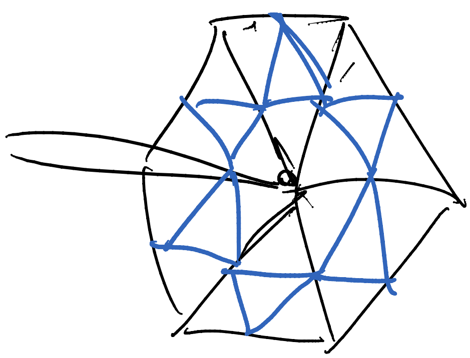
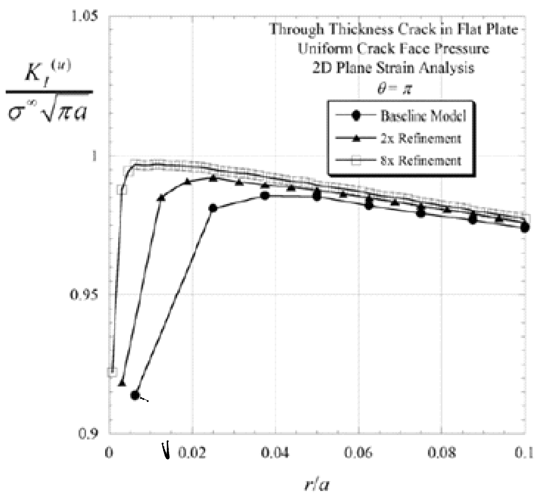
2 important points:
By improving accuracy of FEM solutions we often CANNOT improve the accuracy of displacement- and stress-based methods

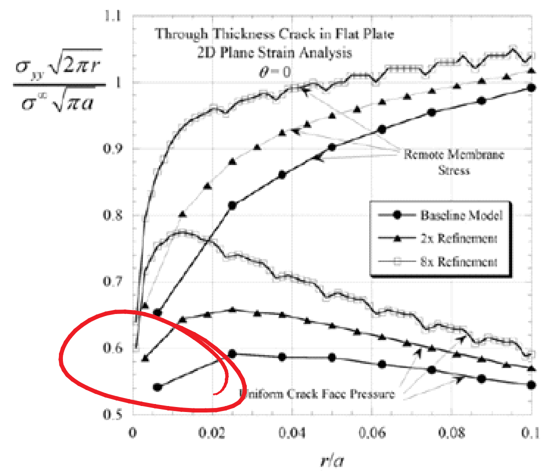



h-refinement
using smaller elements



$h \to h/2$




K from displacement ***u***

$$K_I = \lim_{r \to 0} \left[ \frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$

K from stress σ

$$K_I = \lim_{r \longrightarrow 0} \left( \sqrt{2\pi r}\, \sigma_{22}|_{\theta=0} \right)$$

Displacement- and stress-based methods can produce NONCONSERVATIVE RESULTS

$K_{\text{from FEM}}$ = .5 of exact K

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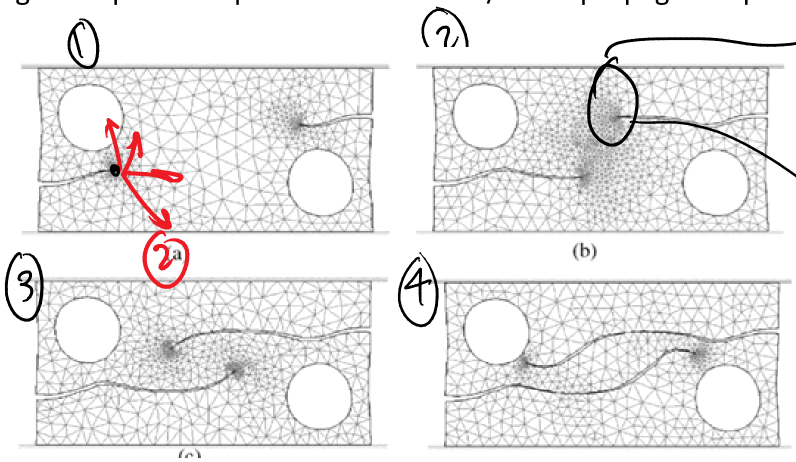
Obtaining crack path as a part of the solution / crack propagation problems

0. solution around the CTs is singular (LEFM) or very high gradient

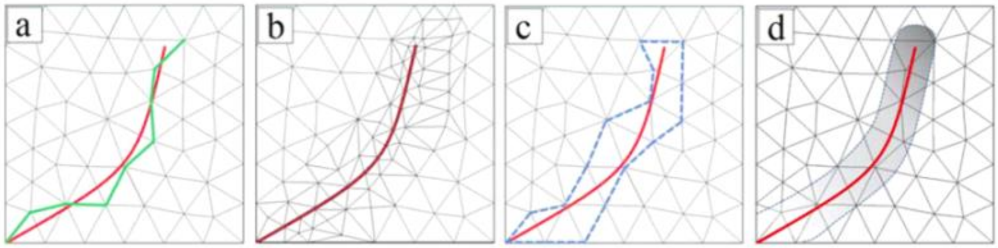



often very refined elements are needed around the crack tip

For crack propagation:

1. Whether the crack can propagate
2. What direction propagates

General approaches to model cracks with FINITE ELEMENT METHODS

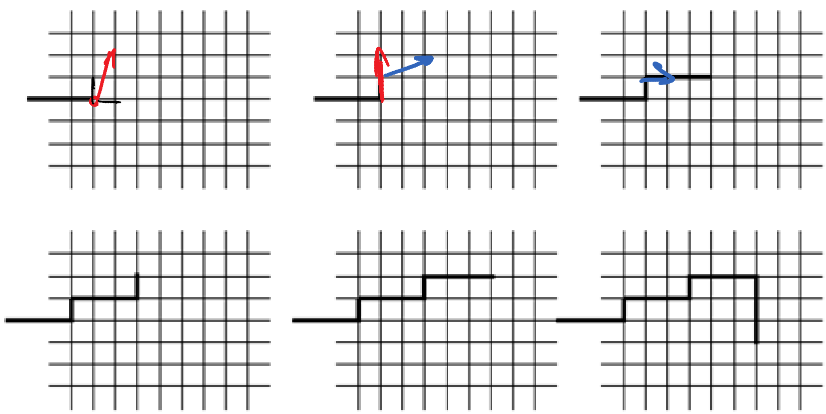



Fixed mesh — Crack tracking — XFEM enriched elements — Crack/void capturing by bulk damage models

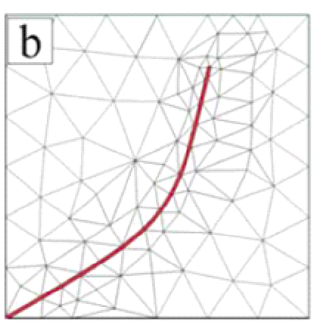
XFEM enriched elements: Brief overview in the next section

Crack/void capturing by bulk damage models: Brief overview in continuum damage models

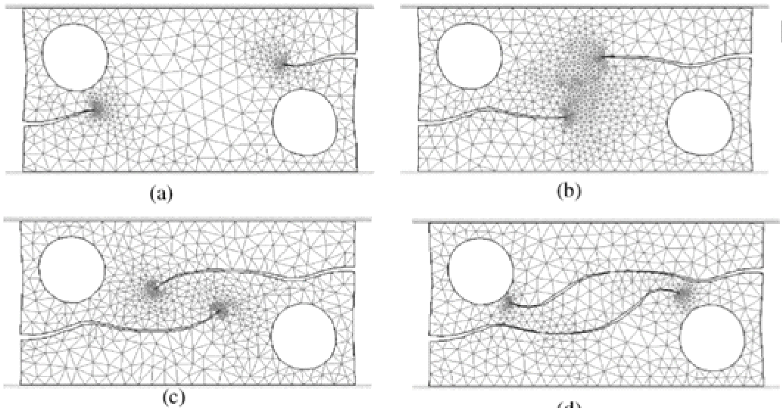
a) Is the worst one (and the easiest one)



b) When we use adaptive operations to align the crack path with the element boundaries



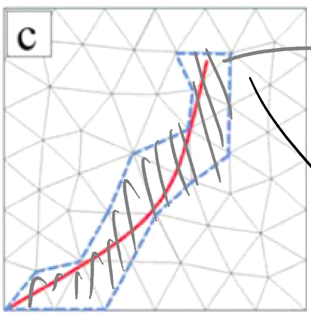

Very accurate BUT VERY DIFFICULT to do: to align element boundaries with crack edges




Bouchard et al.
CMAME 2003

This is an example of adaptive meshing to TRACKING CRACK path

c)



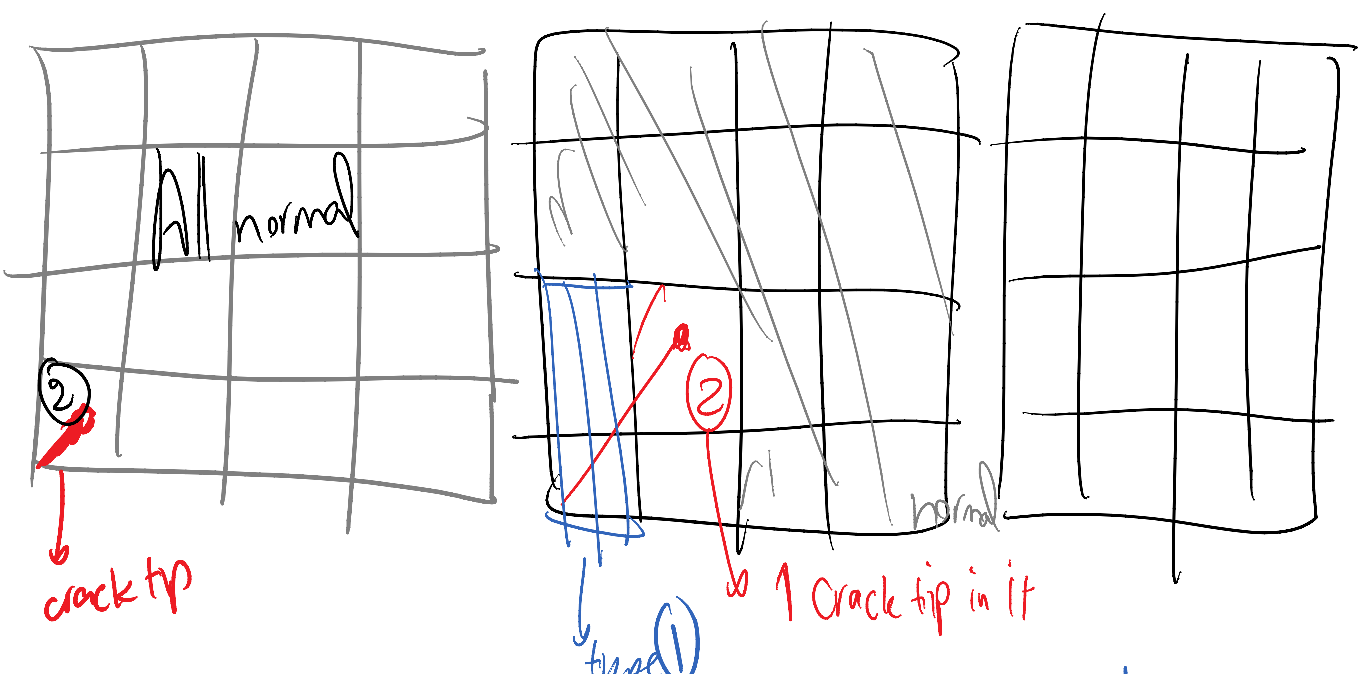

normal elements

inside these elements crack fields are added so we do NOT need to modify (adapt) the mesh YET we have a very accurate representation of cracks

XFEM enriched elements

Brief overview in the next section

A very popular method from around 1999

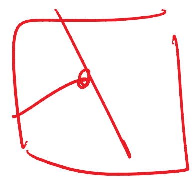


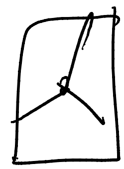
crack tip

type (1) 1 Crack tip in it

the have 1 crack segment in them

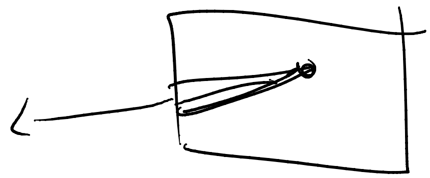
We need to have the exact
- solutions for the stress field around 1 crack tip (we have from asymptotic solutions (type 2)
- Solution when there is the discontinuity from 1 crack surface

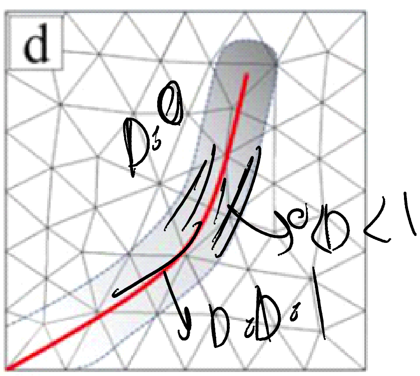




Challenges:
- For each new crack topology inside an element we need to have the exact (or at least a reasonable) solution
- Even for simple cases (e.g. one crack tip in the elements) solutions are generally limited to LEFM

Cohesive model






Crack/void capturing by bulk damage models

## Brief overview in continuum damage models

Continuum damage / bulk damage
Or recently phase-field model

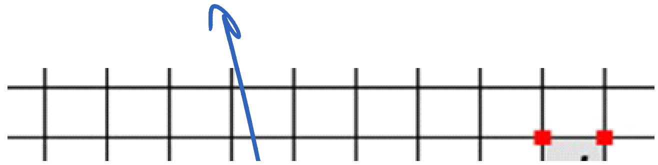
Easy to implement: Do not have many problems that the three previous approaches have

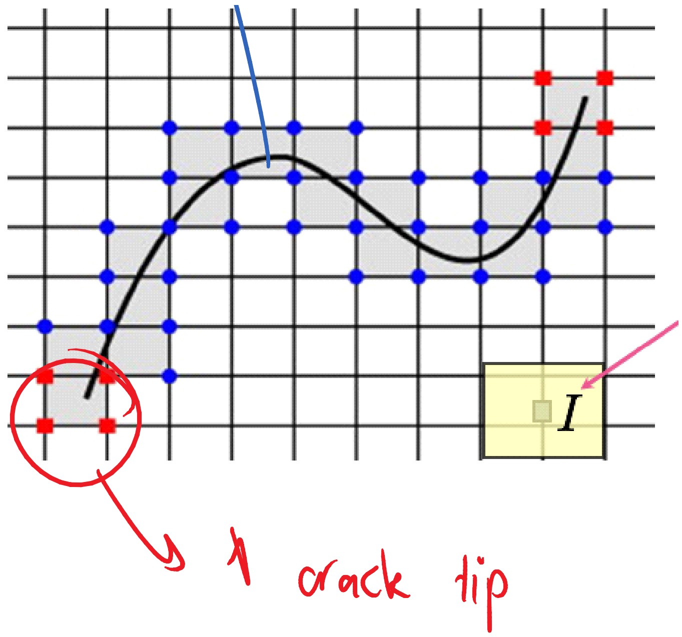
- Challenge: It not a very accurate representation of Cracks

Whenever these models can be used (i.e. not very unrealistic) they are much easier to use

X-FEM

1 crack edge





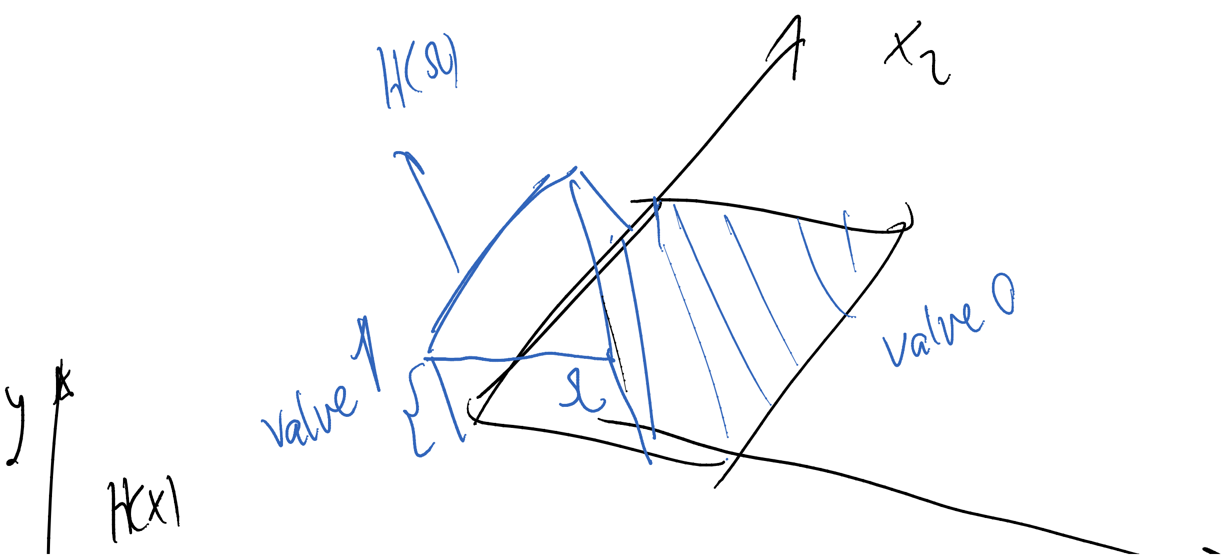

Displacement solution
around the crack tip from asymptotic
expansion

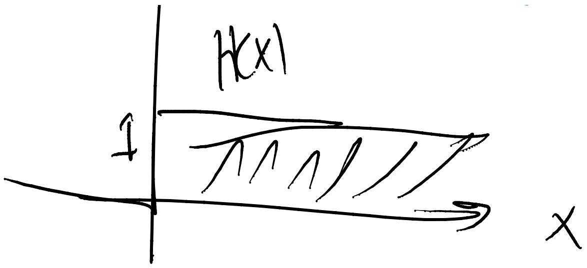
$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$
$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

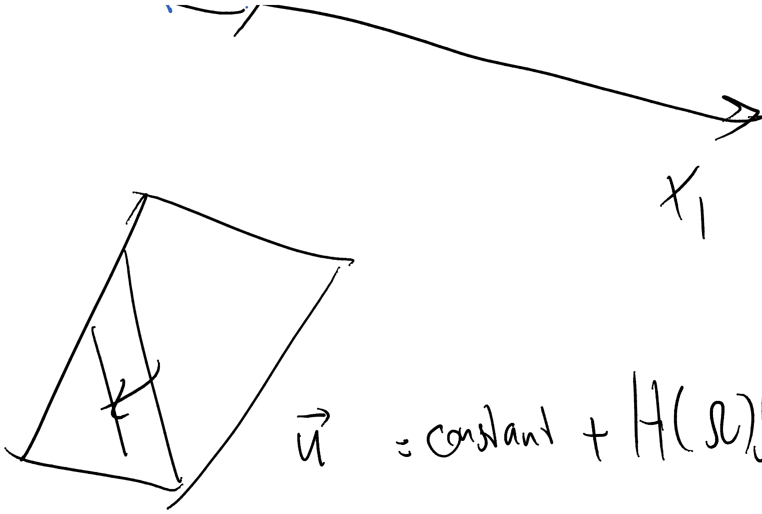
We add the following CRACK TIP DISPLACEMENT SHAPE FUNCTIONS to elements' regular polynomial shape functions

$$[B_\alpha] = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

For elements that have cracks inside we enrich them with Heaviside function






$\vec{u}$ = constant + H(x) jump

Crack tip enrichment functions:

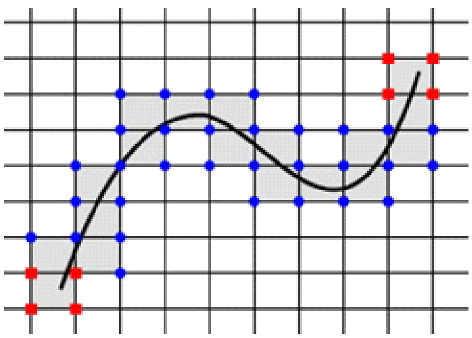
$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin\right.$$
$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos\right.$$

$$[B_\alpha] = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

Crack edge enrichment functions:

$$\boxed{H(\mathbf{x}) = \begin{cases} +1 & \text{if } (\mathbf{x}-\mathbf{x}^*)\cdot\mathbf{n} \geq 0 \\ -1 & \text{otherwise} \end{cases}}$$

| | |
|---|---|
| $S^c$ | blue nodes |
| $S^t$ | red nodes |



$$\mathbf{u}^h(\mathbf{x}) = \sum_{I\in\mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J\in\mathcal{S}^c} N_J(\mathbf{x})H(\mathbf{x})\mathbf{a}_J + \sum_{K\in\mathcal{S}^t} N_K(\mathbf{x})\left(\sum_{\alpha=1}^{4} B_\alpha \mathbf{b}_K^\alpha\right)$$

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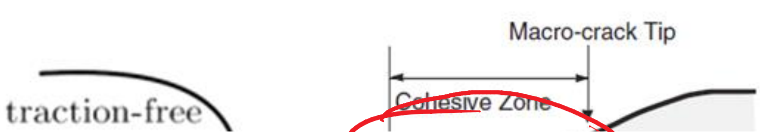
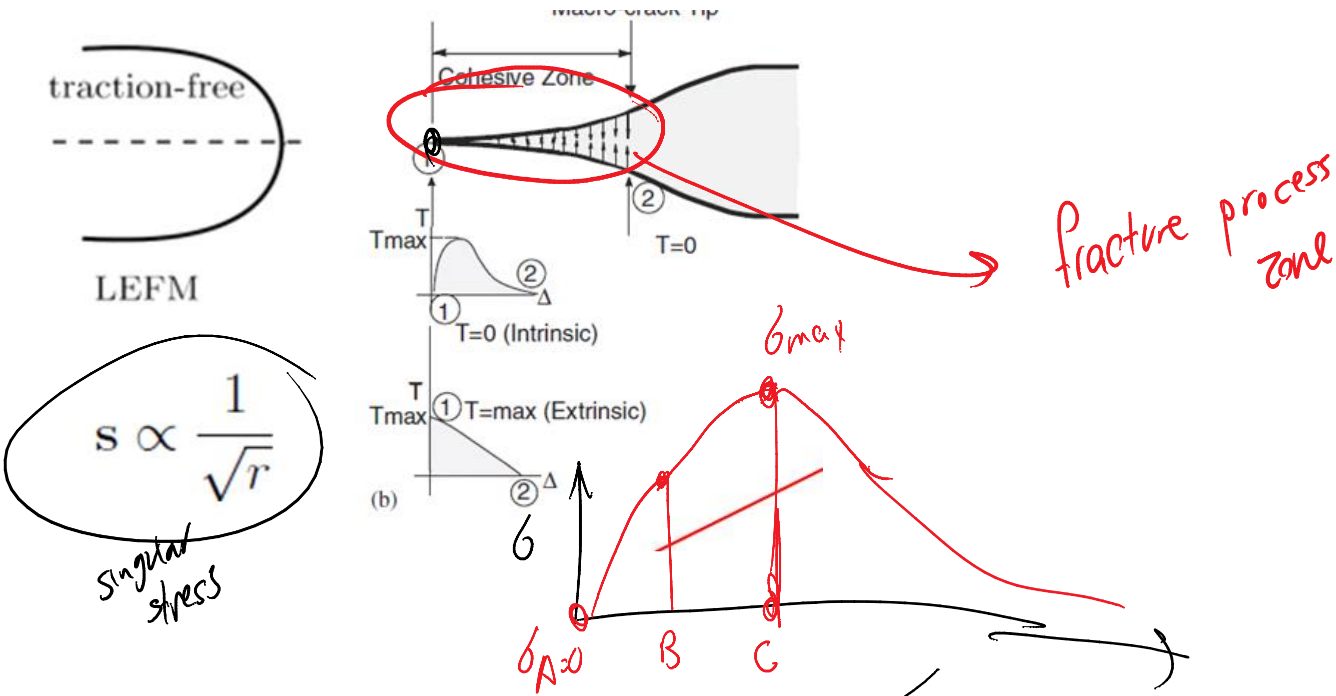
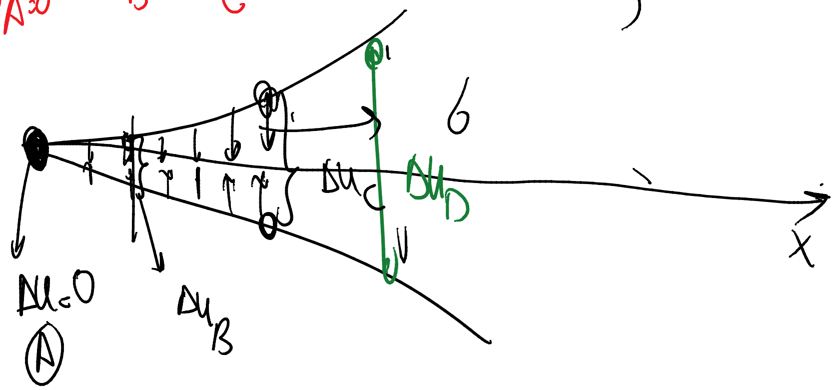
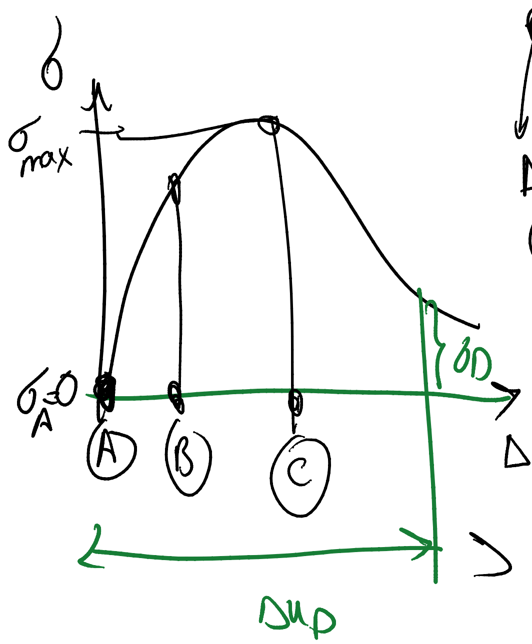
Traction separation Relations

# 6.2. Traction Separation Relations (TSRs)






$\Delta u \rightarrow$ displacement jump