2015/11/17

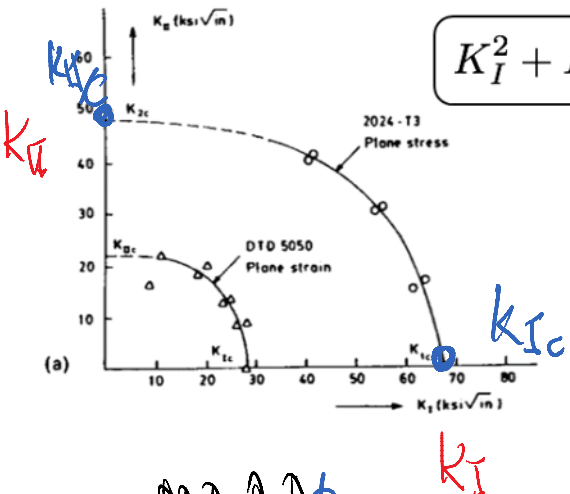
Tuesday, November 17, 2015
1:08 PM

Crack propagation criterion

1. If the crack propagates
2. What direction it propagates

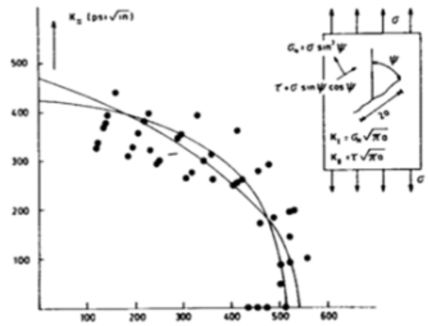
$K_{Ic} = K_{IIc}$

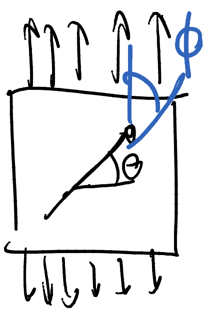
Question 1:



$$\boxed{K_I^2 + K_{II}^2 = K_{Ic}^2}$$

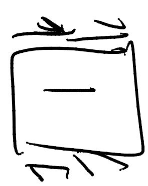
a circle in KI, KII plane





by changing θ we can create Mode I & mixed mode loading

Mode II

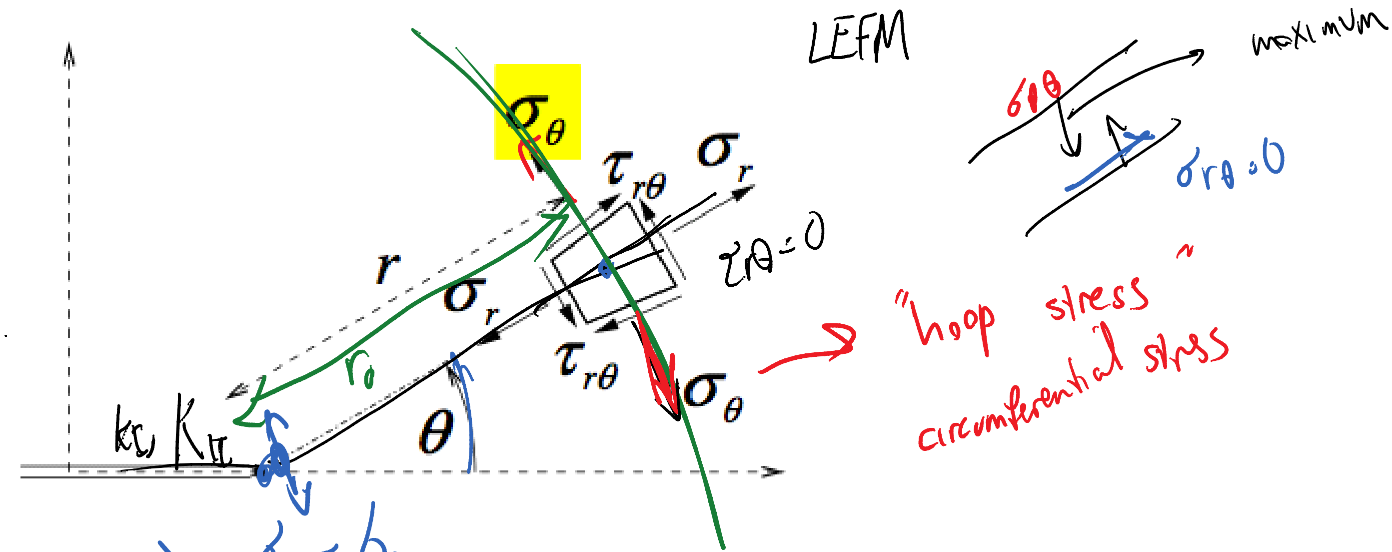


ϕ is the angle of propagation

Different criteria for answering questions 1 (if the crack propagates) and 2 (the direction of propagations)

Maximum circumferential stress criterion

Erdogan and Sih



$\times$ $\sigma_\theta = \sigma_\theta$

$\sigma_\theta \to \infty$ as $r \to 0$

crack propagates in a direction θ where σ_θ is maximum

$$\sigma_r = \frac{K_\mathrm{I}}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_\mathrm{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right) \quad (7.35a)$$

$$\sigma_\theta = \frac{K_\mathrm{I}}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_\mathrm{II}}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right) \quad (7.35b)$$

$$\tau_{r\theta} = \frac{K_\mathrm{I}}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) + \frac{K_\mathrm{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right). \quad (7.35c)$$

polar coordinate stress solutions

Maximize σ_θ

Or $\Sigma_{r\theta} = 0$

$$\tau_{r\theta} = 0 \longrightarrow K_I\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) + K_{II}\left(\cos\frac{\theta}{2} + 3\cos\frac{3\theta}{2}\right) = 0$$

(A)

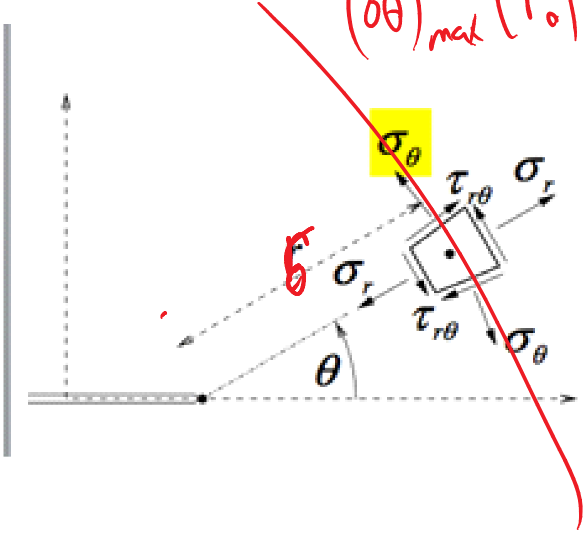
K_I & K_{II} are known θ is unknown

$$\theta_c = 2\arctan\frac{1}{4}\left(K_I/K_{II} \pm \sqrt{(K_I/K_I I)^2 + 8}\right)$$

$$\frac{K_{II}}{K_I} = \text{ratio of } \overset{K \text{ for}}{\text{mode II}} \text{ to mode I} = \alpha$$

$$\theta_c = 2\,tg^{-1}\frac{1}{4}\left(\frac{1}{\alpha} \pm \sqrt{\left(\frac{1}{\alpha}\right)^2 + 8}\right)$$

$(\sigma_\theta)_{max}(r_0) = \sigma_0$ Crack can propagate



σ_0 → strength

r_0 → length scale

P . A

σ_0 from θ

$$\sigma_{\theta max}\sqrt{2\pi r_0} = K_{\mathrm{Ic}} = \underbrace{\cos\frac{\theta_0}{2}\left[K_{\mathrm{I}}\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{\mathrm{II}}\sin\theta_0\right]}_{\sqrt{2\pi r_0}}$$

$\times\sqrt{2\pi r_0}$

$$\sigma_{\theta max}\sqrt{2\pi r_0} = K_{\mathrm{Ic}} = \cos\frac{\theta_0}{2}\left[K_{\mathrm{I}}\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{\mathrm{II}}\sin\theta_0\right]$$

why equal to K_{Ic}?

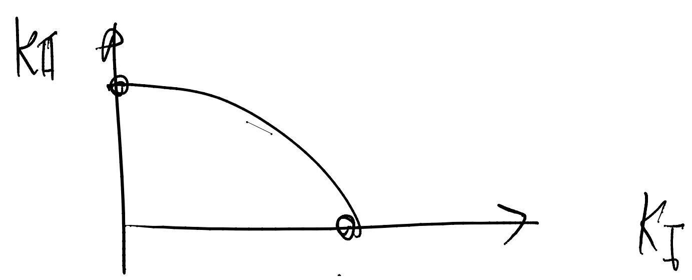
K_I , $K_{II} = 0$

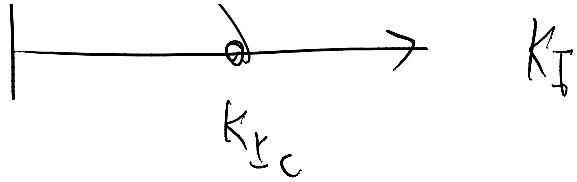
$\theta_0 = 0$

$\sigma_{\theta max}\sqrt{2\pi r_0} = K_I = K_{Ic}$ $\quad \theta = 0$

$$\sigma_{\theta max}\sqrt{2\pi r_0} = K_{\mathrm{Ic}} = \cos\frac{\theta_0}{2}\left[K_{\mathrm{I}}\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{\mathrm{II}}\sin\theta_0\right]$$

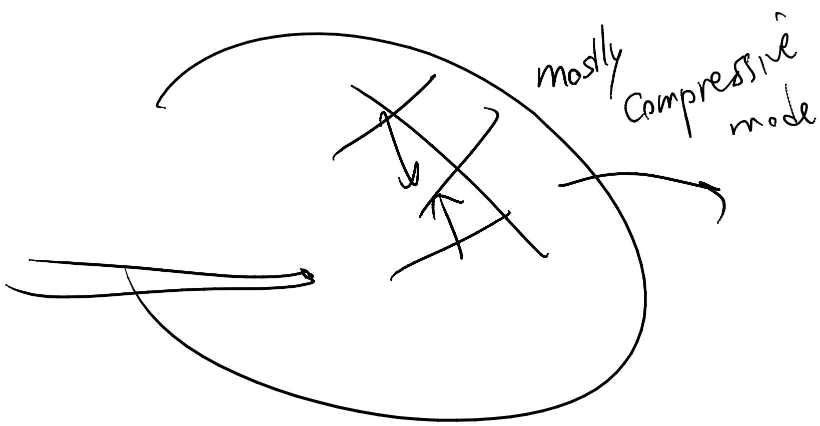
$$\theta_0 = 2\arctan\frac{1}{4}\left(K_I/K_{II} \pm \sqrt{(K_I/K_I I)^2 + 8}\right)$$

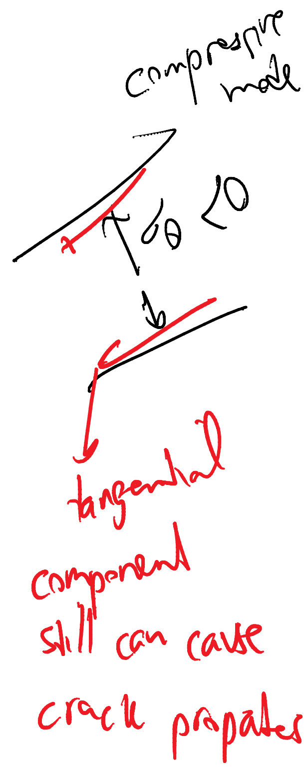




Modifications to maximum circumferential stress criterion

A generalization that is relevant in many fracture applications where is a high value of confinement stress (e.g. rock fracture)





- **Effective traction** can be defined as a function of both normal σ_θ and tangential $\tau r\theta$ components of traction. For example:

normal stress

$$\sigma_{\text{eff}} = \sqrt{\sigma_\theta^2 + (\alpha\tau_{r\theta})^2}$$

tangential stress

combines normal and tangential components through **mode mixity parameter** α.

- Crack propagation direction θ_c can be based on maximizing effective traction:

$\alpha = 0$

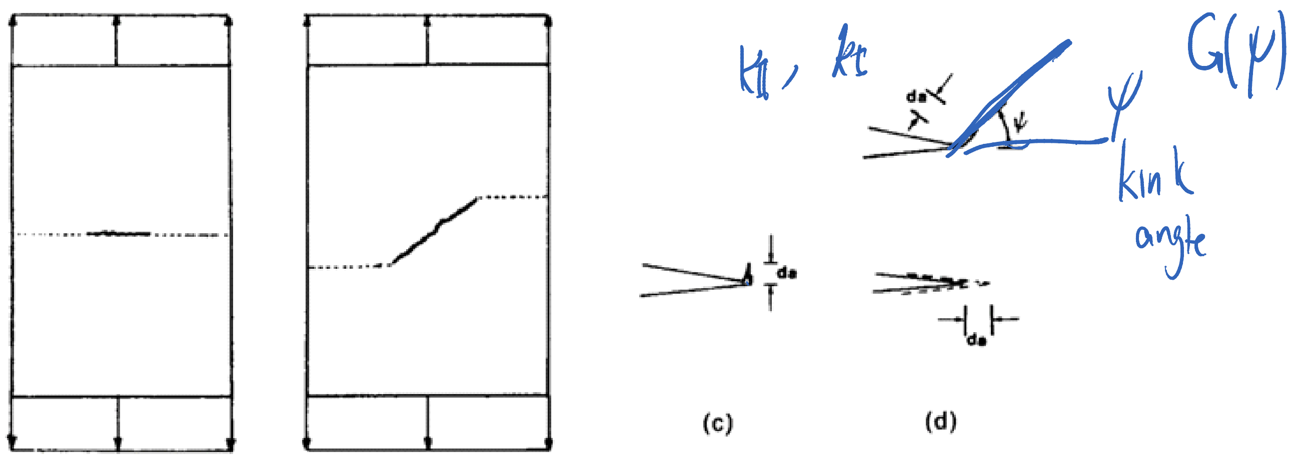
$$\sigma_{\text{eff}}(r, \theta_c) = \max_{-\pi<\theta<\pi} \sigma_{\text{eff}}(r, \theta)$$

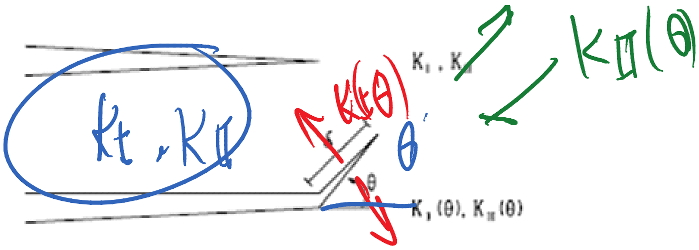
$\sigma_{\text{eff}} = \sigma_\theta$

- For example, in soil and rock applications normal tractions can be compressive for cracks that propagate under high shear tractions.

2 Maximum Energy Release Rate

Rate





Stress intensity factors for **kinked crack extension**:
Hussain, Pu and Underwood (Hussain et al. 1974)

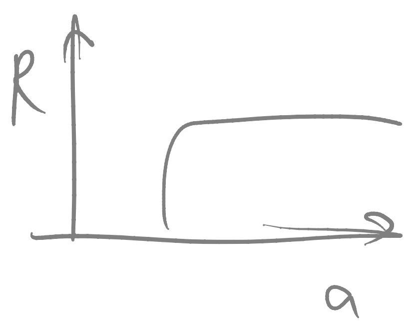
$$\begin{Bmatrix} K_I(\theta) \\ K_{II}(\theta) \end{Bmatrix} = \left(\frac{4}{3+\cos^2\theta}\right)\left(\frac{1-\frac{\theta}{\pi}}{1+\frac{\theta}{\pi}}\right)^{\frac{\theta}{2\pi}} \begin{Bmatrix} K_I\cos\theta + \frac{3}{2}K_{II}\sin\theta \\ K_{II}\cos\theta - \frac{1}{2}K_I\sin\theta \end{Bmatrix}$$

want to find
the optimal angle

energy release rate if the crack propages in direction θ

$$G(\theta) = \frac{K_I^2(\theta) + K_{II}^2(\theta)}{E'}$$

$$\max_{\theta} G(\theta) = R \rightarrow$$ "fracture resistance"



$$G(\theta) = \frac{4}{E'}\left(\frac{1}{3+\cos^2\theta}\right)^2\left(\frac{1-\frac{\theta}{\pi}}{1+\frac{\theta}{\pi}}\right)^{\frac{\theta}{\pi}} \left[(1+3\cos^2\theta)K_I^2 + 8\sin\theta\cos\theta K_I K_{II} + (9-5\cos^2\theta)K_{II}^2\right]$$

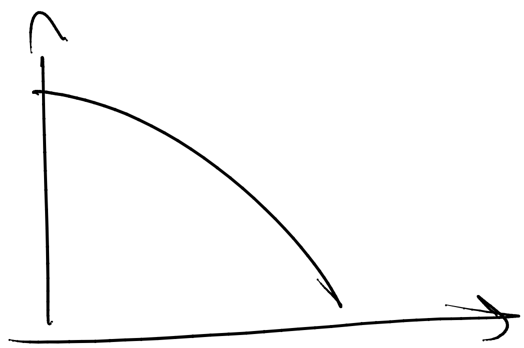
$G'(\theta)$ θ → θ

$G'(\theta_0) = 0 \Rightarrow \theta_0$

$$G(\theta) = \frac{4}{E'}\left(\frac{1}{3+\cos^2\theta}\right)^2\left(\frac{1-\frac{\theta}{\pi}}{1+\frac{\theta}{\pi}}\right)^{\frac{\theta}{\pi}} \left[(1+3\cos^2\theta)K_I^2 + 8\sin\theta\cos\theta K_I K_{II} + (9-5\cos^2\theta)K_{II}^2\right]$$

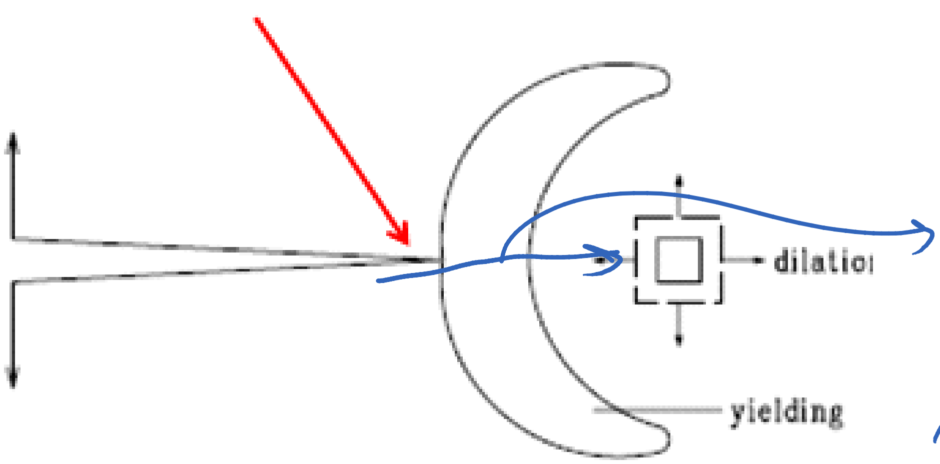
$$\boxed{4\left(\frac{1}{3+\cos^2\theta_0}\right)^2\left(\frac{1-\frac{\theta_0}{\pi}}{1+\frac{\theta_0}{\pi}}\right)^{\frac{\theta_0}{\pi}} \left[\left(1+3\cos^2\theta_0\right)\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}\right)^2 + 8\sin\theta_0\cos\theta_0\left(\frac{K_{\mathrm{I}}K_{\mathrm{II}}}{K_{\mathrm{Ic}}^2}\right) + \left(9-5\cos^2\theta_0\right)\left(\frac{K_{\mathrm{II}}}{K_{\mathrm{Ic}}}\right)^2\right] = 1}$$

K_I, K_{II} fracture locus



Strain Energy Density (SED) criterion

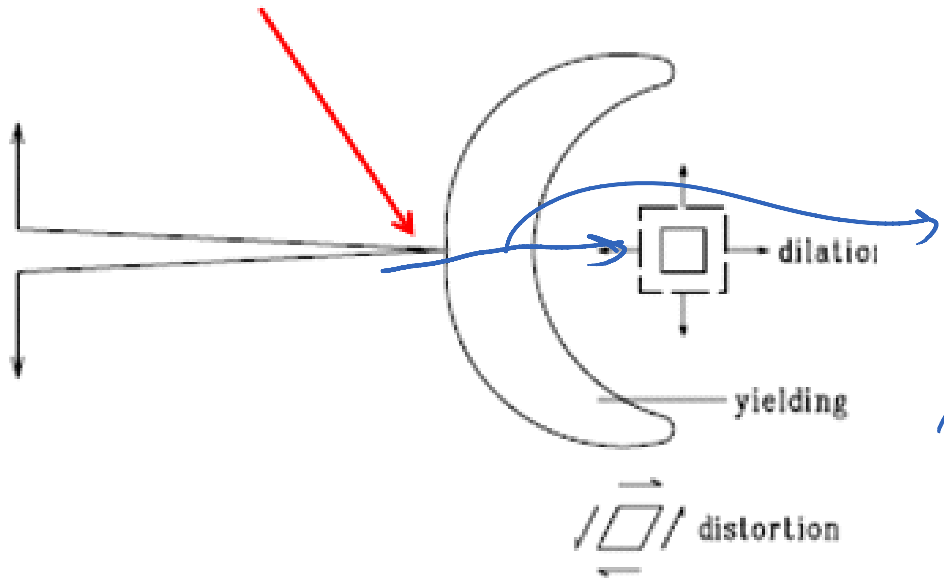
Sih 1973



minimum

strain energy density

$\sim \frac{1}{2}\sigma\varepsilon$ (1D)



minimum

strain energy density

$\sim \frac{1}{2}\sigma\varepsilon$ (1D)

$$U_i = \int_0^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij} \qquad U_i = \frac{1}{4\mu}\left[\frac{\kappa+1}{4}(\sigma_x^2+\sigma_y^2) - 2(\sigma_x\sigma_y - \tau_{xy}^2)\right]$$

$$\sigma_x = \frac{K_{\rm I}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - \frac{K_{\rm II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_{\rm I}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \frac{K_{\rm II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (7.13)$$

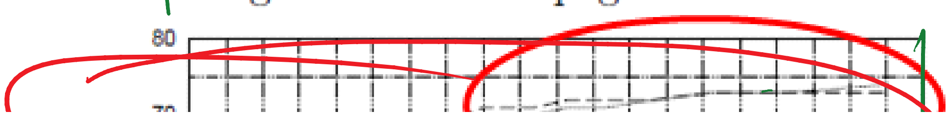
$$\tau_{xy} = \frac{K_{\rm I}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} + \frac{K_{\rm II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right).$$

$$\boxed{\frac{8\mu}{(\kappa-1)}\left[a_{11}\left(\frac{K_{\rm I}}{K_{\rm Ic}}\right)^2 + 2a_{12}\left(\frac{K_{\rm I}K_{\rm II}}{K_{\rm Ic}^2}\right) + a_{22}\left(\frac{K_{\rm II}}{K_{\rm Ic}}\right)^2\right] = 1}$$

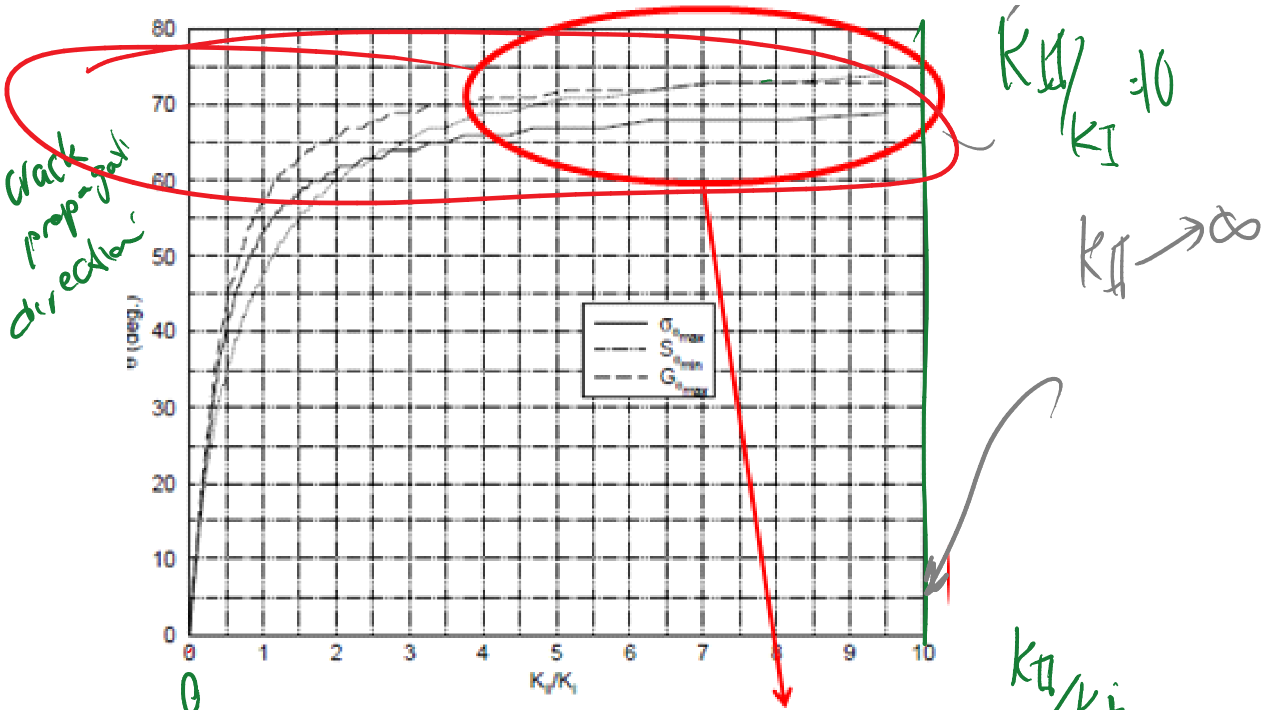
$$\begin{aligned}
a_{11} &= \frac{1}{16\mu}\left[(1+\cos\theta)(\kappa-\cos\theta)\right] \\
a_{12} &= \frac{\sin\theta}{16\mu}\left[2\cos\theta - (\kappa-1)\right] \\
a_{22} &= \frac{1}{16\mu}\left[(\kappa+1)(1-\cos\theta) + (1+\cos\theta)(3\cos\theta-1)\right]
\end{aligned}$$

$$\begin{aligned}
\kappa &= \tfrac{3-\nu}{1+\nu} \quad \text{(plane stress)} \\
\kappa &= 3-4\nu \quad \text{(plane strain)}
\end{aligned}$$

pure mode I



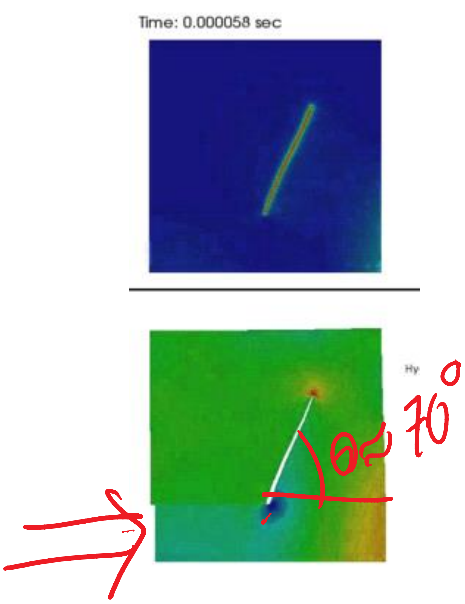
$K_{II} = 0$

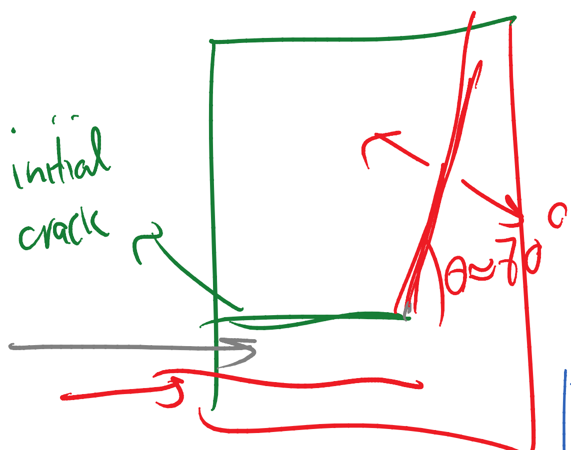


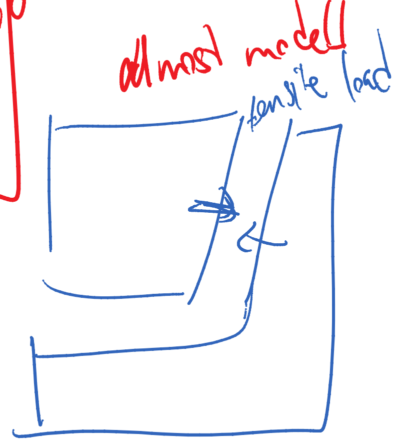
~ 70 degree angle for mode II !

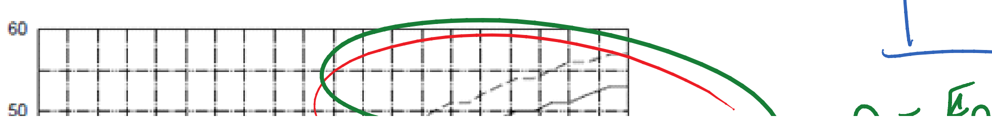
mode II dominated crack propagates in $\theta \approx 70^\circ$

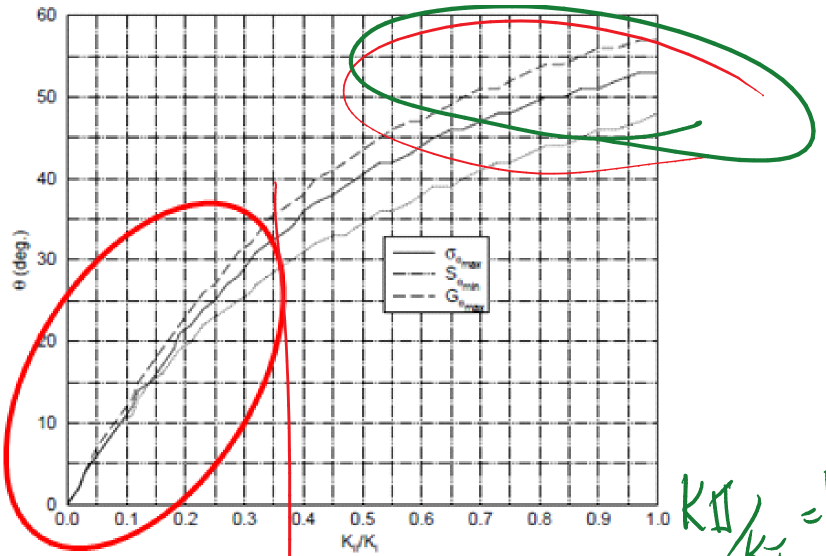
Kalthoff example:











$\theta = 50$ -

$K_{II}/K_I = 1$

Zoom view (low K_{II} component)

mode II dominant

mode I

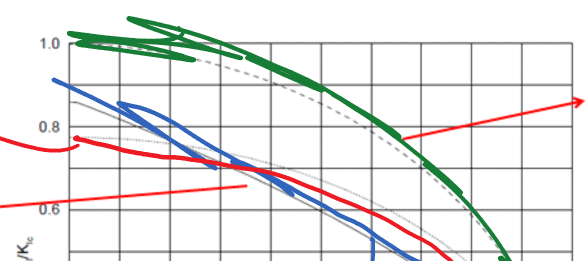
Cracks align themselves such that their subsequent loading is mostly in mode I

K_{II}/K_I high

Question 1) when the crack propagates?

b) Locus of crack propagation

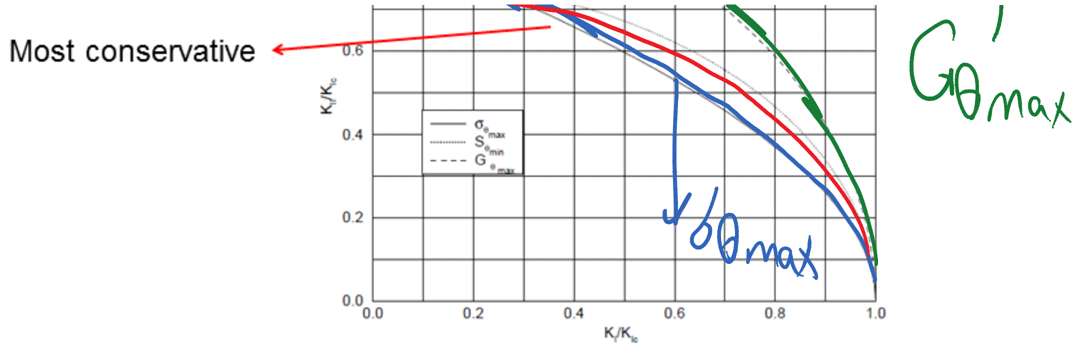
Minimum strain energy density



Least conservative

Most conservative

$G_{\theta max}$



Nucleation:
Cracks nucleate from material microdefects (such as microcracks)

Crack nucleation criterion

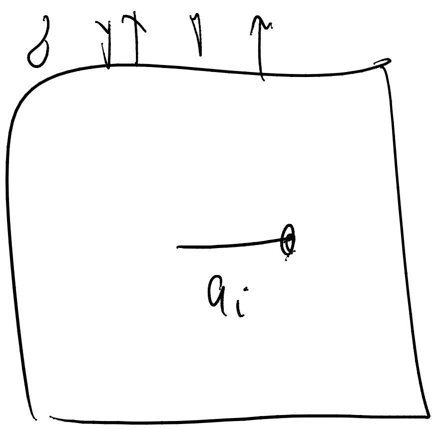
- For Maximum Energy Release Rate Criterion if we assume there are no defects, there will be no crack nucleation. However, assuming that local stress field generates a tensile maximum principal stress of σ_1 a "microscopic" initial crack (defect) of length a_{ini} perpendicular to σ_1 direction generates,

$$G = \frac{K_I^2 + K_{II}^2}{E'} = \pi a_{\text{ini}} \sigma_1^2$$

so the microcrack propagates (*i.e.*, a "macroscopic" crack nucleates) when,

$$G = G_c \quad \Leftrightarrow \quad \sigma_1 = \sqrt{\frac{G_c}{\pi a_{\text{ini}}}}$$

  - Initial crack direction perpendicular to σ_1 is chosen to maximize G.
  - We have assumed the initial crack to be small enough to use the infinite domain SIF formula of $K_I = \sqrt{\pi a}\bar{\sigma}$.



$$G = \frac{K_I^2}{E} = \frac{(\sqrt{\pi a_i}\,\sigma)^2}{E}$$

$$= \frac{\pi a_i \sigma^2}{E} = R$$

$$\frac{\pi a_i \sigma^2}{E} = R$$

the larger material defects smaller σ

the larger material defects smaller δ is needed for crack nucleation

Fatigue examples

Key Idea: Fluctuating loads are more dangerous than monotonic loads.

Example: *Comet Airliner* (case study). The actual cabin pressure differential when the plane was in flight was ≈ **8.5** pounds per square inch (psi). The design pressure was ≈ **20** psi (a factor of safety greater than 2). Thought to be safe! However, crack growth due to *cyclic* loading caused catastrophic failure of the aircraft.

Fatigue fracture is prevalent!

- Deliberately applied load reversals (e.g. rotating systems)
- Vibrations (machine parts)
- Repeated pressurization and depressurization (airplanes) flight, ground
- Thermal cycling (switching off electronic devices)
- Random forces (ships, vehicles, planes)

(source: Schreurs fracture notes 2012)

Fatigue occurs occur always and everywhere and is a major source of mechanical failure

- Fatigue occurs when a material is subjected to repeated loading and unloading (cyclic loading).
- Under cyclic loadings, materials can fail (due to fatigue) at stress levels well below

loading).

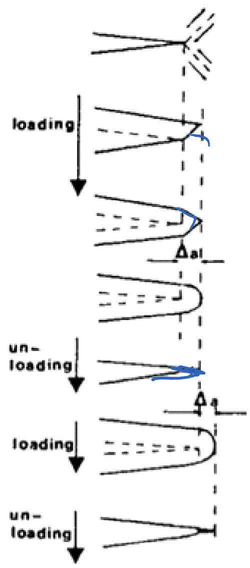
- ~~Under cyclic loadings, materials can fail~~ (due to fatigue) at stress levels well below their yield strength or crack propagation limit-> fatigue failure.

$\sigma_{\text{fatigue failure}} < \sigma_{\text{yield}}$

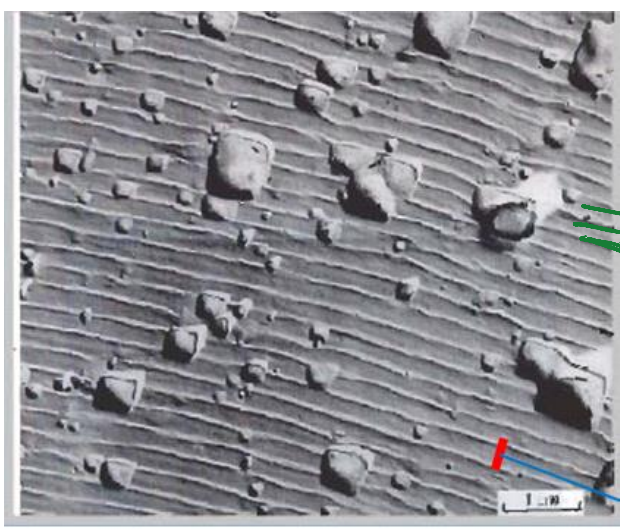
fatigue :

A sequence of very short crack propagation

blunting



resharpening



Fracture surface of a 2024-T3 aluminum alloy
(source S. Suresh MIT)

the amount that the crack propagate in each cycle or ~ every few cycles for very short ... loads

of every
for very short
initial defects

Fatigue Regimes

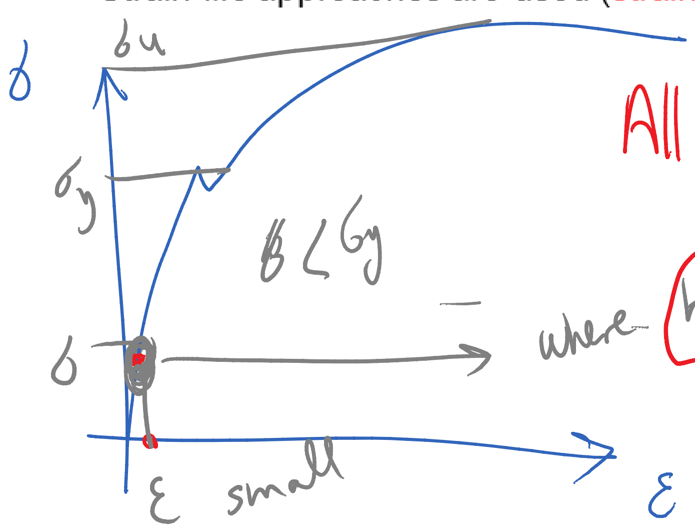
Table 7.1 Classification of fatigue damage

Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$
Very high cycle fatigue	$> 10^7$	$< \sigma_F$	≈ 0	≈ 0
High cycle fatigue	10^5 to 10^6	$< \sigma_Y$	≈ 0	≈ 0
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

Source: Dufailly and Lemaitre (1995)

our focus

- Very high cycle and high cycle fatigue:
  - Stresses are well below yield/ultimate strength.
  - There is almost no plastic deformation (in terms of strain and energy ratios)
  - Fatigue models based on LEFM theory (e.g. SIF K) are applicable.
  - Stress-life approaches are used (stress-centered criteria)
- Low cycle and very low cycle fatigue:
  - Stresses are in the order of yield/ultimate strength.
  - There is considerable plastic deformation.
  - Fatigue models based on PFM theory (e.g. J integral) are applicable.
  - Strain-life approaches are used (strain-centered criteria)



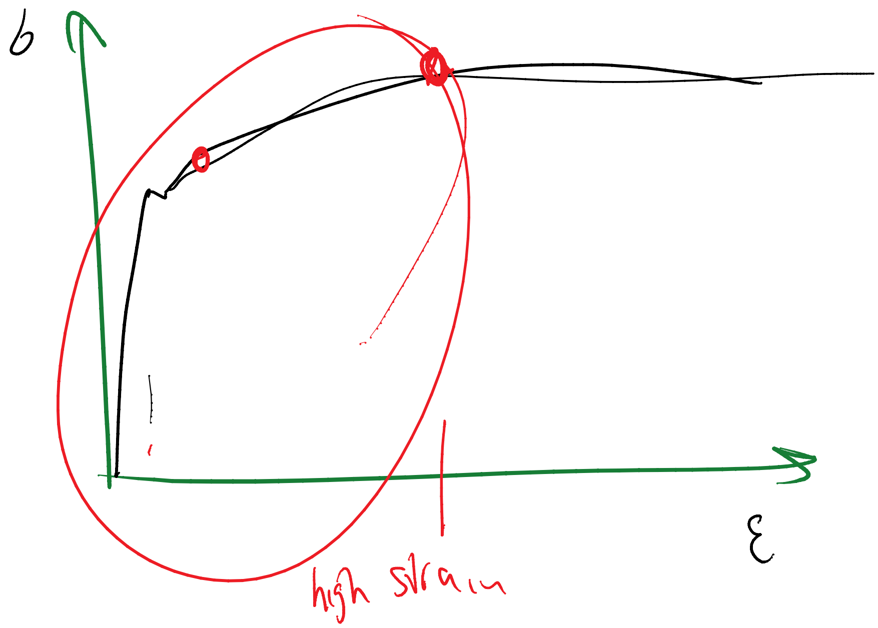
All in elastic mode

where high cycle fatigue is

Most of strain energy is elastic

characterized by σ (stress)

K (stress intensity factor)



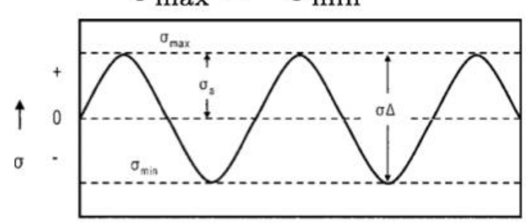
low cycle fatigue

cyclic plastic deformation fatigue

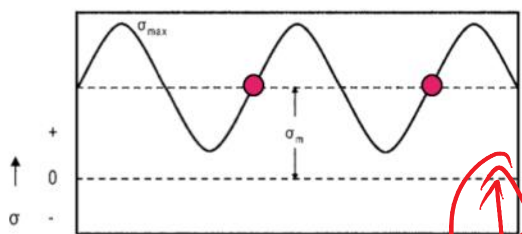
$\cancel{\sigma} \longrightarrow \varepsilon$

$\cancel{K} \longrightarrow J$

$$\sigma_{max} = -\sigma_{min}$$



Fully Reversed Loading



Tension-Tension with Applied Stress

what is important in fatigue?

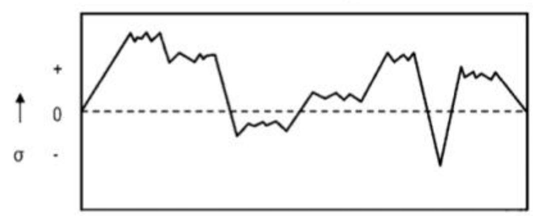
$\Delta\sigma = \sigma_{max} - \sigma_{min}$

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

$$\sigma_a = 0.5(\sigma_{max} - \sigma_{min})$$

$$\boxed{\sigma_m = 0.5(\sigma_{max} + \sigma_{min})}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}} \quad \text{load ratio}$$



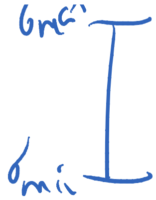
Random or Spectrum Loading

Δσ by large part determines fatigue response

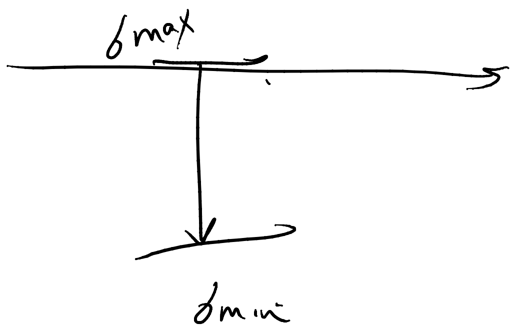
drives fatigue $\longleftarrow$ $\sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

σ_m: .5 $(\sigma_{max} + \sigma_{min})$ "average stress level"

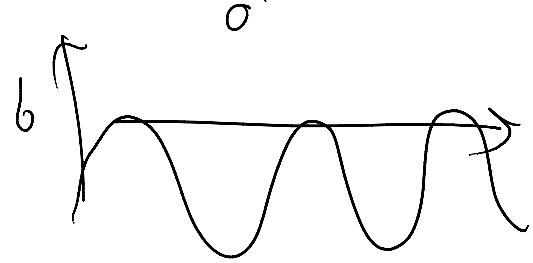
$R = \frac{\sigma_{min}}{\sigma_{max}}$



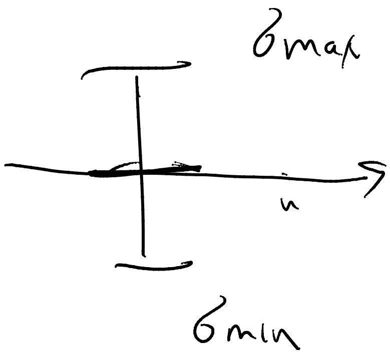
$\Delta\sigma$ fixed



$R = \frac{-\Delta\sigma}{0^+} \rightarrow -\infty$



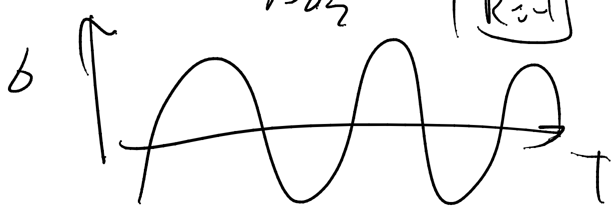
$\boxed{R = -\infty}$

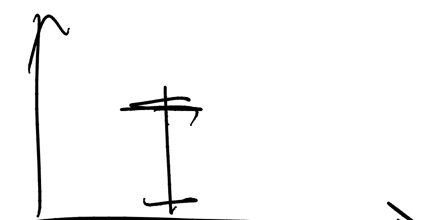


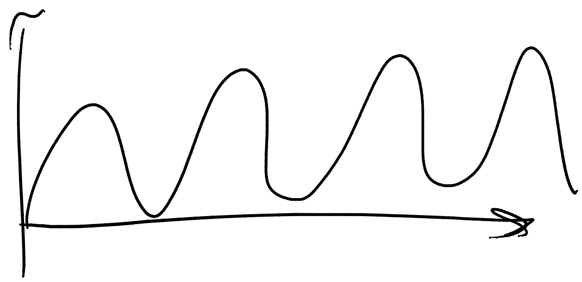
$R = \frac{-\Delta\sigma/2}{\Delta\sigma/2} = -1$

$\boxed{R = -1}$

symmetric loading

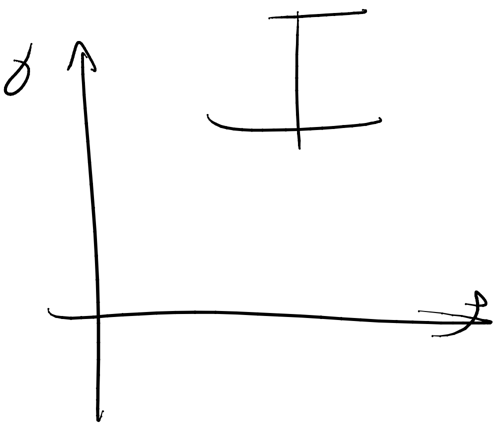


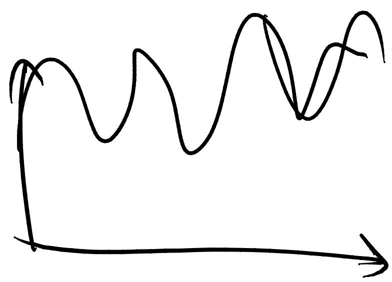




$\boxed{R = 0}$

all the load is tensile



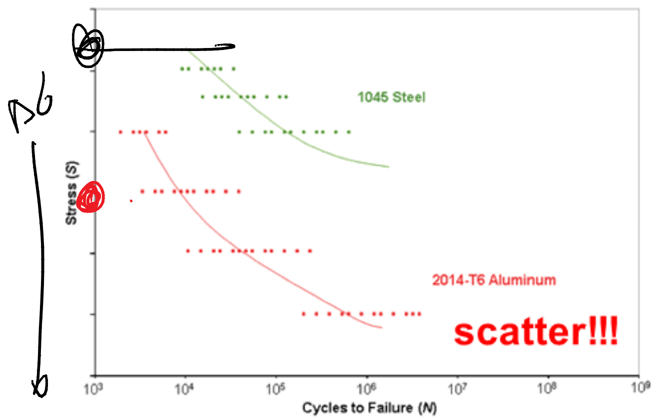


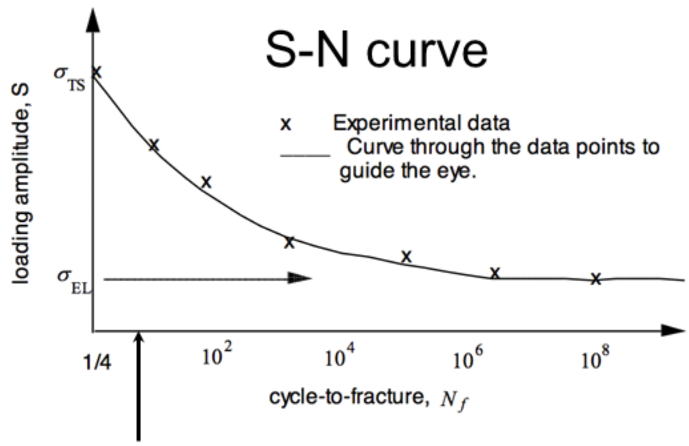
$R \nearrow \infty$

S-N curve

Reminder: ASTM defines *fatigue life*, N_f, as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs.

- Stress->Nf
- Nf->allowable S





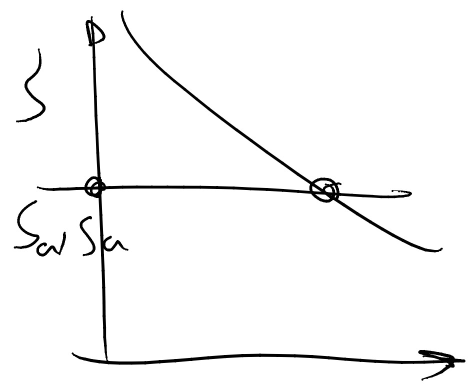
endurance limit (g.han keo dai)

cyc load

# of cycles

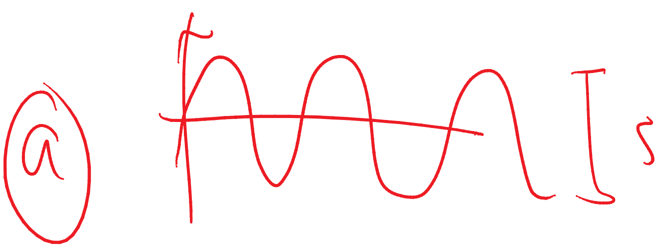
$S = \sigma_a = \sigma_{max} - \sigma_{min}$

fixed S



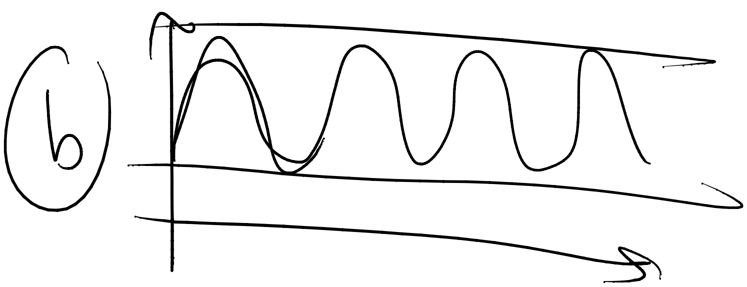
S_a S_a

log N



$R = -1$

$\frac{\sigma_{min}}{\sigma_{max}}$



$R = .2$

more tensile