2016/08/30

Tuesday, August 30, 2016
8:40 AM

Reasons for ductile to brittle transition:

6. Size effect

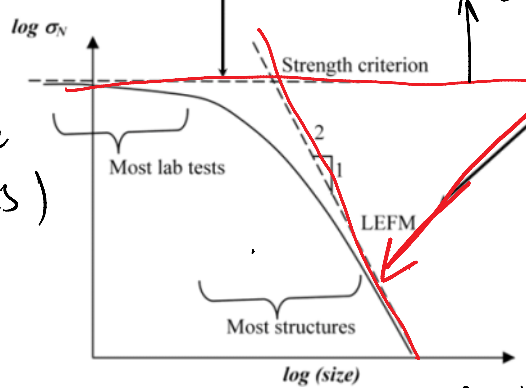
Size effect

$$\sigma_N = f_t = f_t(D)^0$$

by yielding sect.

$$\sigma_N = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{K_{Ic}}{\sqrt{\pi c_N D}}$$

fracture criterion



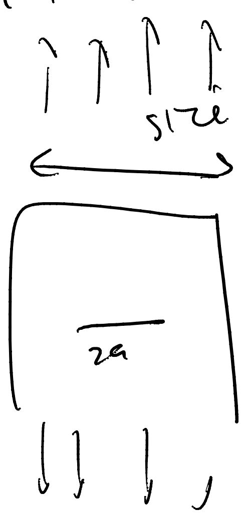
Larger size ⇨ More brittle

log (ultimate stress) = log(σ_u)

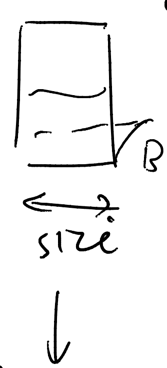
log (size of the domain)

σ ← applied stress

size



P



$\sigma_u = \frac{P}{(\text{size}) B}$ when specimen breaks

crack length

$K \propto \sqrt{a}\ \sigma$

stress intensity factor

crack can propagate when

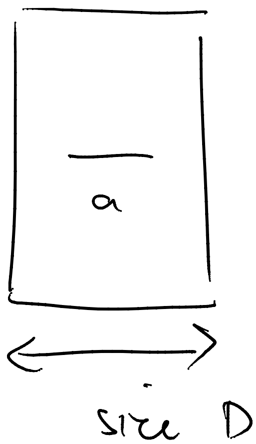
$K = K_c$ ← fracture toughness

ultimate stress

$K_c \propto \sqrt{a}\ \sigma_c \Rightarrow \sigma \propto \frac{K_c}{}$

$$K_c \propto \sqrt{a}\,\sigma_c \Rightarrow \sigma_c \propto \frac{K_c}{\sqrt{a}}$$

$$\sigma_c \propto \frac{K_c}{\sqrt{a}}$$ $\sqrt{a} \rightarrow$ crack length



$a \propto D$

reasonable assumption

"larger domains have larger cracks"

$\Rightarrow$

ultimate stress $\leftarrow$ $\boxed{\sigma_c \propto \frac{K_c}{\sqrt{D}}}$ $\sqrt{D} \rightarrow$ domain size

Examples of size effect equations
From Bazant

Bazant's size effect law

$$(\sigma_N)_u = Af_t\left(1+\frac{W}{B}\right)^{-1/2} \tag{14.8}$$

where

$(\sigma_N)_u$ = Nominal stress at failure of a structure of specific shape and loading condition.

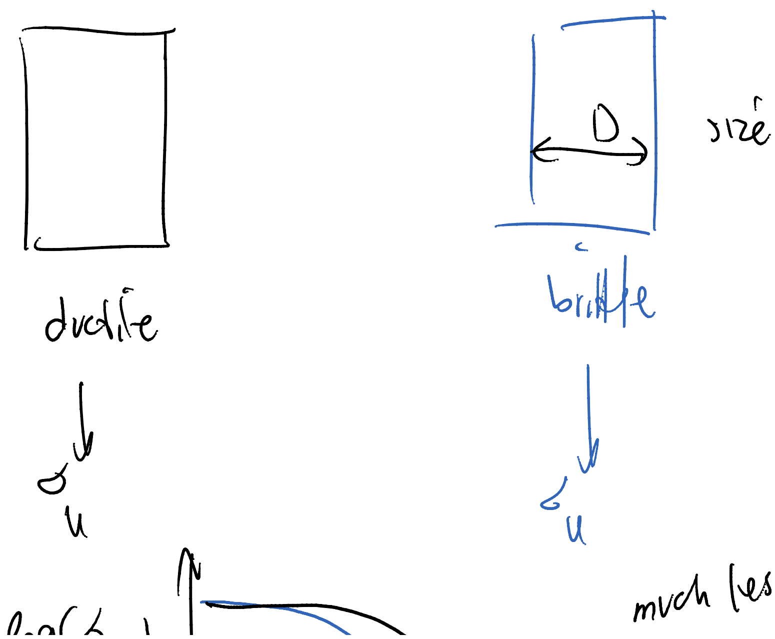
W = Characteristic length of the structure.

A, B = Positive constants that depend on the fracture properties of the material and on the shape of the structure, but not on the size of the structure.

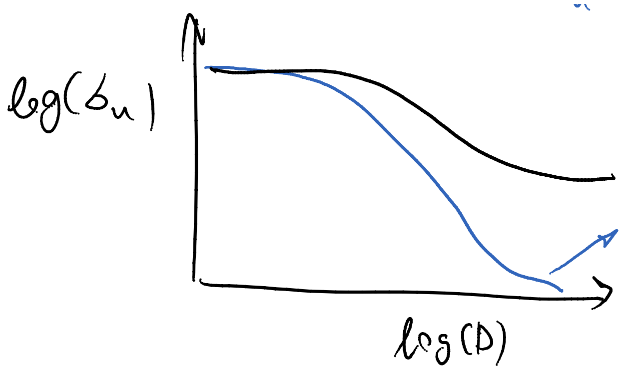
f_t = Tensile strength of the material introduced for dimensional purposes.

Conclusions about size effect:

1. Any structure (intrinsically ductile or brittle) becomes more brittle (sensitive to defects) as its size increases. It means that its ultimate strength decreases as its size increases AND its toughness (energy absorption per unit volume) decreases as the size increases
2. Size effect is more important for brittle materials



much less size effect for ductile material

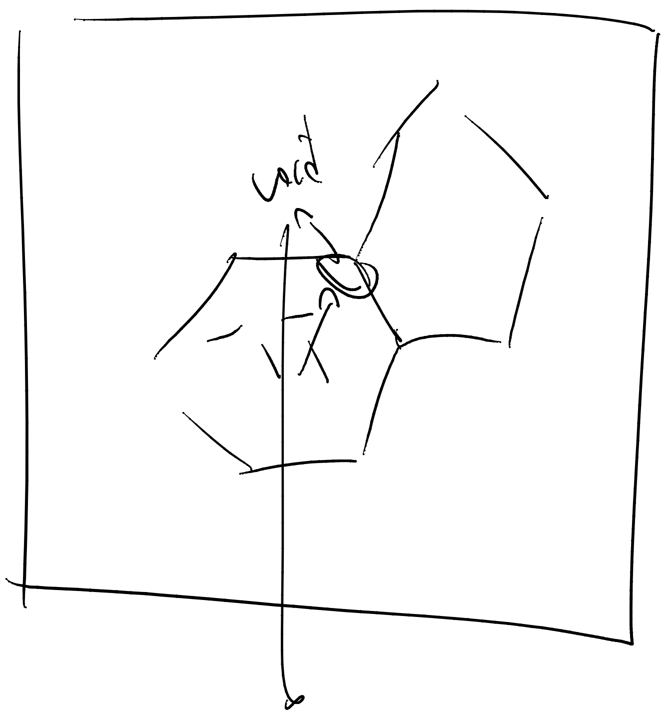


much less size for ductile material

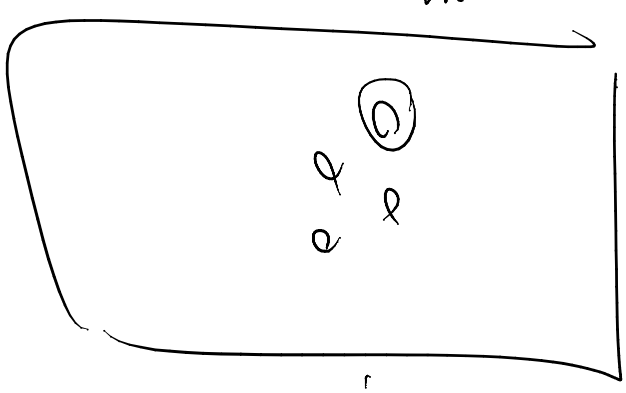
brittle material

log (size)

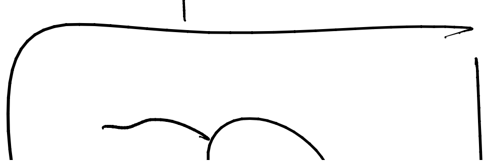
ductile material

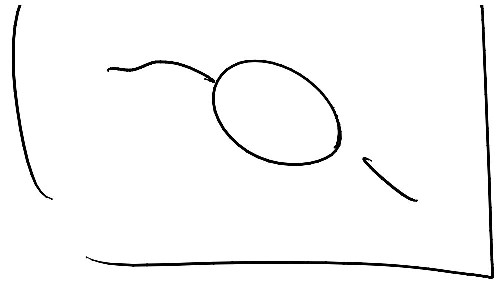


micro voids



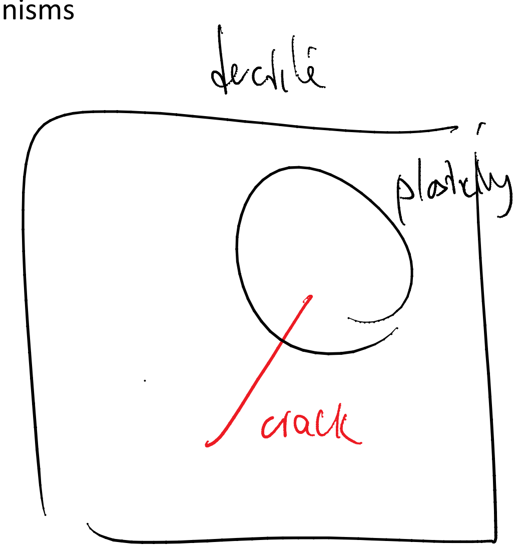
plastically



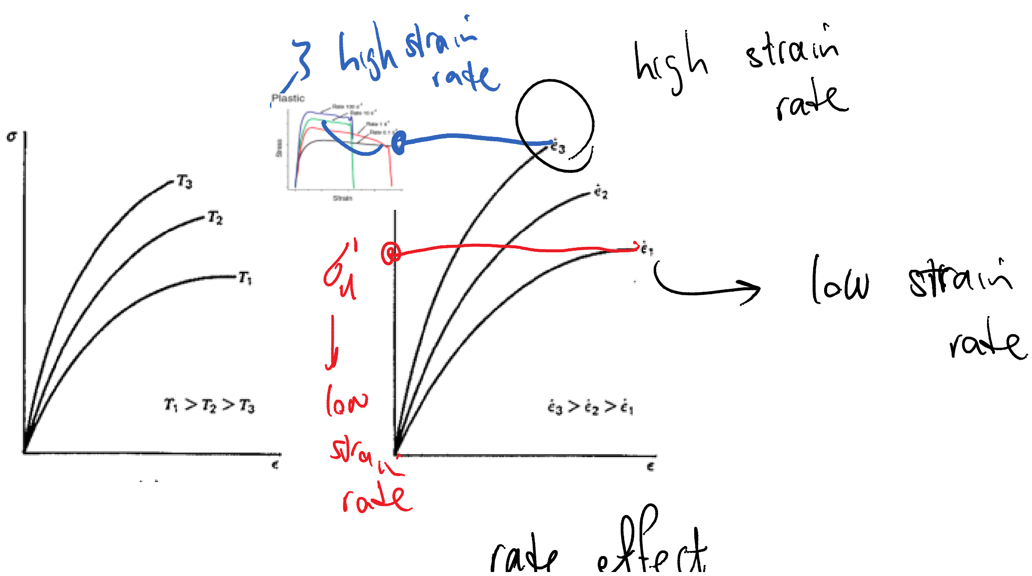


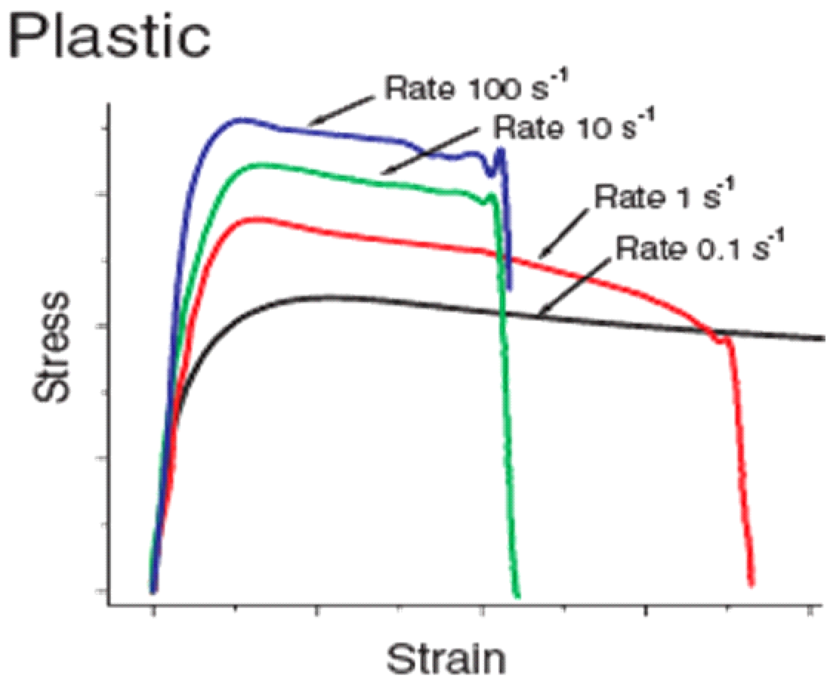
All energy dissipative mechanisms (from dislocation motion, void formation, and plastic deformation) make the stress field more uniform and shield defects ->

A critical defect may not result in catastrophic crack propagation due to all energy dissipative mechanisms



7. Rate effects on ductility

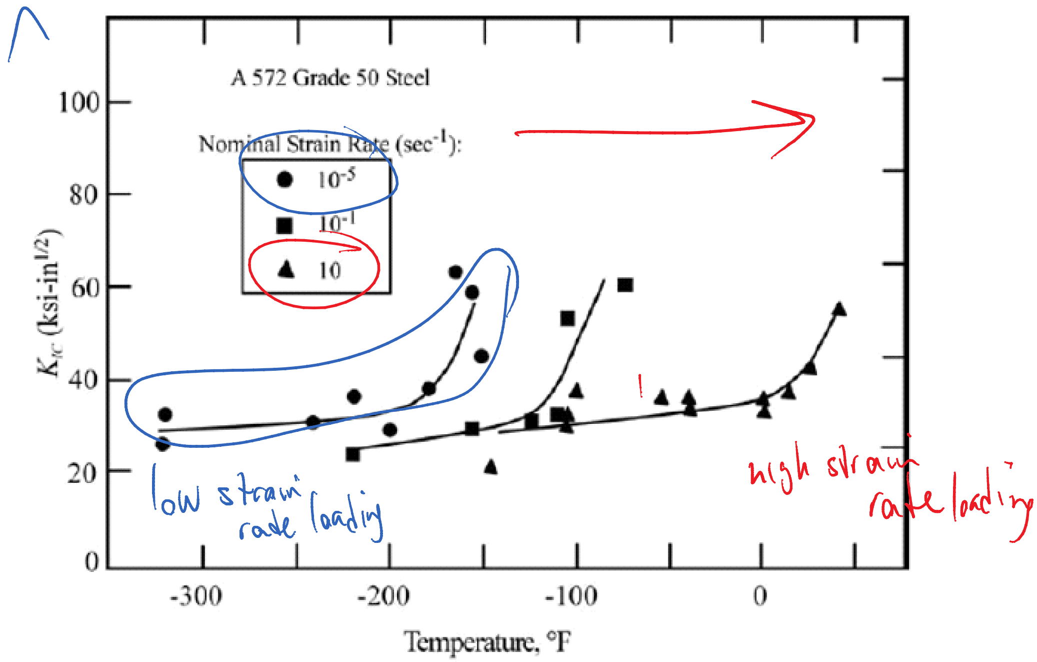




Higher loading rates -> inhibit dislocation motion -> less plasticity and higher ultimate strength

Strain rate effects on Impact toughness

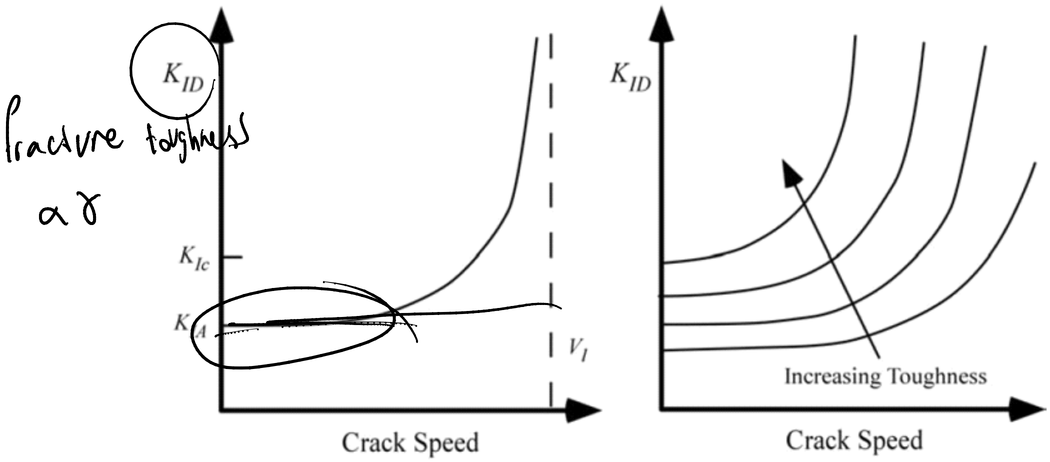
Strain rate ↗ ⇨
DBTT ↗ (more brittle in impact)



Ki Computed using quasistatic relations (Anderson p.178)

High strain rate loading & higher crack speeds may have opposing effects in terms of strengthening / thoughening of the material

Crack speed effect on dynamic crack propagation resistance



(a) (b)

- Strain speed ↗
- K_{ID} ↗ (Insensitive at low speeds, quick increase approaching V_l)
- Increasing toughness makes K_{ID} more sensitive and grow faster

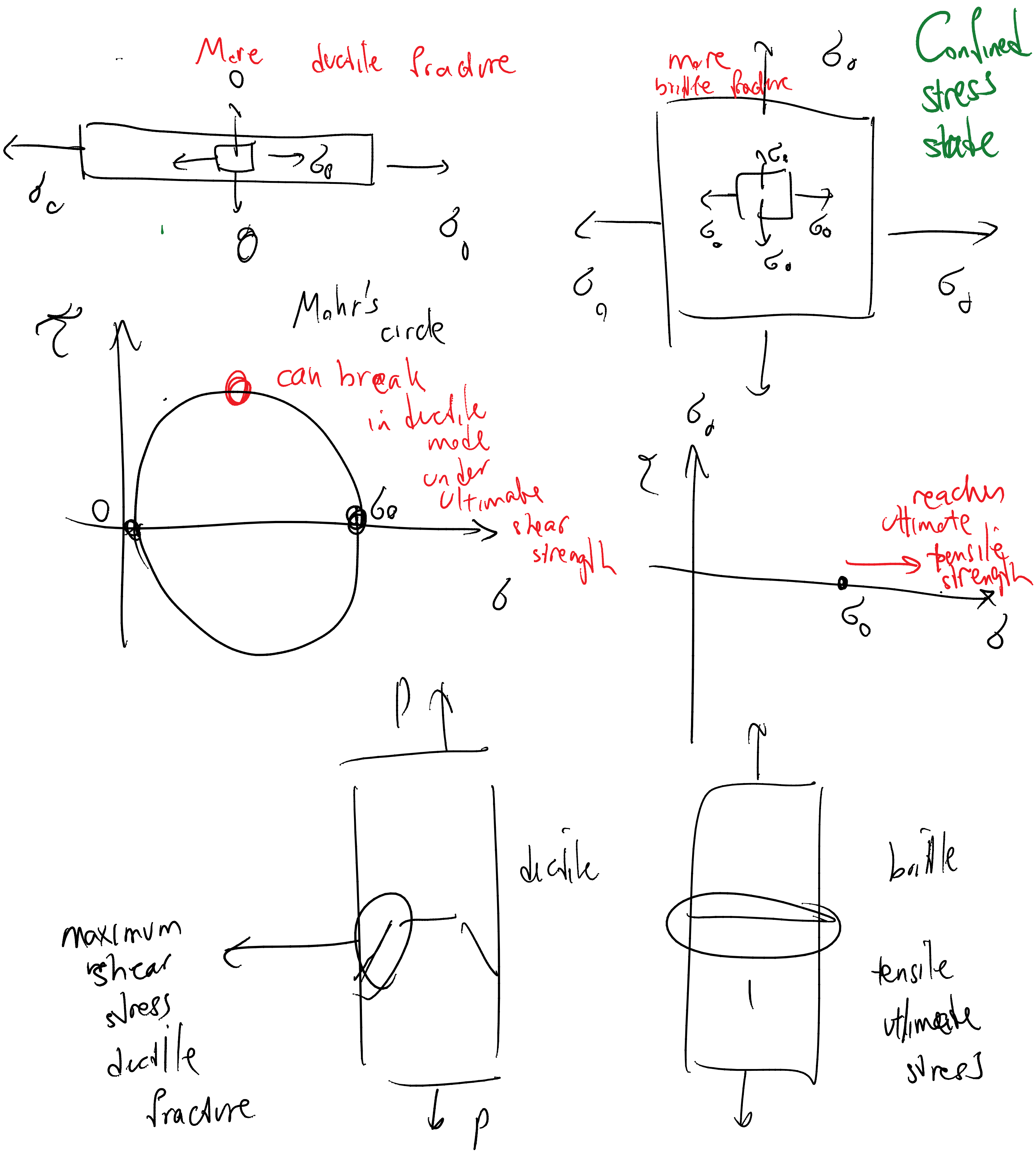
2γ energy needed per unit surface of crack

As the crack accelerates it takes much higher energy to crack a unit surface of crack

Dynamic effects:

1. Higher strain rate loading -> higher strength material
2. Higher strain rate loading -> More brittle response (lower toughness)
3. As a crack accelerates it takes higher energy to create unit surface of crack (higher speed cracks result in higher toughness)

8. Triaxial stress and confinement



Specimens with higher triaxial stress state (almost the same stress applied from all directions) - also called confined stress state - exhibit more brittle fracture

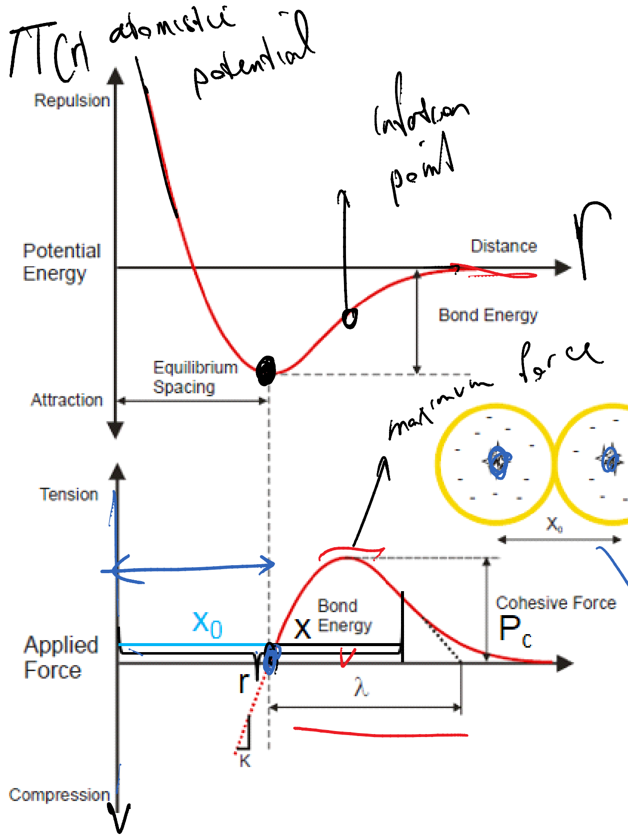
4. Linear Elastic Fracture Mechanics (LEFM)

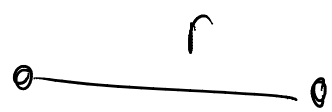
4.1Griffith energy approach

Goal:
Want to compute ultimate strength from atomistic model and show that ultimate strength should be very close to elastic modulus

In reality ultimate strength is much smaller than elastic modulus

Next we describe what the cause for this is



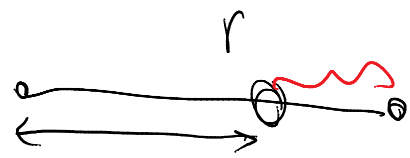


distance between two atoms

$$\mathbf{P} = -\frac{\partial \Pi_a}{\partial \mathbf{r}} \quad \Rightarrow |P| = \left|\frac{\partial \Pi_a}{\partial r}\right| \text{ (1D)}$$

equilibrium distance "when atoms exert no mutual force"

- P: Force between atoms (tensile positive cancels - sign)
- Position r: Distance from other atom
- x_0: Equilibrium position, $P = \frac{\partial \Pi_a}{\partial r} = 0$
- Displacement $x = r - x_0$.
- λ: Length scale where atomistic force is too small.
- P_c: Max force at $\frac{\partial^2 \Pi_a}{\partial r^2} = 0$.

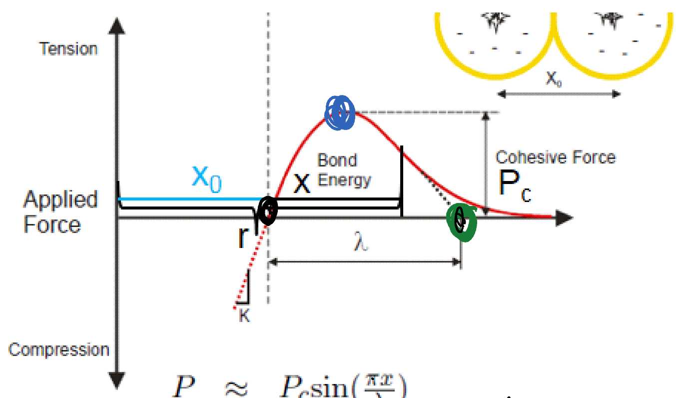


x_0
equilibrium
distance

force between atoms

displacement $x = r - x_0$

$x = 0 \Rightarrow P = 0$



$$P \approx P_c \sin\left(\frac{\pi x}{\lambda}\right)$$

$x = r - x_0$

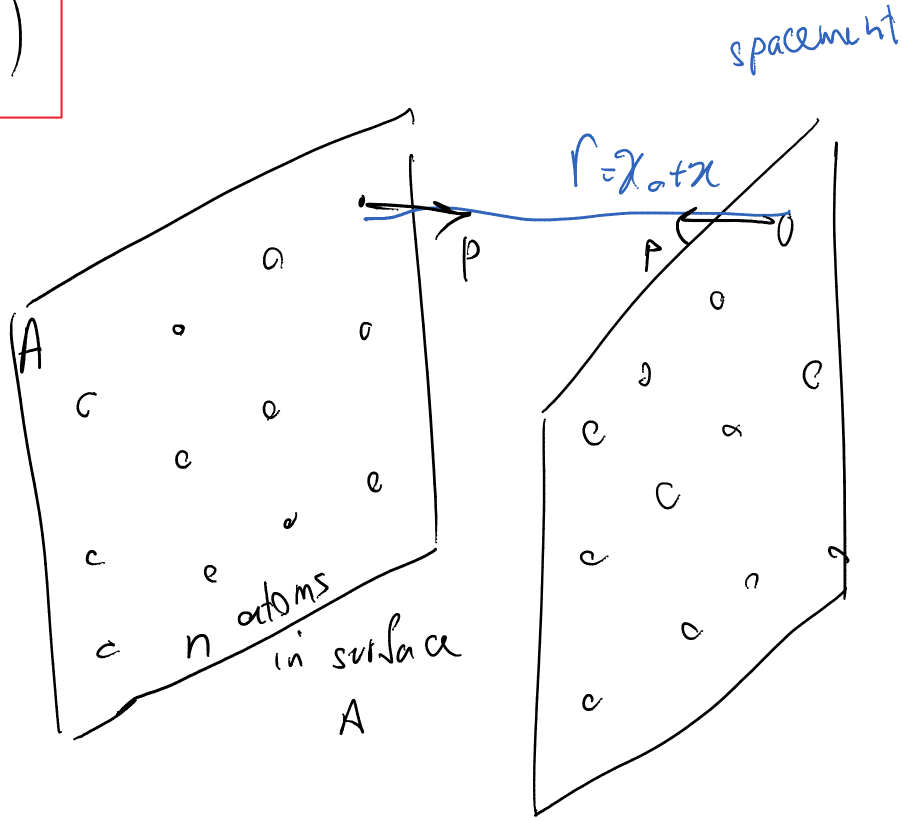
we approximate P vs x curve by a sine wave

$$P(x=0) = P(r = x_0) = 0 \qquad P_c \sin\left(\frac{\pi}{\lambda} \cdot 0\right) = 0$$

$$P\left(x = \frac{\lambda}{2}\right) = P_c \sin\left(\frac{\pi}{\lambda} \frac{\lambda}{2}\right) = P_c \sin\left(\frac{\pi}{2}\right) = P_c$$

$$P(x = \lambda) = P_c \sin\left(\frac{\pi}{\lambda} \cdot \lambda\right) = P_c \sin(\pi) = 0$$

$$\boxed{P = P_c \sin\left(\frac{\pi x}{c}\right)}$$



$$\sigma = \frac{\sum \text{forces}}{\text{Area}}$$

$$= \frac{n \cdot P}{A} =$$

$$\underbrace{\left(\frac{n}{A}\right)} P$$

density of atoms per surface area

$$\sigma = \frac{n}{A} P(x)$$

$$\boxed{\sigma = \frac{n}{A} P_c \sin\left(\frac{\pi x}{\lambda}\right)}$$

$$\left.\begin{array}{l}\text{Atomistic:}\\ x_0\\ \lambda\\ P_c\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\text{Continuum:}\\ E\\ \gamma_c(\text{surface energy})\end{array}\right.$$

σ_c

new length $= x + x_c$

$$\text{strain} = \frac{\text{change of length}}{\text{original length}} = \frac{(x_0 + x) - x_0}{x_0} = \frac{x}{x_0}$$

$\downarrow x_0$

$$\boxed{\epsilon = \frac{x}{x_0} \qquad \sigma = \underbrace{\frac{n}{A} P_c}_{\sigma_c} \sin\left(\frac{\pi x}{\lambda}\right)}$$

σ_c = maximum stress

$= \frac{n}{A} P_c$

$$\sigma = \sigma_c \sin\left(\frac{\pi x}{\lambda}\right), \quad \sigma_c = \frac{n}{A} P_c$$

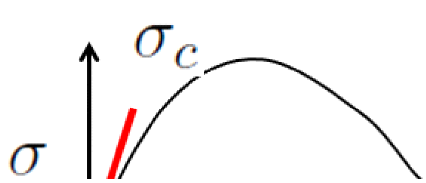
$$\epsilon = \frac{x}{x_0} \Rightarrow x = \epsilon x_0 \quad \Rightarrow$$

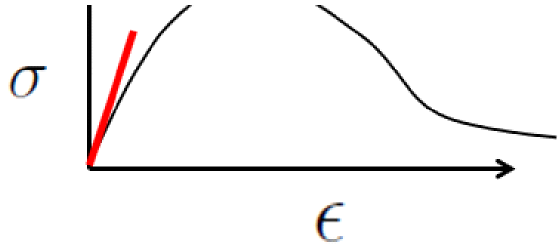
$$\boxed{\sigma(\epsilon) = \sigma_c \sin\left(\frac{\pi x_0}{\lambda} \epsilon\right)}$$

Is it linear

$\sigma = E \epsilon$

No!





No!

How about when ϵ is small?

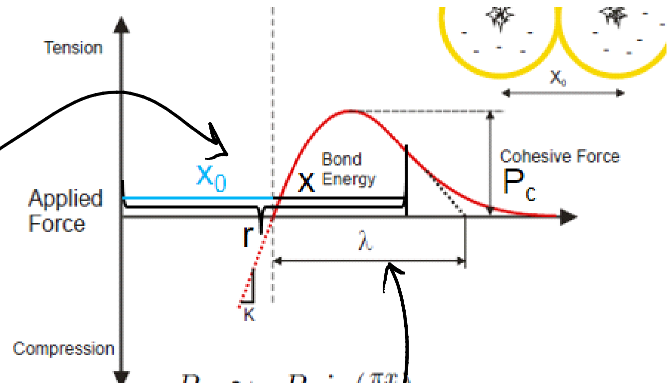
$$E = \frac{\partial \sigma}{\partial \epsilon}\bigg|_{\epsilon=0} = \frac{\partial\, \sigma_c \sin\left(\frac{\pi x_c}{\lambda}\epsilon\right)}{\partial \epsilon}\bigg|_{\epsilon=0}$$

↳ Elastic modulus

$$E = \frac{\sigma_c \pi x_0}{\lambda} \Rightarrow$$

$\approx$

$$\boxed{\sigma_c = \frac{E}{\pi}\frac{\lambda}{x_0} \approx \frac{E}{\pi}}$$



$$\boxed{\sigma_c \approx \frac{E}{\pi}}$$

This is not realistic! For steel $\sigma_c \approx 250\text{MPa}$, $E = 200\text{GPa}$