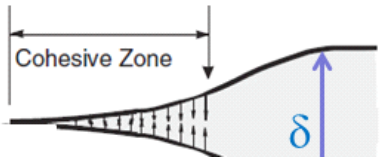
2015/11/12

Thursday, November 12, 2015
1:09 PM

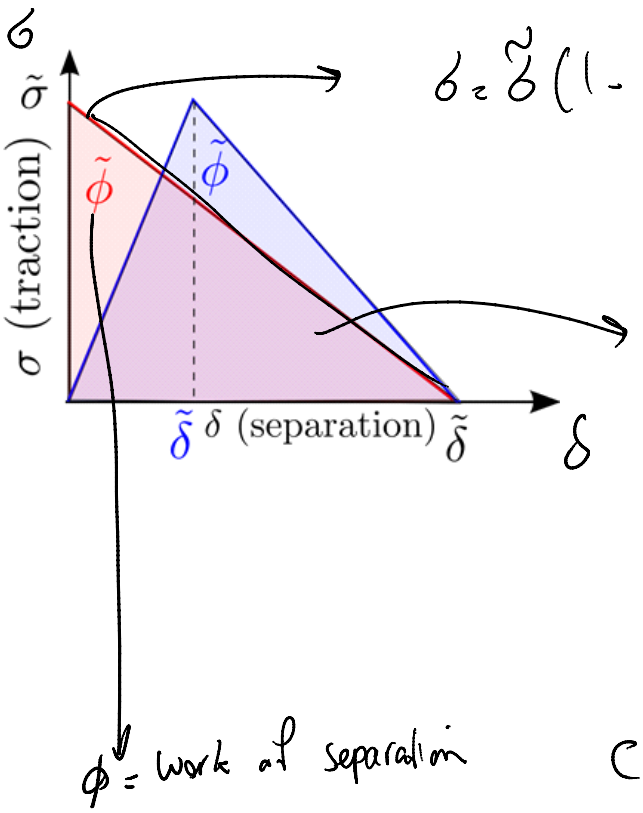
Cohesive models

- Traction Separation Relation (TSR): Relation between traction (stress) and displacement jump

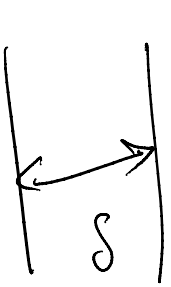
$$\sigma = \tilde{\sigma} f(\delta/\tilde{\delta})$$



Parameters of a cohesive model



$\sigma = \tilde{\sigma}\left(1 - \frac{\delta}{\tilde{\delta}}\right)$



unit $\sigma\delta$: $\frac{F}{L^2} \cdot L = \frac{\text{energy}}{\text{area}}$

Compare it with fracture toughness

R

ϕ = work of separation

Cohesive model

$G = R$ crack can propagate in LEFM

} DIFFERENT APPROACHES

TSR $\rightarrow \phi$

LEFM, PFM $\rightarrow$ R

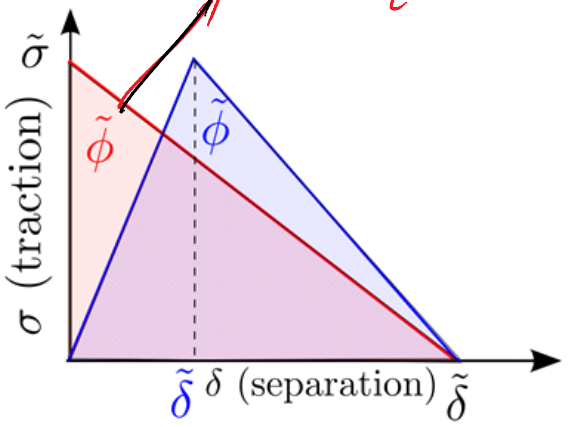
If we have the value of R (fracture toughness) and want to calibrate a cohesive model a good starting point value for ϕ is R

If we have the value of K (fracture toughness) and want to calibrate a cohesive model a good starting point value for ϕ is R

$\tilde{\phi} = \tilde{\delta}\cdot\tilde{\sigma}$

$\tilde{\phi} = \tilde{\sigma}\,\tilde{\delta}\,f$

factor depends on the shape of the cohesive model



- Stress (traction) scale $\tilde{\sigma}$: Maximum traction in TSR
- Displacement scale $\tilde{\delta}$: Separation corresponding to maximum traction (extrinsic models) or maximum nonzero traction
- Work of Separation $\tilde{\phi}$: Area under $\sigma-\delta$ curve is the work needed to complete debond a unit surface area. This can be associated with G_c in LEFM theory.

need 2 out of 3 parameters

(compare with LEFM we only need R)

plane stress ← R

R constant

plane strain

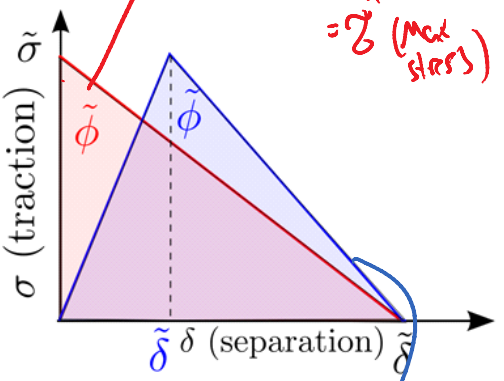
R

G_c

in weakening mode from beginning

extrinsic $G(\delta=0)\neq 0$ & often $=\tilde{\phi}$ (most users)

- Intrinsic cohesive models:
  - It has and initial hardening $\sigma-\delta$ part
  - σ starts from 0
  - Can be inserted in FEM mesh from the start of the simulation (along certain lines or between all elements)
- Extrinsic cohesive models:
  - Generally has only softening $\sigma-\delta$ behavior.
  - σ starts from maximum stress ($\tilde{\sigma}$)
  - Should be adaptively inserted between elements when traction between elements approach $\tilde{\sigma}$



Intrinsic there is an initial hardening

behavior

$\sigma(0) = 0$

The use of knowing fracture process zone size
Assigning sufficient number of elements in that zone to capture fracture response accurately. Often have > 4-5 elements along the fracture process zone

element size $\leftarrow h \leq r_p/5$

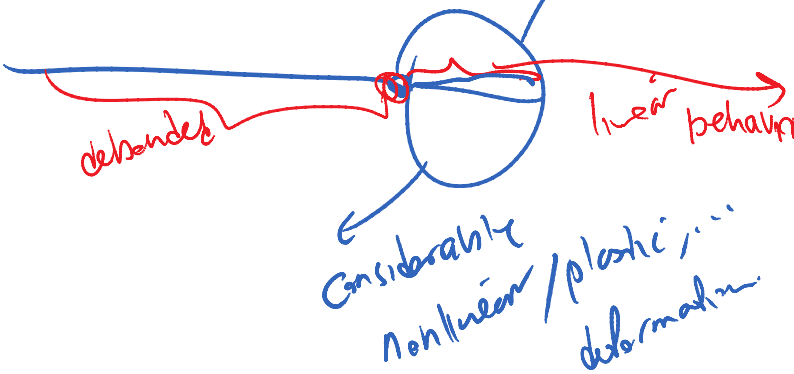
Process zone (Cohesive zone)

model

Remember PFM

$r_p \propto \left(\frac{K_I}{\sigma_y}\right)^2$

PFM estimate



this is where fracture is happening

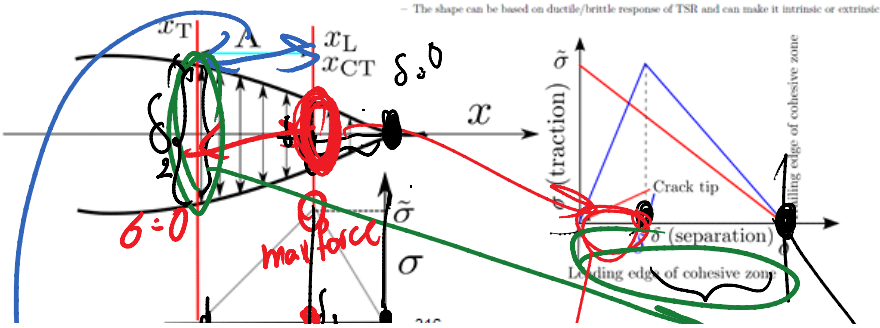
How can we get estimates for fracture process zone size (PZS) USING COHESIVE MODELS

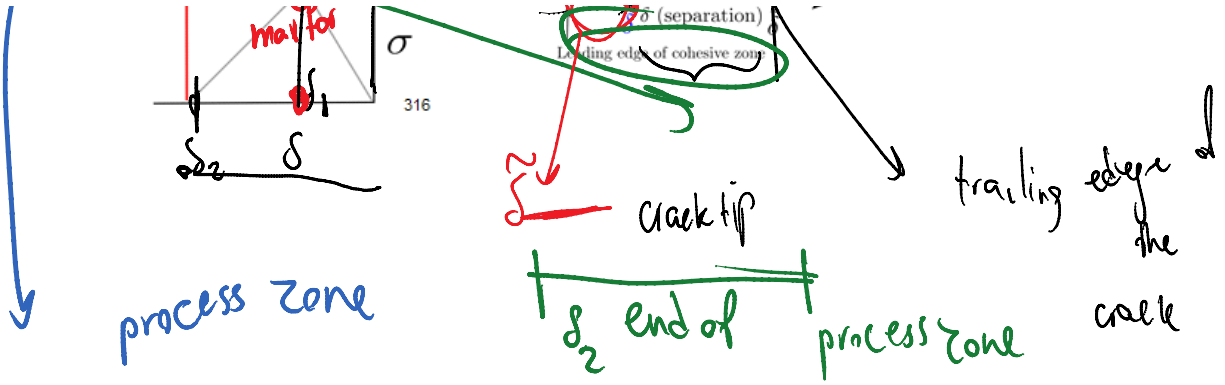
Process zone (Cohesive zone)

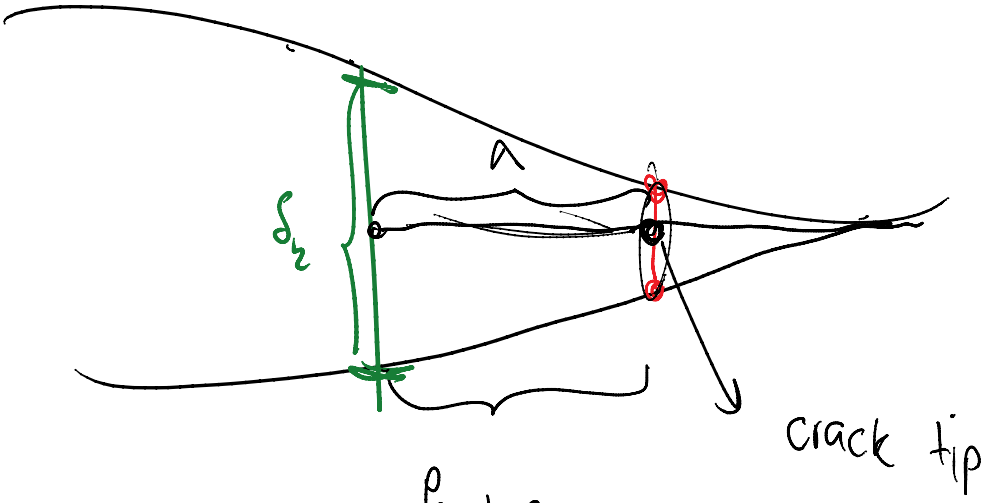
- Process zone Important Points and lengths
  - Nominal crack tip x_{CT} Generally corresponds to the point with maximum traction ($\tilde{\sigma}$)
  - Nominal trailing edge of the process zone x_T: The point where traction goes to zero (or if asymptotically goes to zero taken when stress is arbirary small *e.g.*, 0.01 or 0.001 *sigma*.
  - Nominal leading edge of the process zone x_L: When general crack-like (*e.g.*, highly nonlinear response) starts. Often, x_L is set to x_{CT}.
  - Process zone size $\Lambda = |x_L - x_T|$: Characteristic length scale corresponding to cohesive models ($\bar{L}$)

$\delta = 0$ Max Cohesive model shape

- Can have important influence on the response of cohesive model
- The shape can be based on ductile/brittle response of TSR and can make it intrinsic or extrinsic







TSR: its size is fracture process zone size = Λ

Why process zone size is important?

- Importance of process zone size Λ
  - Static estimate:

work of separation

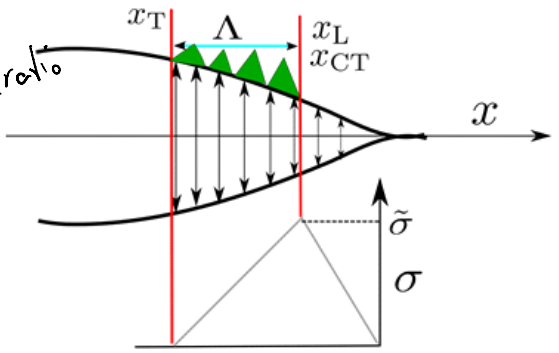
shear modulus = $\frac{E}{2(1+\nu)}$, ν: Poisson ratio

process zone size estimate

$$\Lambda = \varsigma\pi \frac{\mu}{1-\nu} \frac{\tilde{\phi}}{\tilde{\sigma}^2} \propto \tilde{L}$$

stress scale

$$\varsigma = \begin{cases} \frac{1}{4} & \text{Dugdale model} \\ \frac{9}{16} & \text{Potential-based TSRs} \end{cases}$$



  - Minimum number of elements in process zone size: There should be at least 4-10 elements along the PZ

Traction separation relation scales

Scales of cohesive model

$$\tilde{\phi} = \tilde{\sigma}\tilde{\delta} \qquad \text{Energy}$$

2 out of the three are independent

$$\tilde{p} = \rho\tilde{v} = \frac{\tilde{\sigma}}{c_\mathrm{d}} \qquad \text{Linear momentum}$$

$$\tilde{E} = \frac{\tilde{v}}{c_\mathrm{d}} = \frac{\tilde{\sigma}}{\rho c_\mathrm{d}^2} \propto \frac{\tilde{\sigma}}{\|\mathbf{C}\|} \qquad \text{Strain}$$

$$\tilde{v} = \frac{\tilde{\delta}}{\tilde{\tau}} = \frac{\tilde{\sigma}}{\rho c_\mathrm{d}} \qquad \text{Velocity}$$

$$\tilde{L} = c_\mathrm{d}\tilde{\tau} = \frac{\rho c_\mathrm{d}^2 \tilde{\delta}}{\tilde{\sigma}} \propto \Lambda^0 \qquad \text{Length}$$

$\Lambda = \varsigma\pi \frac{\mu}{1-\nu}\frac{\phi}{\tilde{\sigma}^2} \propto \tilde{L}$

Process zone size Λ

$$\tilde{\tau} = \frac{\rho c_\mathrm{d}\tilde{\delta}}{\tilde{\sigma}} \qquad \text{Time}$$

Influences time step for time marching methods

process zone size

TSR

displacement scale

Stress scale

time scale

mass density

longitudinal wave speed $c = \sqrt{\frac{\lambda + 2\mu}{\rho}}$

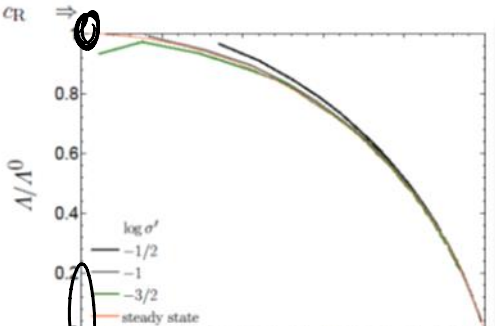
The time scale gives you an idea what time step size should be chosen for dynamic fracture analysis

Side note:
In dynamic fracture PZS goes to zero (at least theoretically)

– Dynamic estimate: PZS decreases as crack speed $\hat{v}$ approaches Rayleigh wave speed c_R

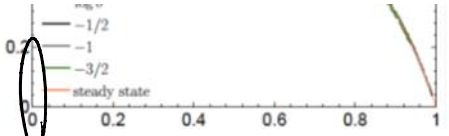
$$\Lambda(\hat{v}) = \frac{\Lambda}{A(\hat{v})}, \quad A(\hat{v}) \to 0 \text{ as } \hat{v} \to c_\mathrm{R} \Rightarrow$$

Smaller elements are needed in PZT as crack accelerates!



normalized process zone size

process zone size

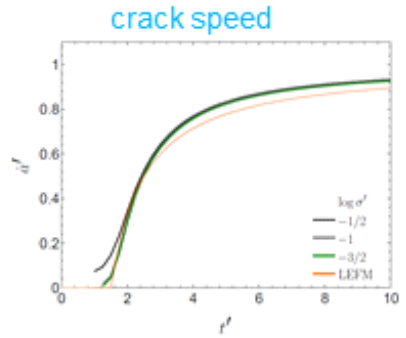


normalized crack speed

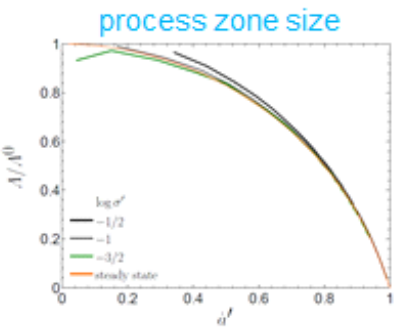
crack speed ← static $\hat{V} = 0$

SSY $\quad \left(\frac{r_p}{r_s}\right) \propto \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 \ll 1$

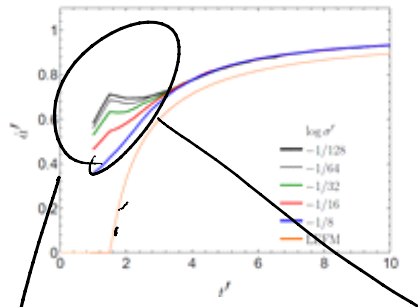
when applied fractions are (much) smaller than yield/ultimate strength LEFM results are "accurate"



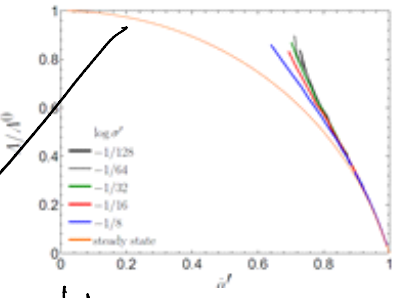
Low $\sigma' = \frac{\bar{\sigma}}{\sigma}$
SSY
LEFM ≈ CFM



$\frac{\bar{\sigma}}{\sigma_y}$ small
CFM ≈ LEFM
SSY is satisfied



High $\sigma' = \frac{\bar{\sigma}}{\sigma}$
LSY
LEFM ≠ CFM



$\frac{\bar{\sigma}}{\sigma_y}$ high

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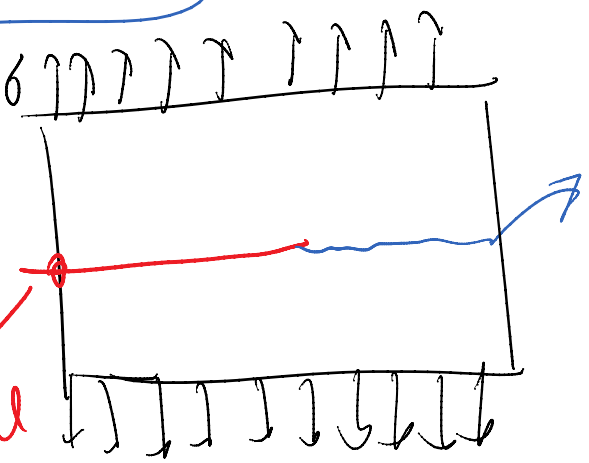
cohesive simulations ← LEFM solution

Cohesive models

What is artificial compliance?

(AC)

How cohesive models are used



insert cohesive models

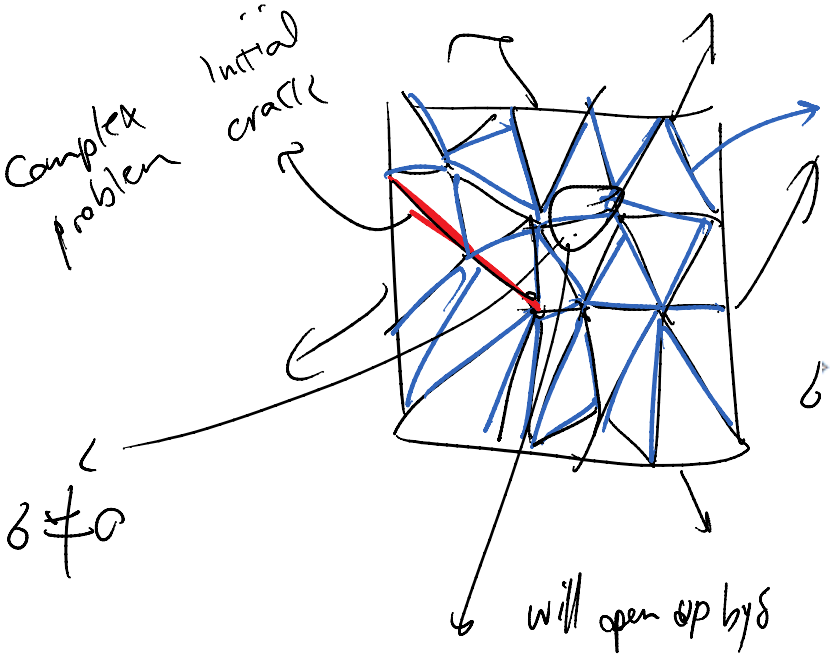
initial crack "fully debonded"

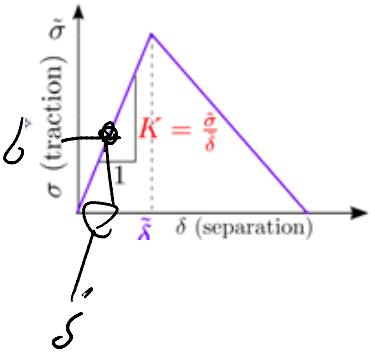
"AC is not a problem"

Complex problem

Initial crack

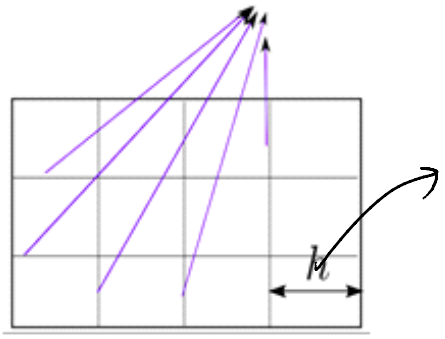
Cohesive surface





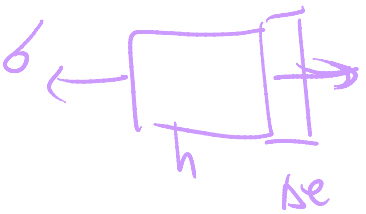
$\delta \neq 0$

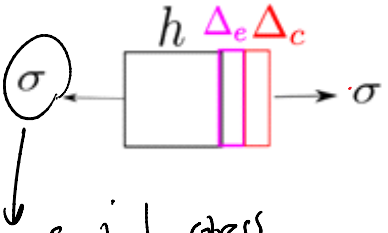
will open up by δ



element size

elastic response



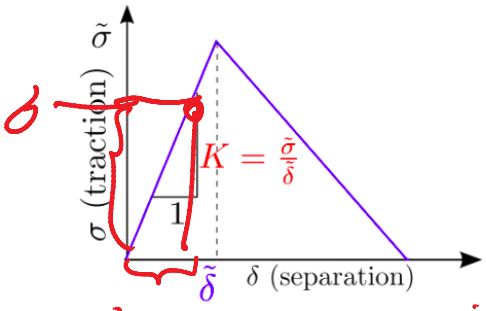


Applied stress

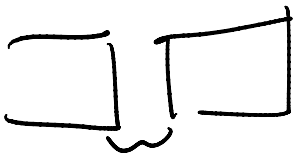
since $\varepsilon = \frac{\sigma}{E}$

$\Delta_e = \varepsilon h$

displacement from elastic deformation



σ/K K is cohesive model stiffness



Cohesive model

$\Delta u = \Delta_e + \Delta_{cohesive}$

$\frac{\sigma}{E_{effective}} h := \Delta u = \frac{\sigma}{E} h + \frac{\sigma}{K}$

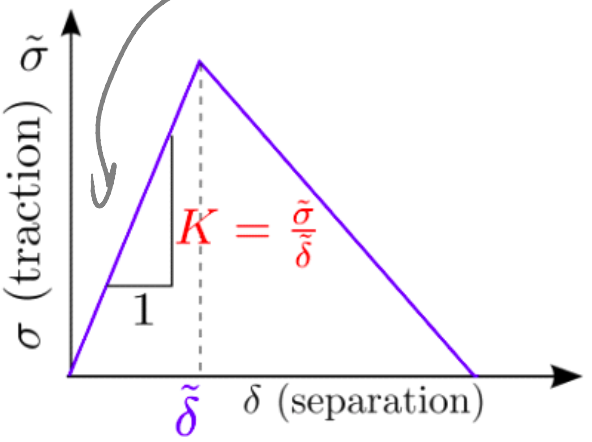
effective compliance $\frac{1}{E_{eff}} = \frac{1}{E} + \frac{1}{Kh}$

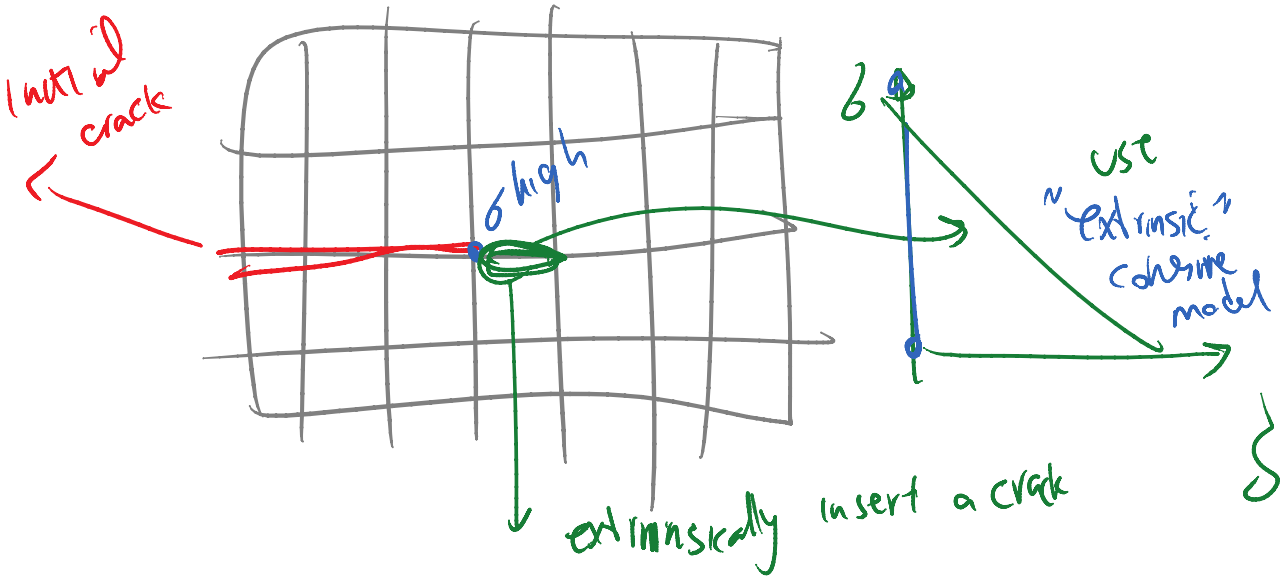
$\frac{1}{E}$ physical compliance

$\frac{1}{Kh}$ Compliance from cohesive model

$h \to 0$ dominates

This is the problem with "intrinsic" cohesive models

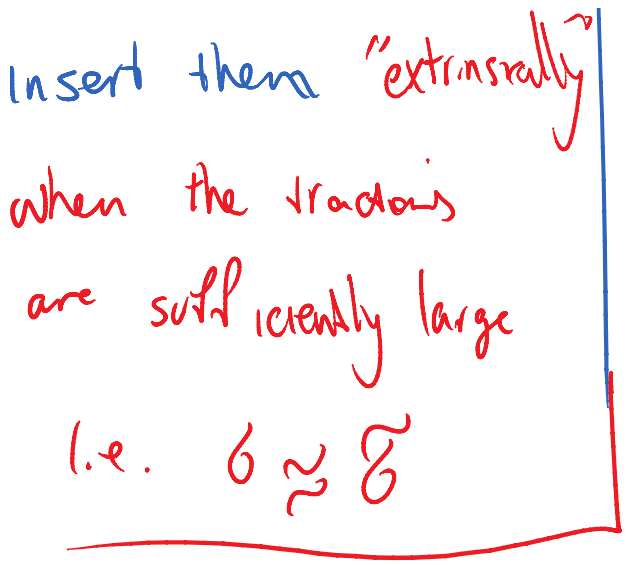




on the other hand we can insert them "extrinsically"

$\delta(u=0) = \tilde{\delta}$

Cannot insert them at the beginning because "initially" $u \approx 0$



"initially" $u \simeq 0$

$\sigma \simeq 0$

$\sigma = \tilde{\sigma}$ does not make sense

4.3 Mixed mode fracture

4.3.1 Crack propagation criteria

a) Maximum Circumferential Tensile Stress

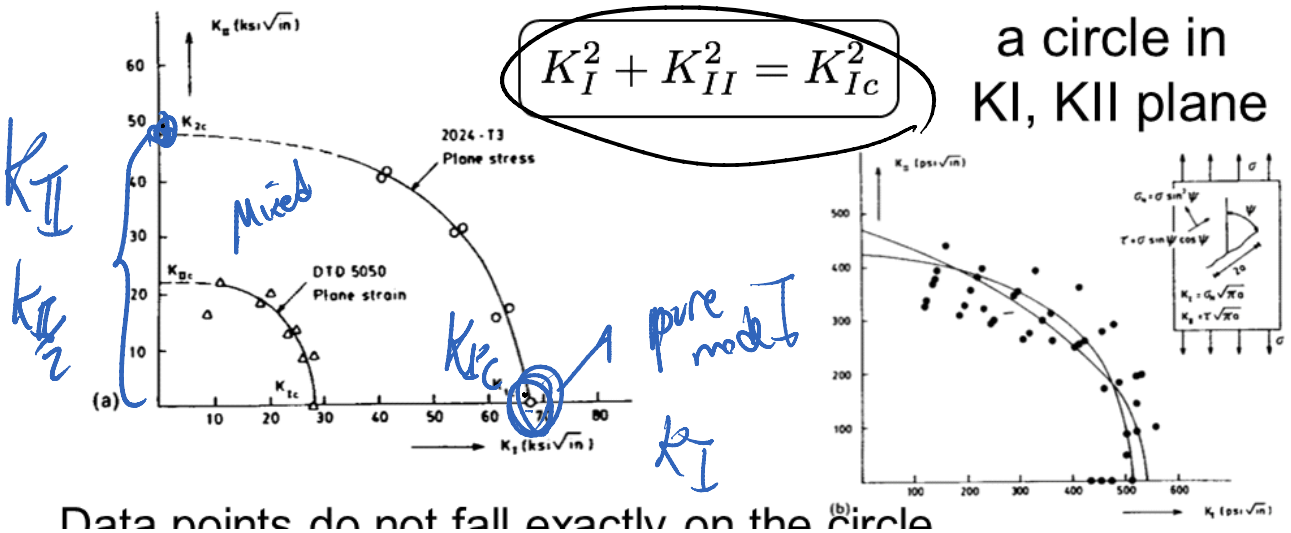
b) Maximum Energy Release Rate

c) Minimum Strain Energy Density

4.3.2 Crack Nucleation criteria

Motivation: Mixed mode crack propagation criteria

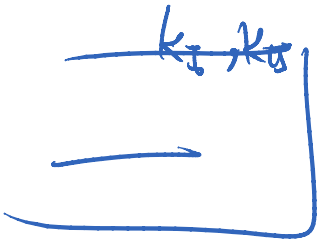
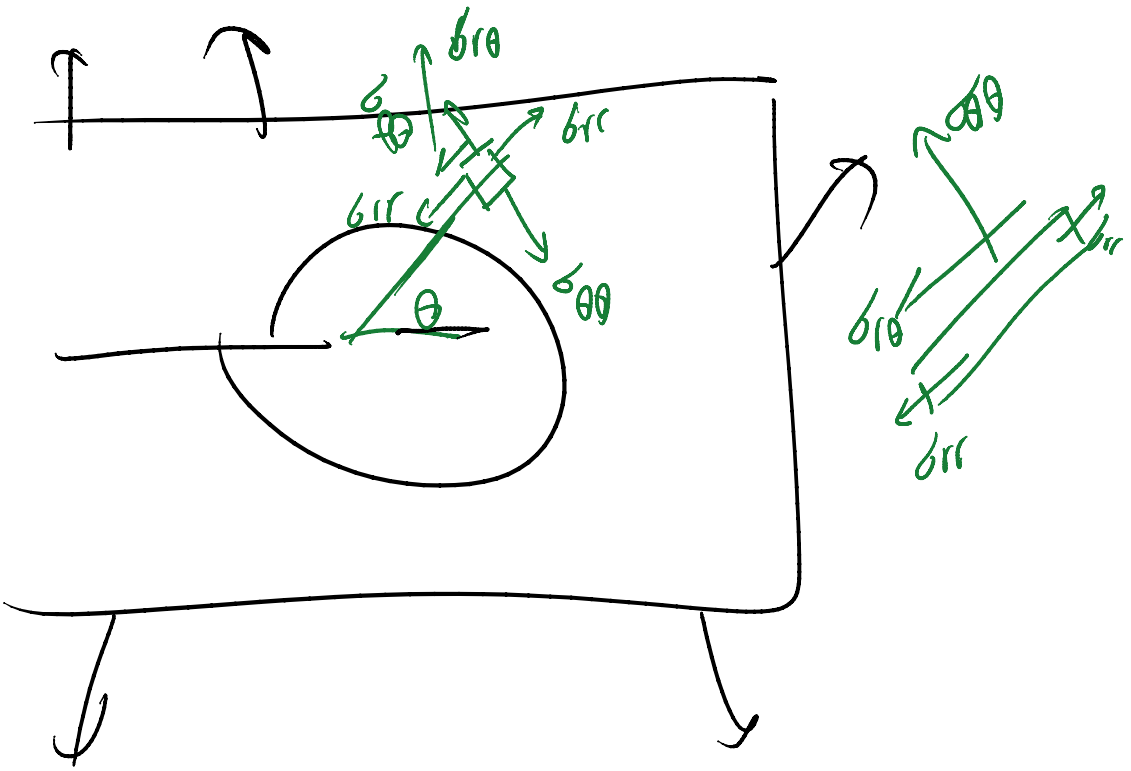
Motivation: Experiment verification of the mixed-mode failure criterion



$$K_I^2 + K_{II}^2 = K_{Ic}^2$$

a circle in KI, KII plane

Data points do not fall exactly on the circle



$$\left(\frac{K_I}{K_{Ic}}\right)^2 + \left(\frac{K_{II}}{K_{IIc}}\right)^2 = 1$$