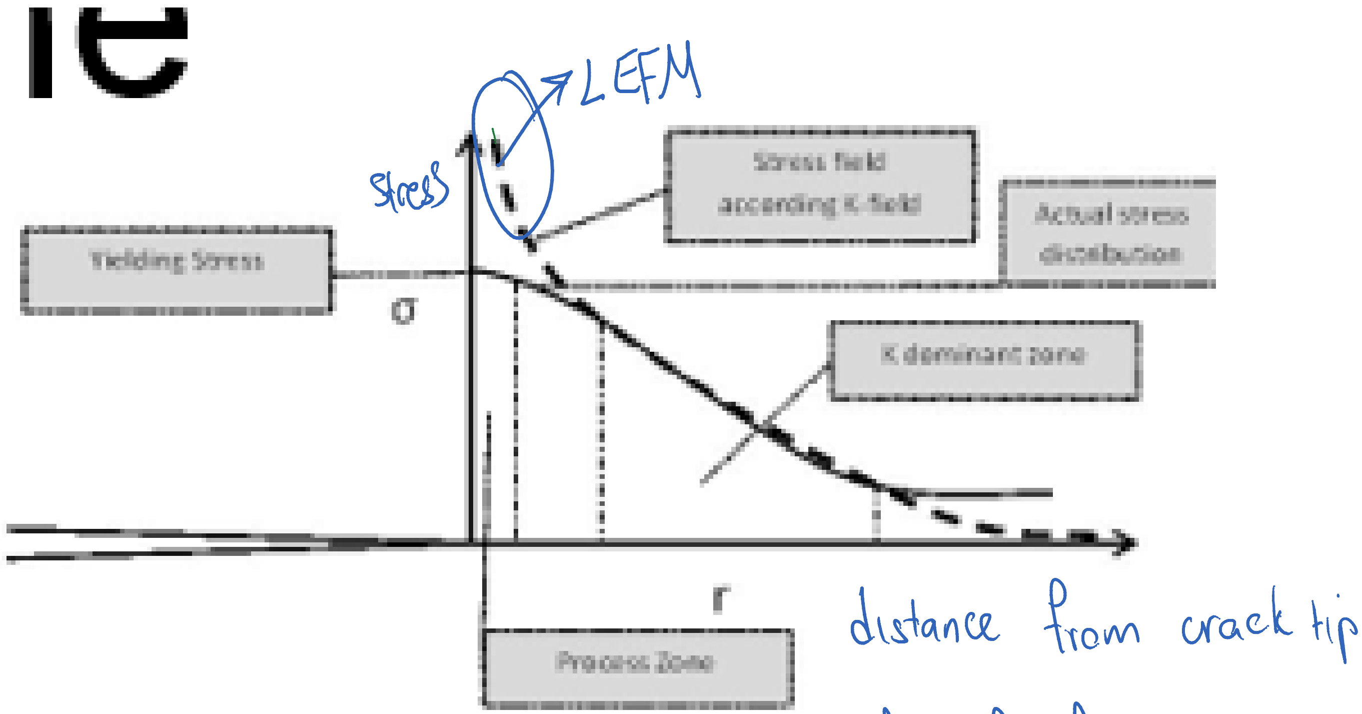
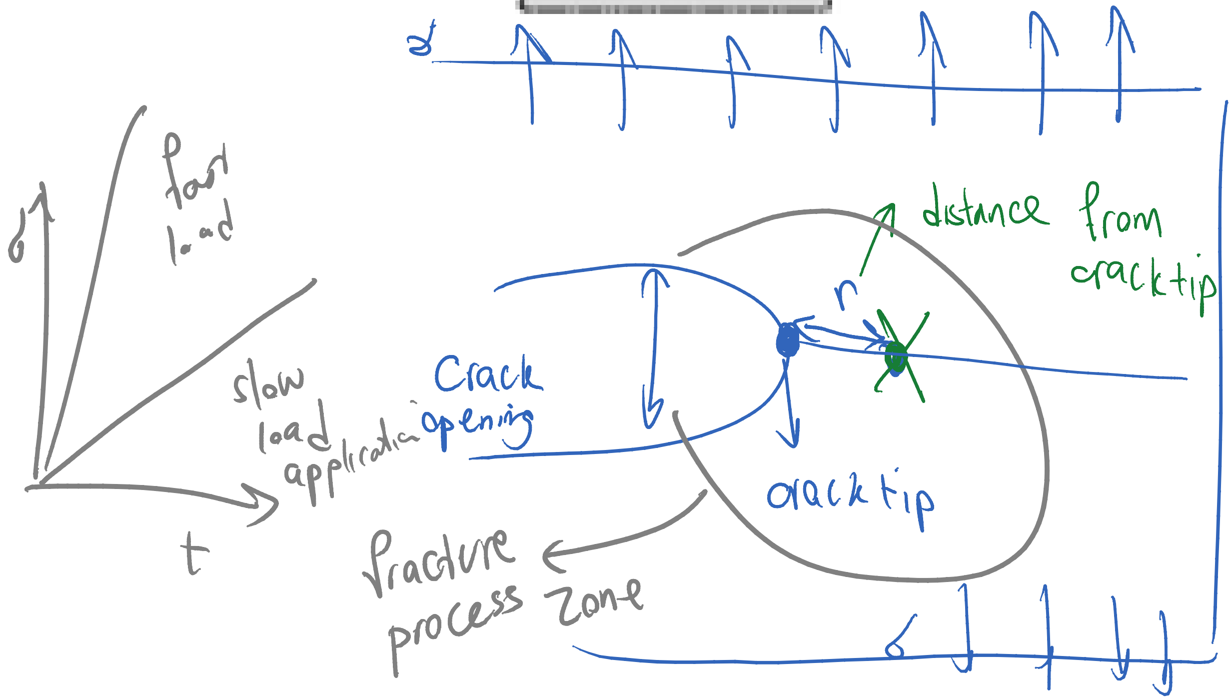
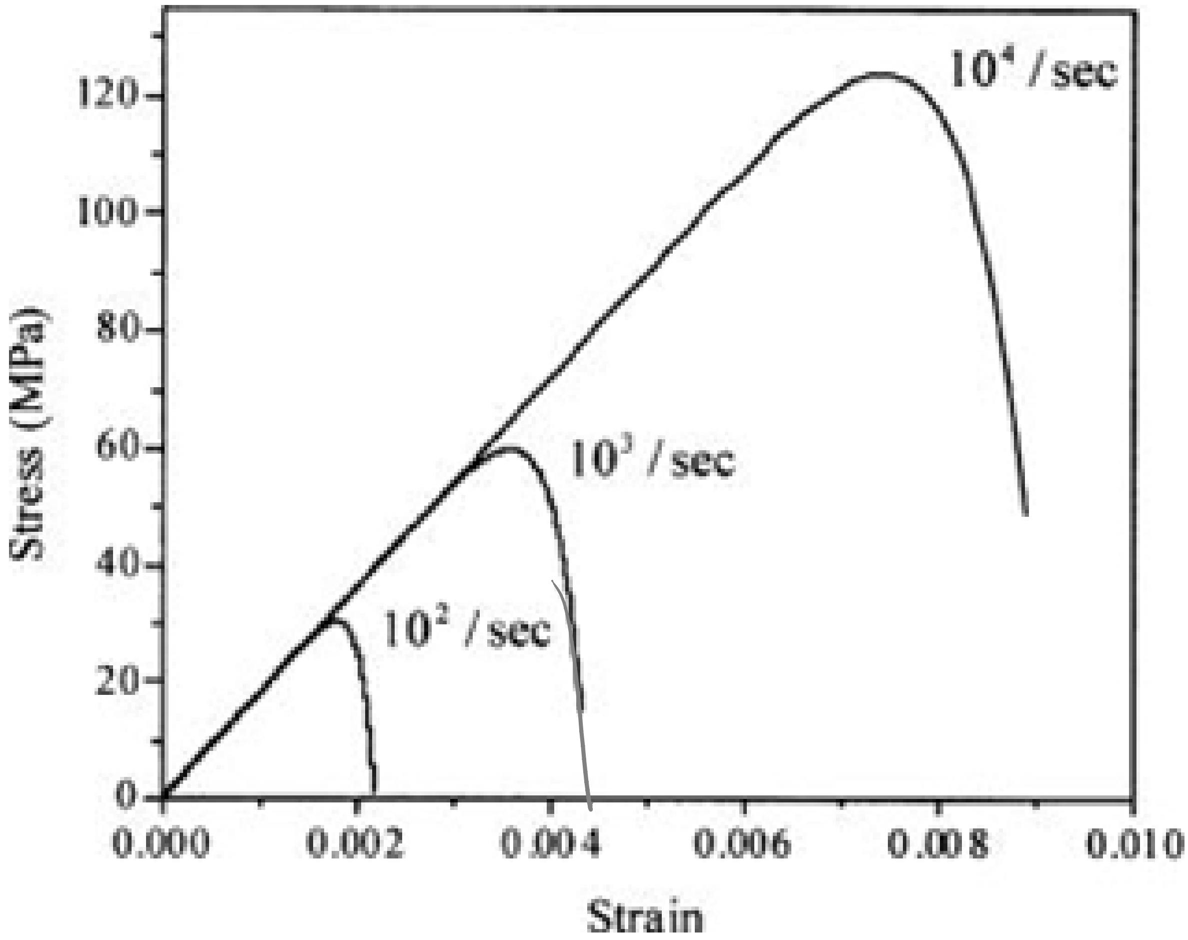
2016/08/23

Tuesday, August 23, 2016
8:40 AM



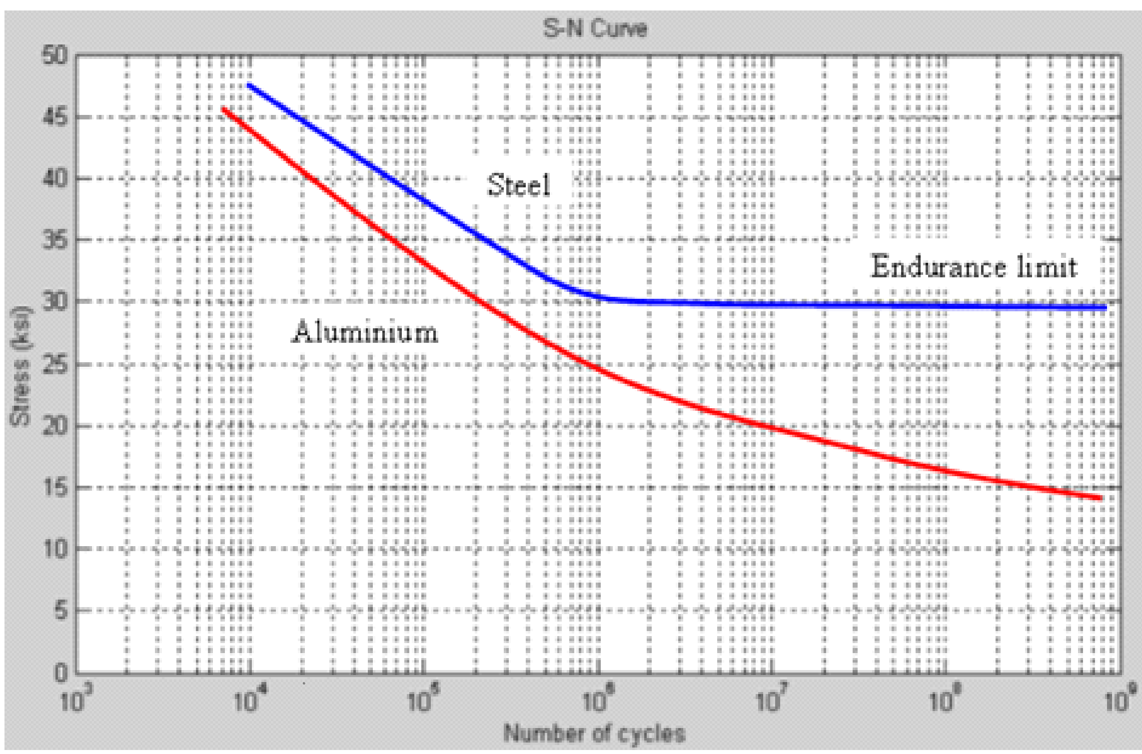


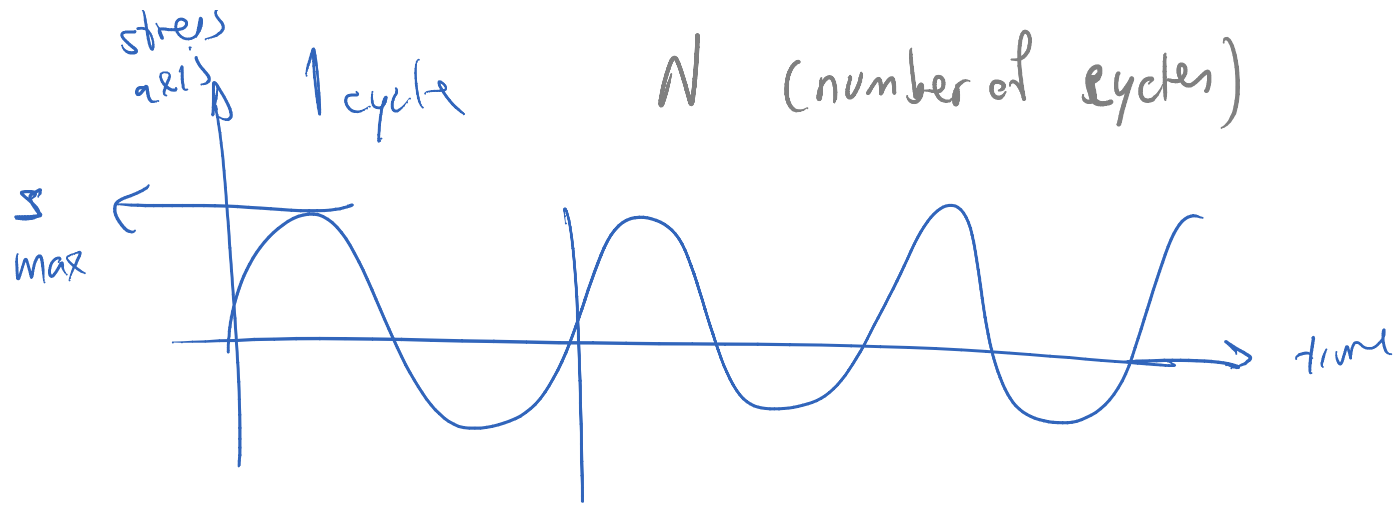
Dynamic fracture
Rate effect



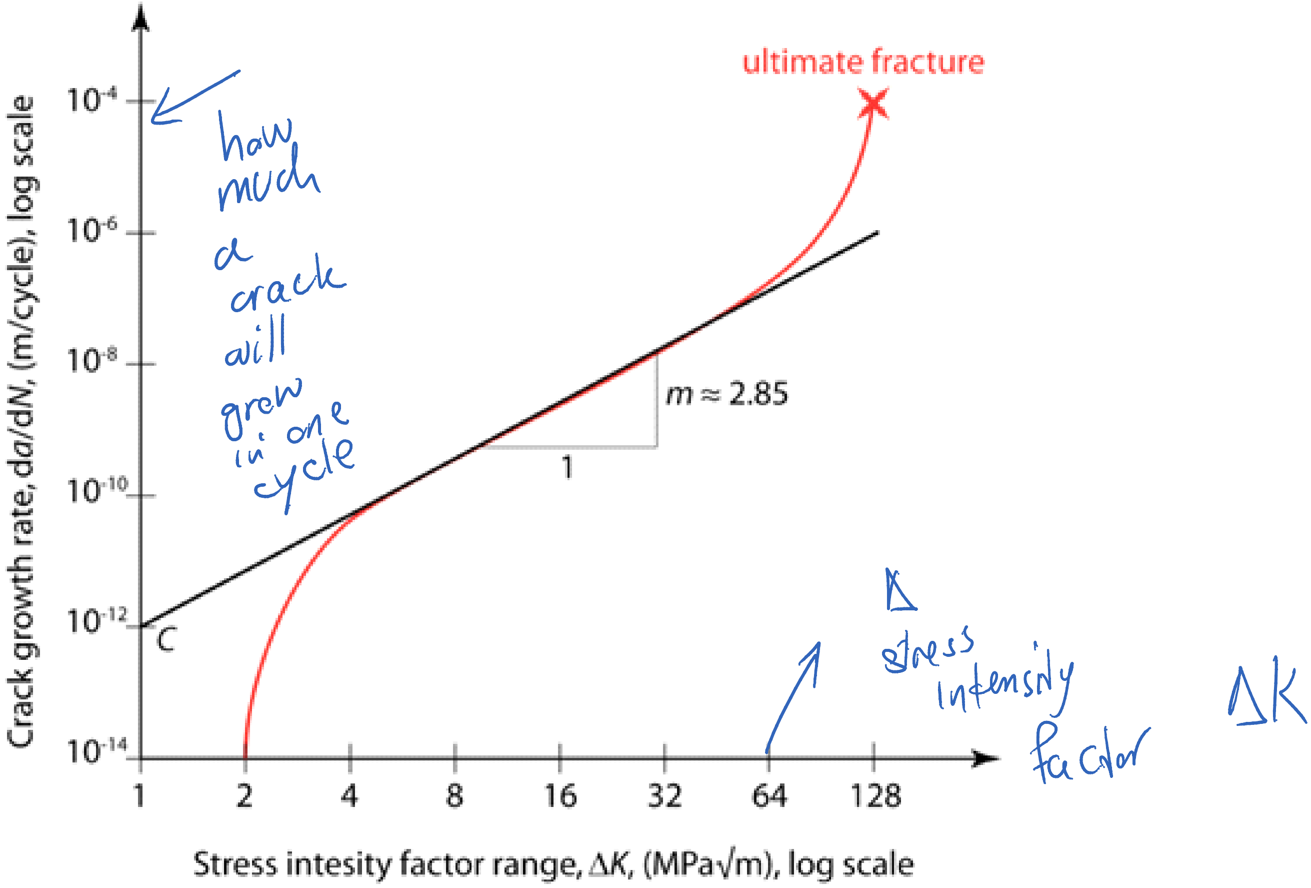
Last section of the course:
Fatigue

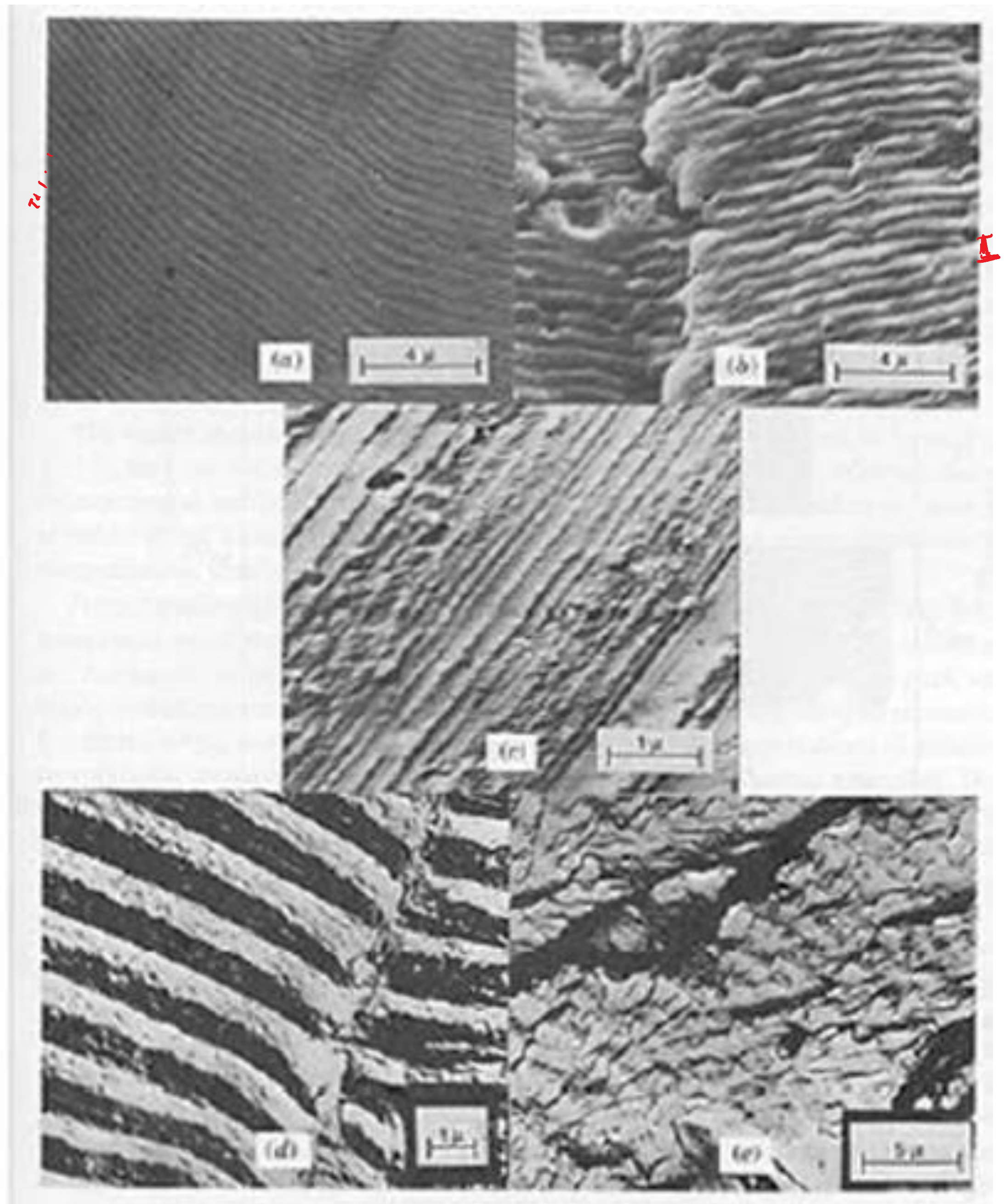
S
stress





Newer approach is Paris law





Front of the crack

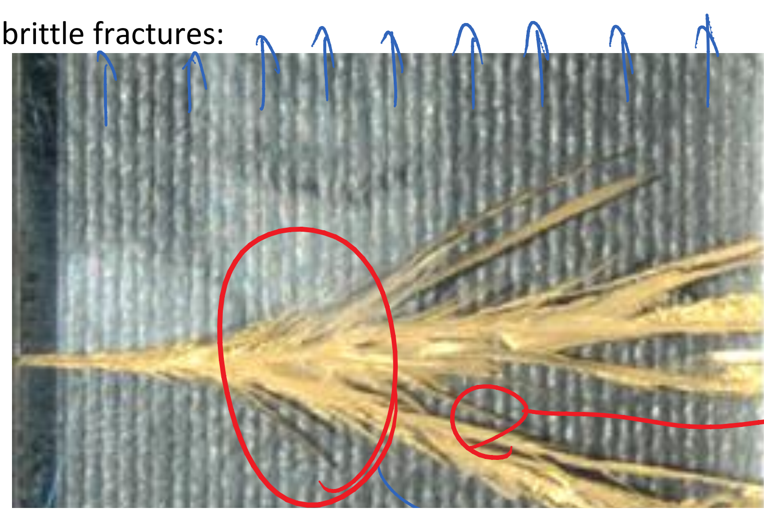
$N=0$

$N=10^3$

crack

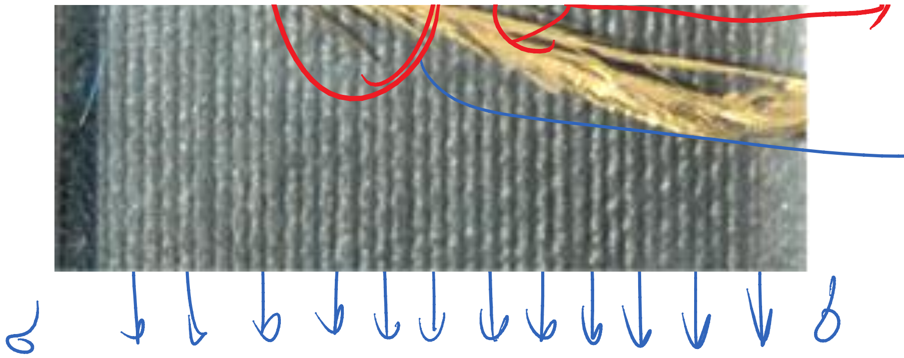
$N=10^6$

Typical brittle fractures:



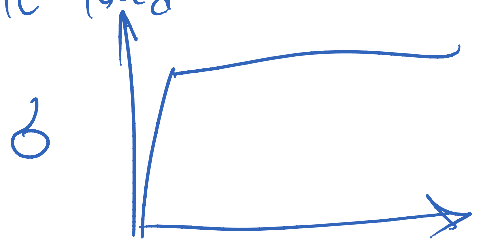
microcracks

macrocracking



macrocracking (bifurcation)

Dynamic load

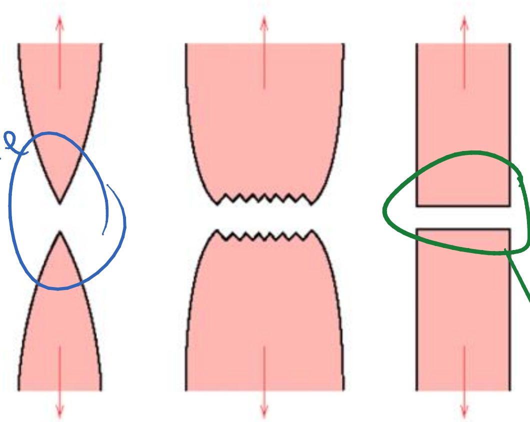


Difference between ductile and brittle fracture

Brittle vs. Ductile Fracture

quasi-static loading

ductile

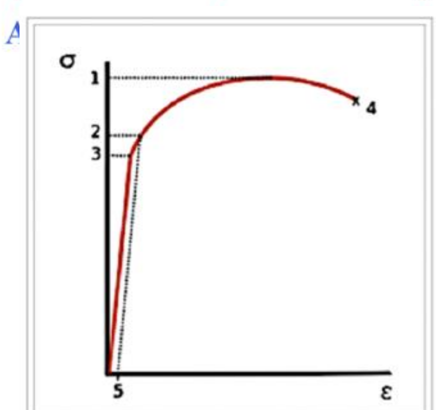


A. **Very ductile**, soft metals (e.g. Pb, Au) at room temperature, other metals, polymers, glasses at high temperature.

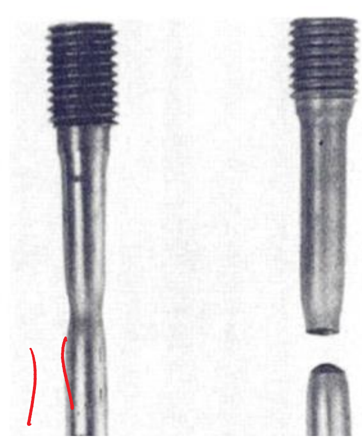
B. **Moderately ductile fracture**, typical for ductile metals

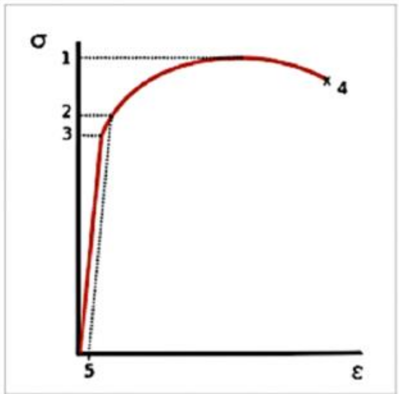
C. **Brittle fracture,** cold metals, ceramics.

brittle

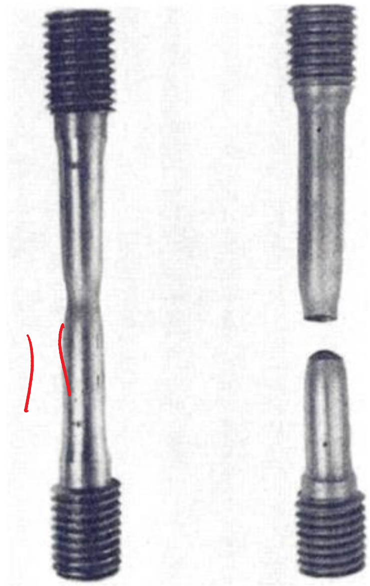


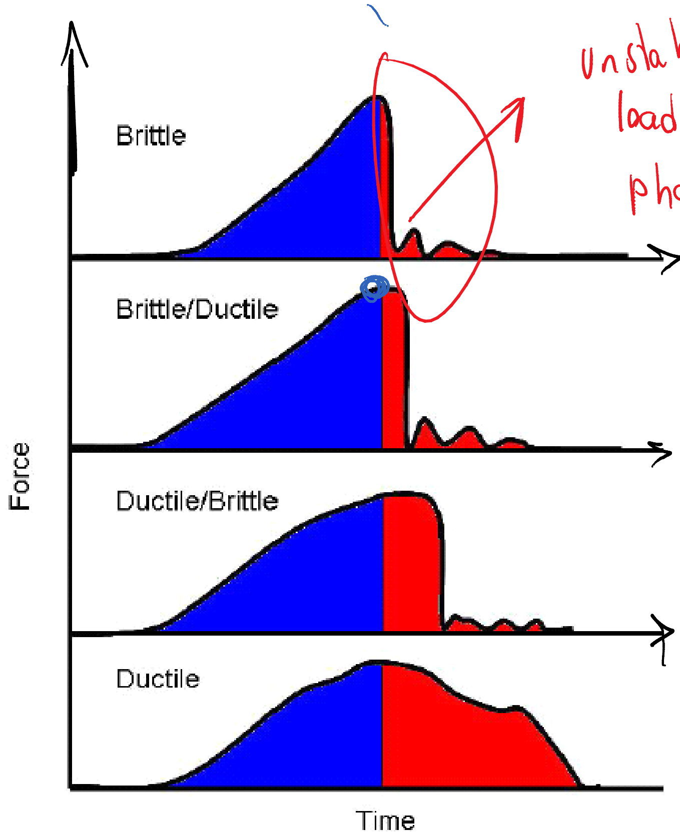
Stress vs. Strain curve typical





Stress vs. Strain curve typical of aluminum
1. Ultimate strength
2. Yield strength
3. Proportional limit stress
4. Fracture
5. Offset strain (typically 0.2%)





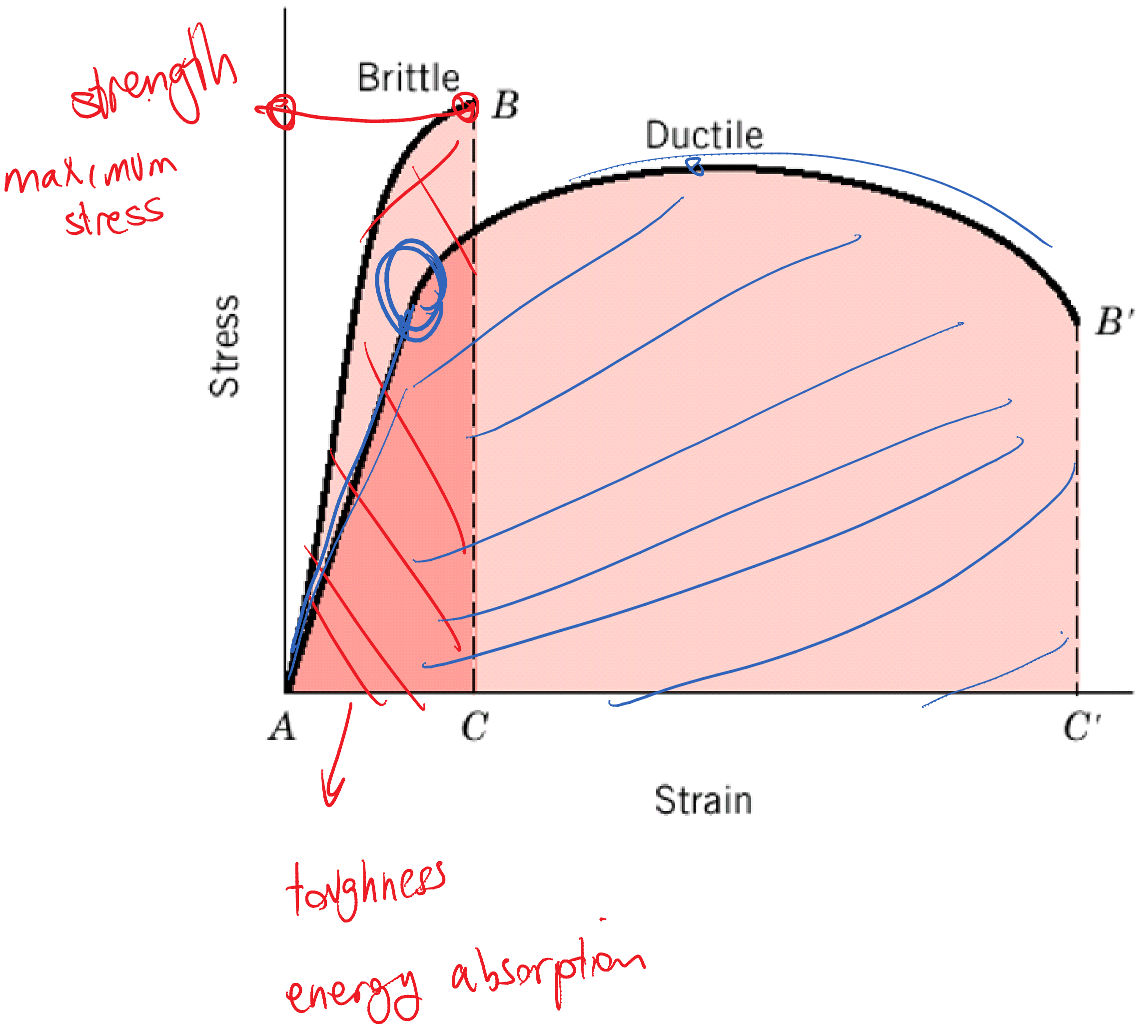
unstable load decreasing phase

Ductile vs. brittle

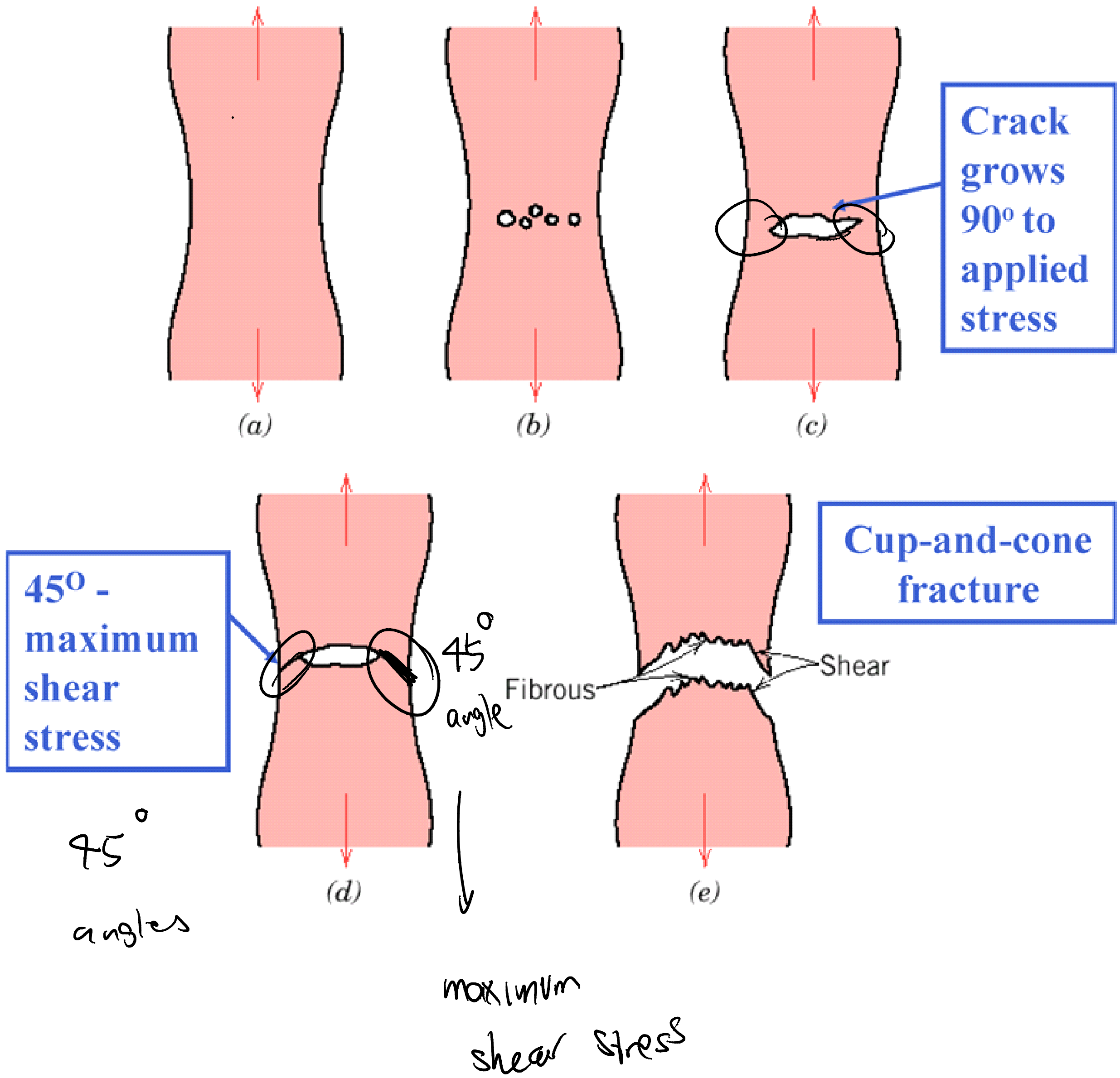
energy absorption properties

Brittle vs. Ductile Fracture

- **Ductile materials** - extensive plastic deformation and energy absorption ("toughness") before fracture
- **Brittle materials** - little plastic deformation and low energy absorption before fracture

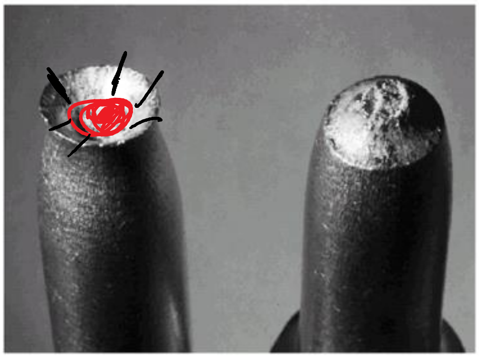


Ductile Fracture (Dislocation Mediated)

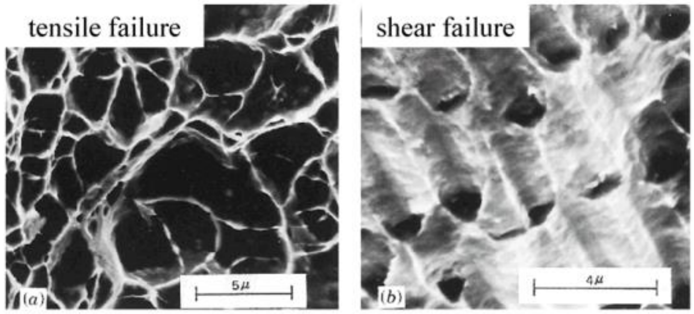


Ductile fracture

Ductile Fracture



(Cup-and-cone fracture in Al)

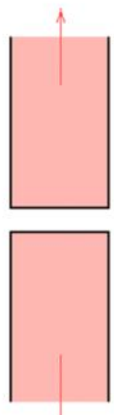


Scanning Electron Microscopy: *Fractographic* studies at high resolution. Spherical "dimples" correspond to microvoids that initiate crack formation.

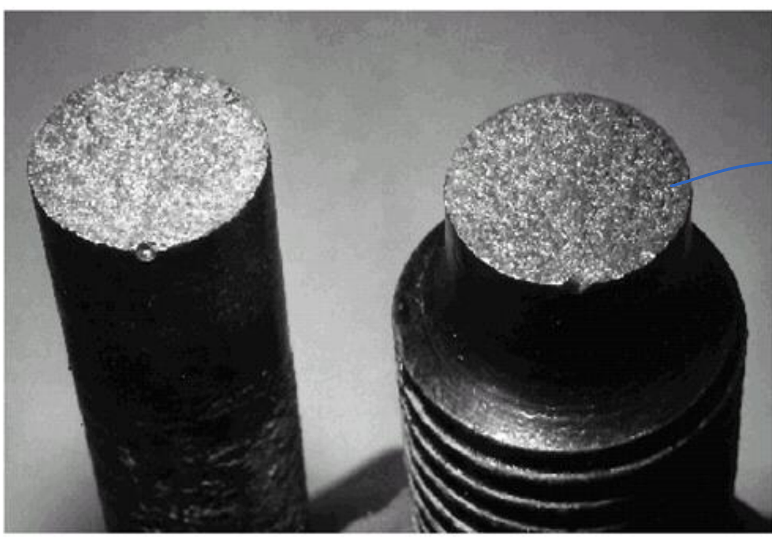
Brittle fracture:

Brittle Fracture (Limited Dislocation Mobility)

- No appreciable plastic deformation
- Crack propagation is very fast
- Crack propagates nearly perpendicular to the direction of the applied stress
- Crack often propagates by cleavage - breaking of atomic bonds along specific crystallographic planes (cleavage planes).



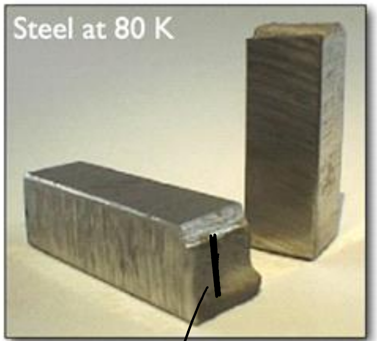
maximum tensile stress



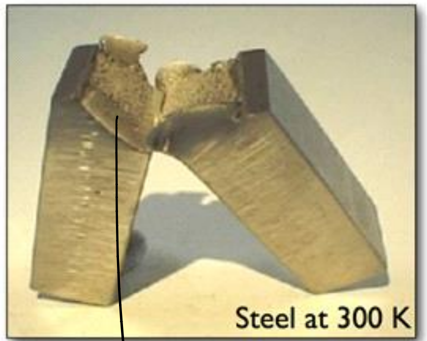
Brittle fracture in a mild steel

experiment on brittle fracture

Brittle Fracture

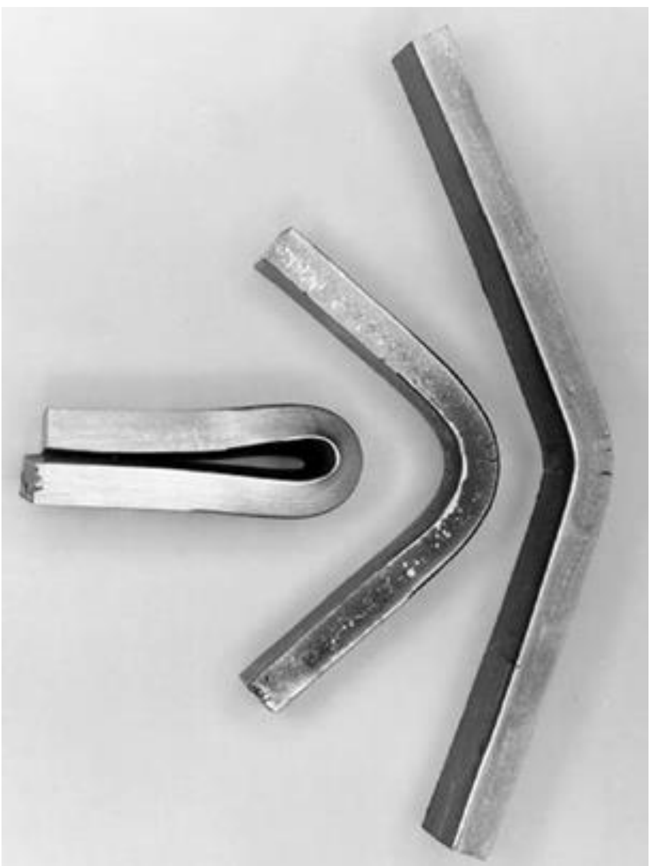


Ductile Fracture



smooth surface

a large amount of plastic deformation befor final failure



Because of all energy absorption mechanisms ductile materials are much more resistant to (manufacturing) defects and have much less scatter in their 1) ultimate load 2) toughness (energy absorption) much more resistant to (manufacturing) defects and have much less scatter in their 1) ultimate load 2) toughness (energy absorption)

Background information

- Indicial notation

$$a_i \geq 0 \qquad i = 1,2,3$$

all the components are positive

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

stress tensor

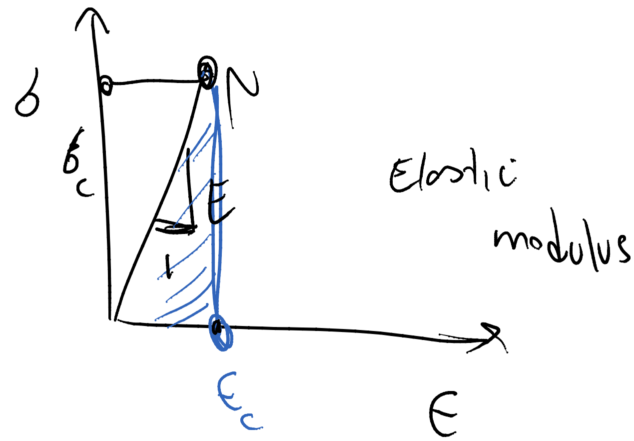
$$\sigma_{ii} = \sum_{i=1}^{3} \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

repeated index

$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

$$\epsilon = \begin{bmatrix} \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

1D case



what is this area?

$$U = \int_0^{\epsilon_c} \sigma \, d\varepsilon = \text{energy density per unit volume}$$

for linear elastic material $\quad U = \frac{1}{2} \sigma \varepsilon$

what is energy density in 2D & 3D?

$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \qquad \sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

loading

Initial state $\epsilon = 0 \longrightarrow \epsilon \neq 0$

How much energy is absorbed per unit volume?

$$U = \frac{1}{2}\,\epsilon : \sigma = \frac{1}{2}\big(\epsilon_{11}\sigma_{11} + \epsilon_{12}\sigma_{12} + \epsilon_{13}\sigma_{13} + \epsilon_{21}\sigma_{21} + \epsilon_{22}\sigma_{22} + \epsilon_{23}\sigma_{23} + \epsilon_{31}\sigma_{31} + \epsilon_{32}\sigma_{32} + \epsilon_{33}\sigma_{33}\big)$$

$$= \frac{1}{2}\big(\epsilon_{ij}\,\sigma_{ij}\big)$$

repeated

repeated

$$= \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\epsilon_{ij}\sigma_{ij}$$

Voigt notation

a more concise way to express stress & strain

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \qquad \epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

$$\begin{bmatrix} \cdots \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \qquad \begin{bmatrix} \cdots \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

symmetric

$\sigma_{12} = \sigma_{21} \cdots$

has 6 independent values

Voigt notation

$$S = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{normal components} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{shear stresses} \end{matrix}$$

Voigt strain

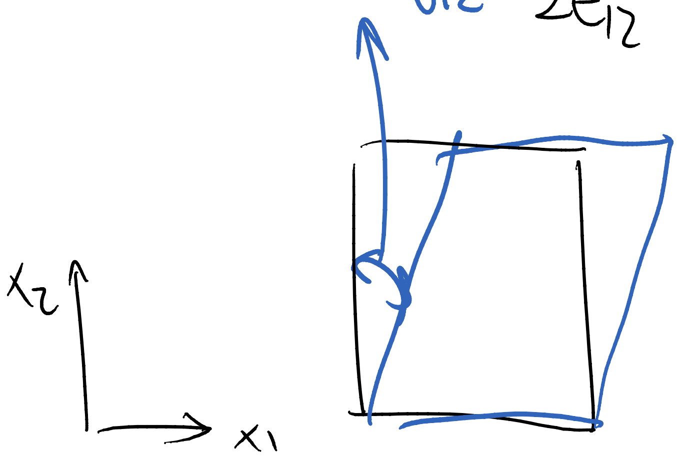
$$\gamma = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \hline 2\epsilon_{12} \\ 2\epsilon_{23} \\ 2\epsilon_{31} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{normal components} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{Engineering shear strain} \end{matrix}$$

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\gamma_{12} = 2\epsilon_{12} = 2 \times \frac{1}{2}(u_{1,2} + u_{2,1})$$

$$= u_{1,2} + u_{2,1}$$

$\gamma_{12} = 2\epsilon_{12}$



$$\sigma_{3\times3} = \underbrace{C_{3\times3\times3\times3}}_{} \, \epsilon_{3\times3}$$

$\downarrow$ 4^{th} order Elasticity tensor

How is this done with Voigt's notation ?

$$S: \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix}_{6\times1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{6\times6} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \hline 2\epsilon_{12} \\ 2\epsilon_{23} \\ 2\epsilon_{31} \end{bmatrix}_{6\times1}$$

$$\mathcal{C}_{6\times6}$$

$\downarrow$ Voigt elasticity matrix

$$u = \frac{1}{2}\,\sigma : \epsilon$$

$$u = \frac{1}{2}\,\epsilon : \sigma = \frac{1}{2}\big(\epsilon_{11}\sigma_{11} + \epsilon_{12}\sigma_{12} + \epsilon_{13}\sigma_{13} + \epsilon_{21}\sigma_{21} + \epsilon_{22}\sigma_{22} + \epsilon_{23}\sigma_{23} + \epsilon_{31}\sigma_{31} + \epsilon_{32}\sigma_{32} + \epsilon_{33}\sigma_{33}\big)$$

$\epsilon_{12} = \epsilon_{21}$

$\sigma_{12} = \sigma_{21}$

$\epsilon_{12} = \epsilon_{21}$

$\sigma_{12} = \sigma_{21}$

$$= \frac{1}{2}\Big(\epsilon_{11}\sigma_{11} + \epsilon_{22}\sigma_{22} + \epsilon_{33}\sigma_{33} + (2\epsilon_{12})\sigma_{12} + (2\epsilon_{23})\sigma_{23} + (2\epsilon_{31})\sigma_{31}\Big)$$

$$= \frac{1}{2}\underbrace{\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix}}_{S} \cdot \underbrace{\begin{bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{33}\\ 2\epsilon_{12}\\ 2\epsilon_{23}\\ 2\epsilon_{31}\end{bmatrix}}_{\gamma}$$

Voigt stress

& strain

$$\boxed{U = \frac{1}{2}\sigma : \epsilon = \frac{1}{2} S \cdot \gamma}$$

Material classification / Tensile test

$\sigma = C\epsilon$