2016/10/04

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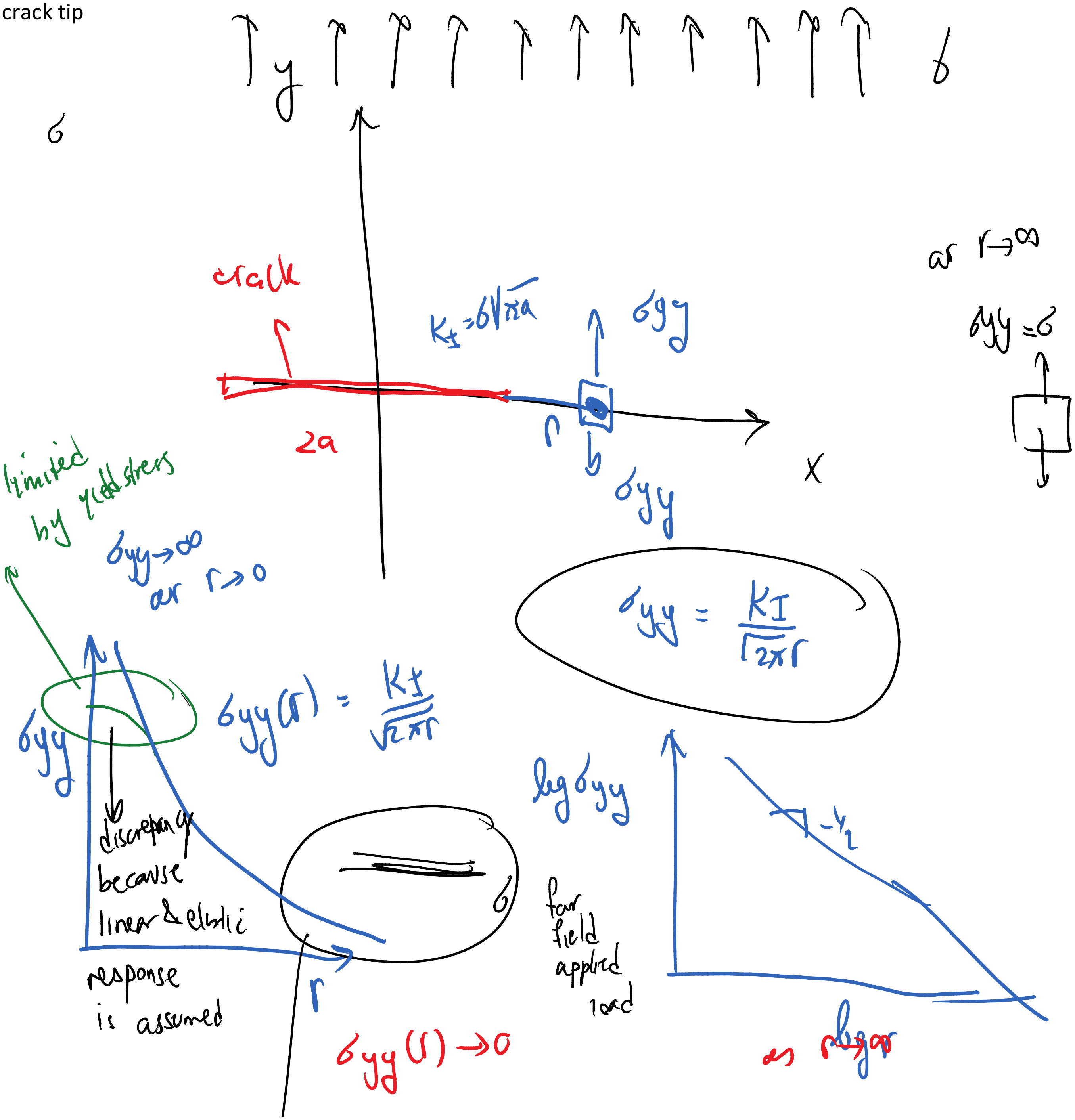
5. Elastoplastic fracture mechanics

LEFM: Linear Elastic (Elastodynamic - for dynamic loading) Fracture Mechanics

PFM: Plastic Fracture Mechanics: Models material yielding around the crack tip



$\sigma_{yy}(0) \to \infty$ as [illegible]

discrepancy here is because

$\sigma_{yy}(r, \theta=0) = \frac{K_I}{\sqrt{2\pi r}}$ is then first term in σ_{yy} expansion

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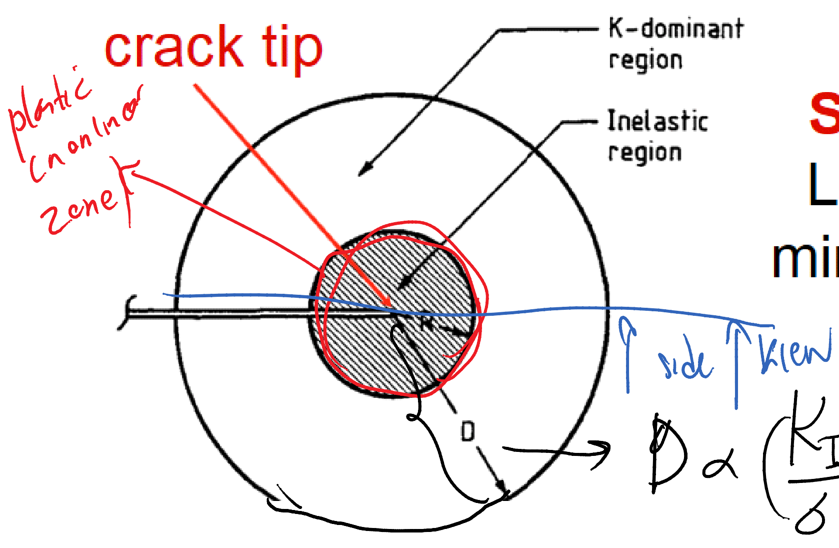
Basically LEFM stress (all field solutions) are accurate:

Close to the crack tip but not very close to the crack tip



$D \propto \left(\frac{K_I}{\sigma}\right)^2$

(SSY)

Small-scale yielding: LEFM still applies with minor modifications done by G. R. Irwin

$$\boxed{R \ll D}$$

K-dominant radius D:

Nonlinear fracture process zone radius R

$$\sigma(r) = \frac{K}{\sqrt{2\pi r}} + \sigma_0 + \dots + c_{\frac{1}{2}} r^{\frac{1}{2}} + c_{\frac{3}{2}} r^{\frac{3}{2}} + \dots$$

$c\,r$

$r \to 0$

first term is dominant as long as it's larger than HO Terms (including 2nd one)

$$\left| \frac{K}{\sqrt{2\pi r}} \right| = \sigma$$

far field stress load

dominant zone radius

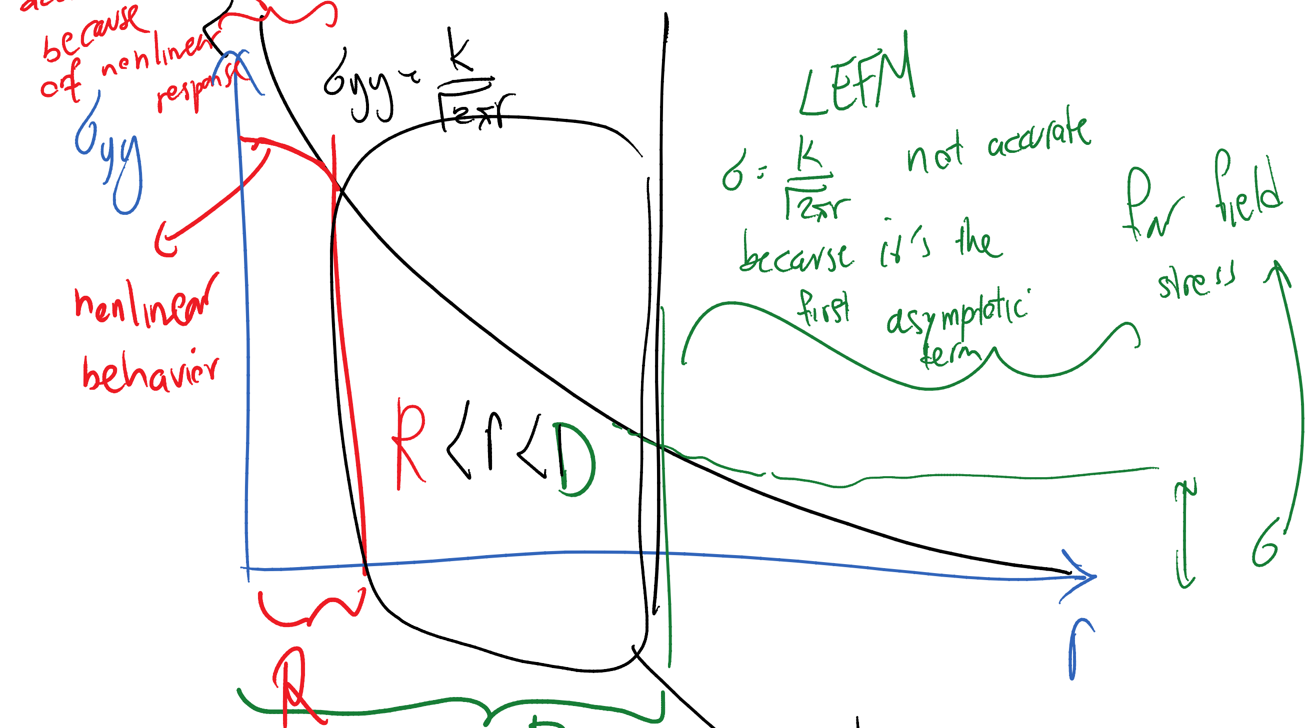
$$D = r = \frac{1}{2\pi}\left(\frac{K}{\sigma}\right)^2$$

$$\boxed{D = \frac{1}{2\pi}\left(\frac{K}{\sigma}\right)^2}$$

far field loading

K - dominant radius

$$\left[\left(\frac{K}{\sigma}\right)^2\right] = \left[\frac{[\sigma][\sqrt{L}]}{[\sigma]}\right]^2 = L$$



only region LEFM theory is accurate

K dominant radius $\propto \left(\frac{K}{\sigma}\right)^2$

far field stress

LEFM theory is accurate if

$$\frac{R}{D} \ll 1 \quad (R \ll D)$$

- Griffith's theory provides excellent agreement with experimental data for brittle materials such as glass. For ductile materials such as steel, the surface energy (*γ*) predicted by Griffith's theory is usually unrealistically high. A group working under G. R. Irwin at the U.S. Naval Research Laboratory (NRL) during World War II realized that plasticity must play a significant role in the fracture of ductile materials.

(SSY)

Small-scale yielding: LEFM still applies with minor modifications done by G. R. Irwin

$$\boxed{R \ll D}$$

SSY means that if the nonlinear response zone is much smaller than the zone that LEFM singular stress response holds then LEFM theory provides an accurate representation of fracture response.

12.7

$$r_\delta \Rightarrow \propto \left(\frac{K}{\sigma}\right)^2$$

LFEM singular radius

applied stress

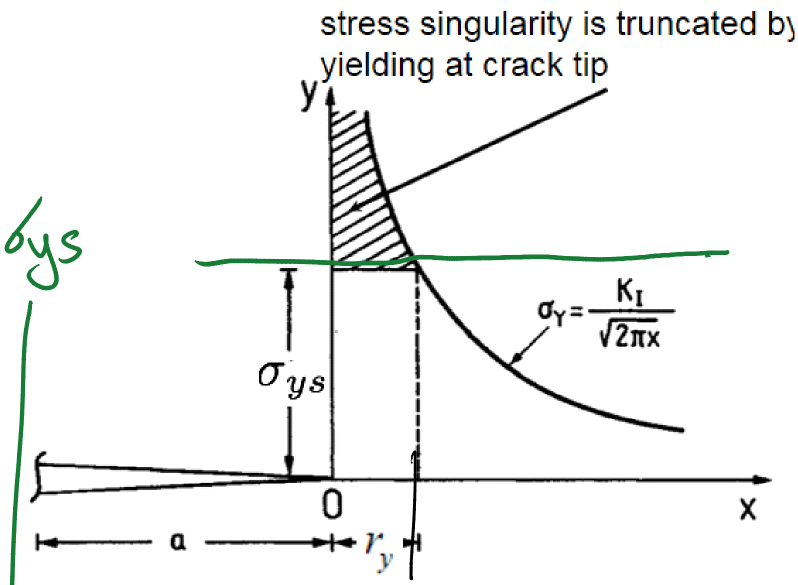
How about R, r_p ?

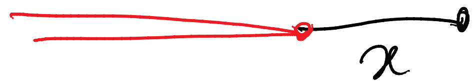
plastic radius

There are many different approaches to estimate R (r_p).
They have different level of accuracy.

5.2.1 Plastic zone shape: 1D models

- 1^{st} order approximation
- 2^{nd} order Irwin model
- Strip yield models (Dugdale, and Barenbolt models)





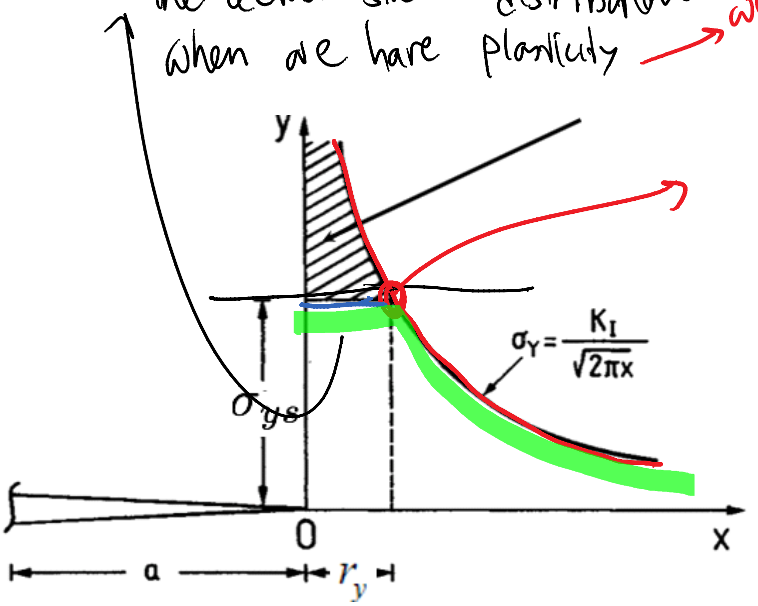
what should σ_{yy} be for the material to yield in plastic mode :

$$\sigma_{ys} = \begin{cases} \sigma_y & \text{p. stress} \\ \dfrac{\sigma_y}{1-2\nu} & \text{p. strain} \end{cases}$$

Poissen ratio

this stress profile is NOT the actual stress distribution when we have plasticity

we really need to solve the problem



$$\sigma = \frac{K_I}{\sqrt{2\pi x}} = \sigma_{ys}$$

for $x = r_y$

from yielding

$$\boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

plastic zone size

$$r_s = D = \frac{1}{2\pi}\left(\frac{K_I}{\sigma}\right)^2$$

applied stress

$$\boxed{\left(\frac{r_y}{r_s}\right) \propto \left(\frac{\sigma}{\sigma_{ys}}\right)^2}$$

$$\frac{r_y}{r_s} \ll 1 \quad \text{SSY hold}$$

when $\frac{\sigma}{\sigma_{ys}} \ll 1$ (applied stresses are much smaller than yield stress) LEFM can provide very accurate representation of fracture / crack tip fields

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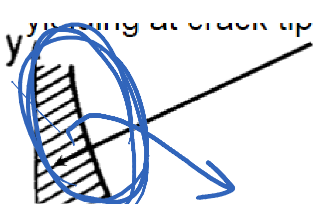
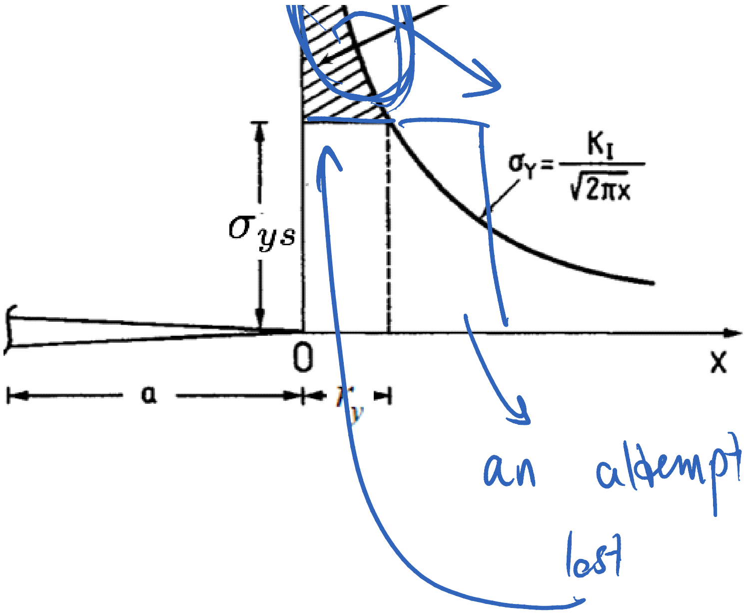
A more accurate estimation of plastic zone size that to some extend take into accurate stress redistribution.



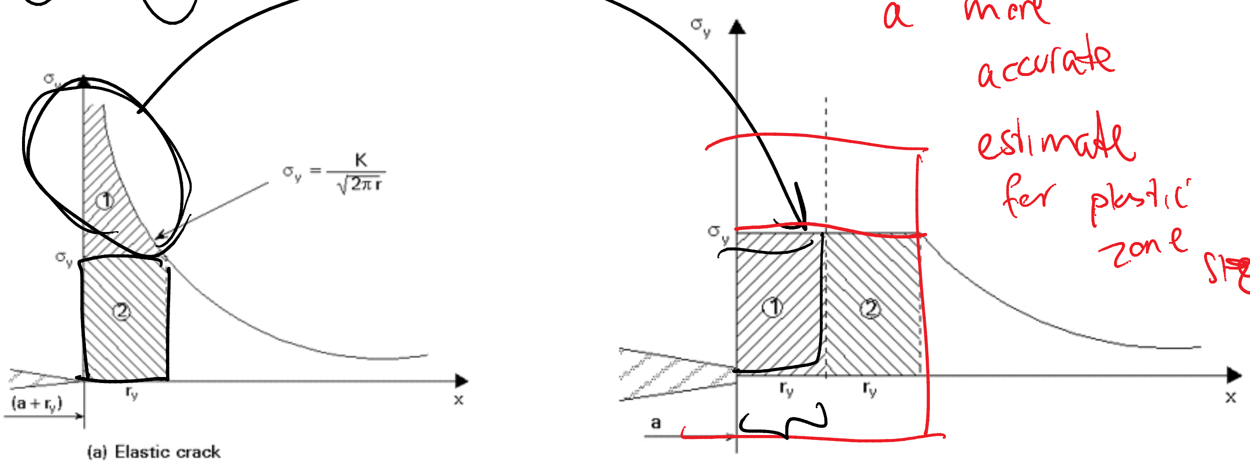


an attempt to redistribute the stress

lost

2.Irwin's plastic correction

area ① = ②



(a) Elastic crack

(b) Real crack with plastic zone

a more accurate estimate for plastic zone size

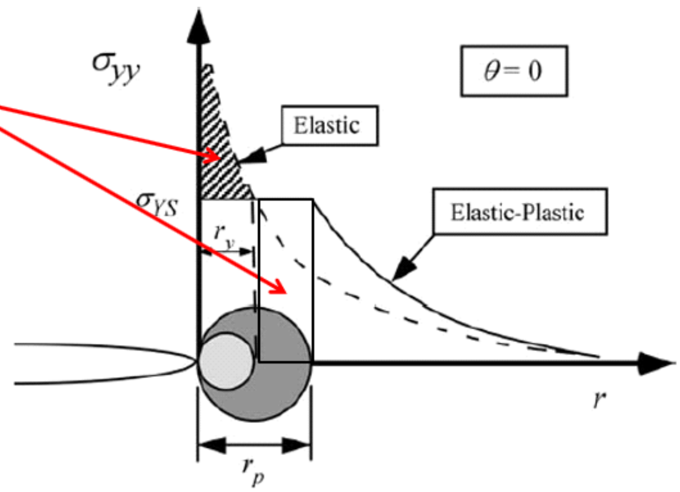
Irwin plastic zone size

$$r_p = 2r_y = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

stress redistribution:

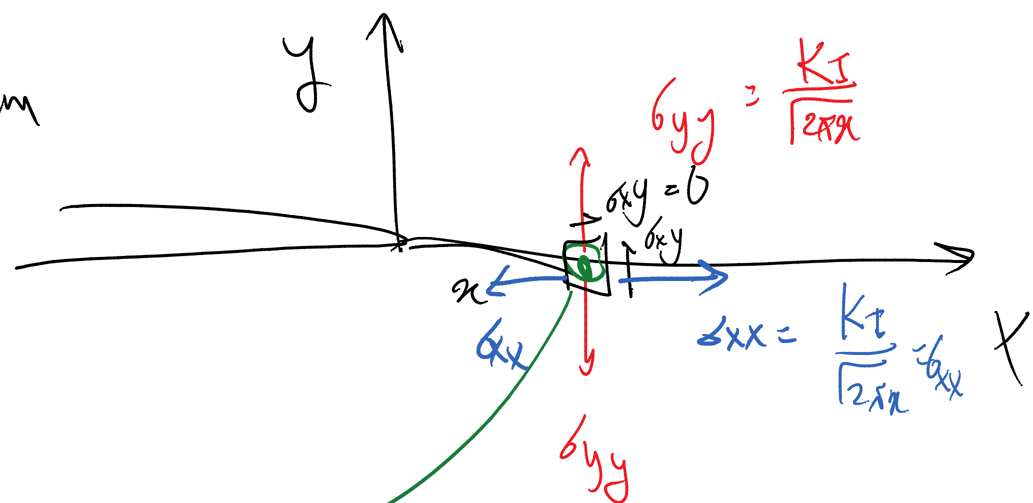
$$\sigma_{ys} r_p = \int_0^{r_y} \sigma_{yy} dr$$

$$\boxed{r_p = 2 r_y = \frac{K_I^2}{\pi \sigma_{ys}^2}}$$



Distinguish between plane stress and plane strain cases:

Mode I problem



out of the plane

$$\sigma_{zz} = \begin{cases} 0 & \text{plane stress} \\ 2\nu\sigma_{xx} & \text{plane strain} \end{cases}$$

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu\sigma_{xx} - \nu\sigma_{yy}\right)$$

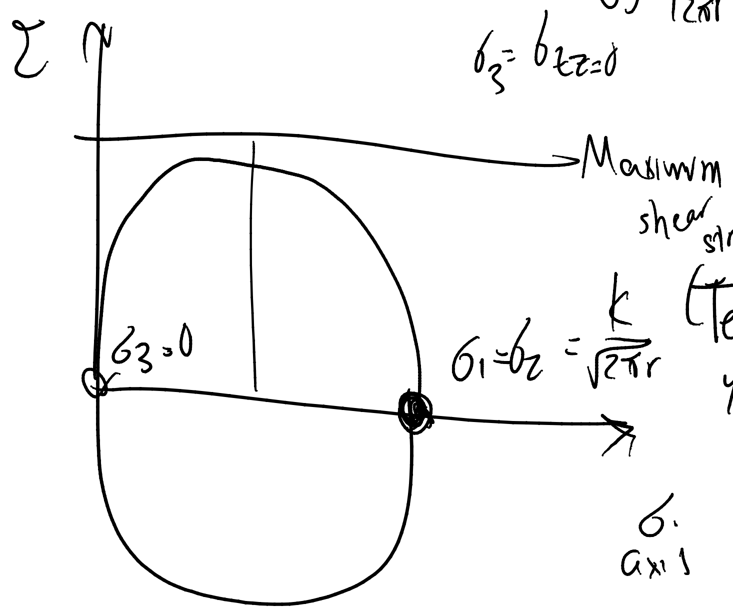
$= 0$

$\sigma_{xx} = \sigma_{yy}$

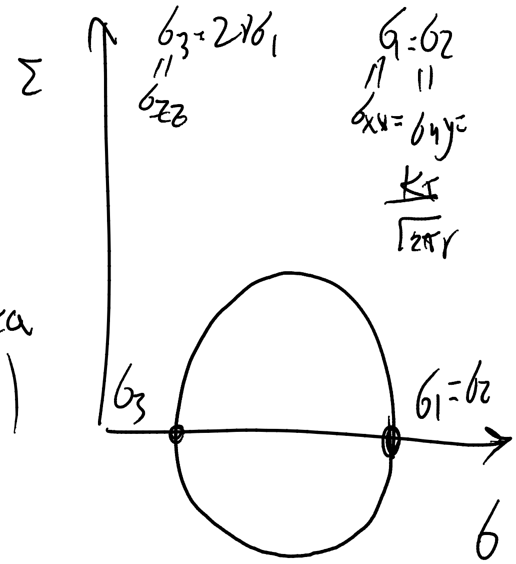
$\sigma_{zz} = 2\nu\sigma_{xx}$

because we don't have shear stresses on cube faces $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ are principal stresses

Moh'r circle



Plane strain



Having principal stresses we can evaluate if a point at distance r yields

More commonly used yield condition than Teresca's condition is Von Misses yield condition

$$\sigma_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

σ_1
σ_2
σ_3
are

equivalent stress

σ_3 are principal stress

$\sigma_e = \sigma_y$ yielding happens

$\leq$ $\sim$

for plane-strain

$\sigma_1 = \sigma_2$

$\sigma_3 = 0$

$$\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$

$$= \sqrt{\frac{0^2+\sigma_1^2+\sigma_1^2}{2}}$$

$$= \sigma_1$$

plane stress $\sigma_e = \sigma_1 = \sigma_y$ when yielding happens

$\sigma_y = \sigma_{ys}$ when yielding happens

for $\sigma_{ys} = \sigma_y$

plane strain condition

$$\sigma_1 = \sigma_2 = \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

$$\sigma_3 = 2\nu\sigma_1$$

$$\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$

$$= \sqrt{\frac{0^2 + ((1-2\nu)\sigma_1)^2 + ((1-2\nu)\sigma_1)^2}{2}}$$

$$= (1-2\nu)\sigma_1$$

in plane strain yielding happens when

$$\sigma_e = \sigma_1(1-2\nu) = \sigma_y$$

$$\Rightarrow \quad \sigma_1 = \sigma_{yy} = \frac{\sigma_y}{1-2\nu}$$

$$\sigma_{yy} = \sigma_{us}$$

for plane strain yielding happens

when $\boxed{\sigma_{yy} = \sigma_{ys} = \frac{\sigma_y}{1-2\nu}}$

plane stress

$\boxed{\sigma_{yy} = \sigma_{ys} = \sigma_y}$

Remember what was in the denominator of r_y, r_p:

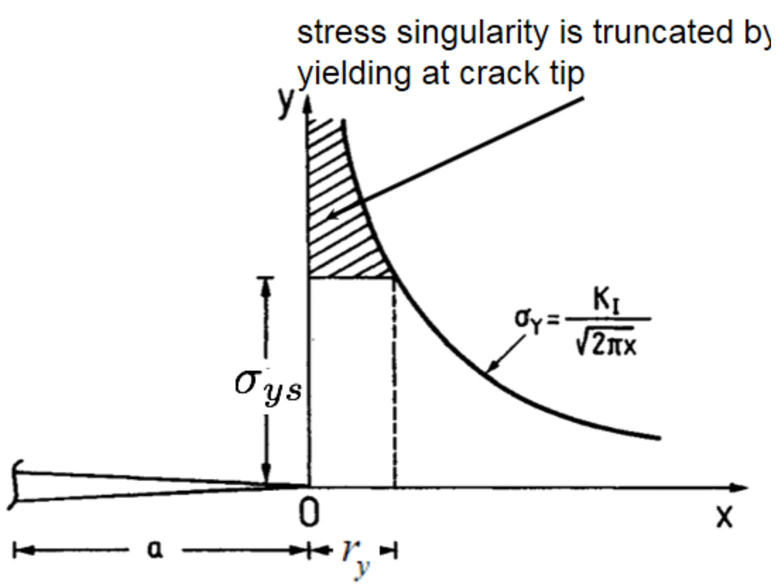
- Material: **elastic perfectly plastic** with yield stress σ_{ys}

On the crack plane $\theta = 0$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

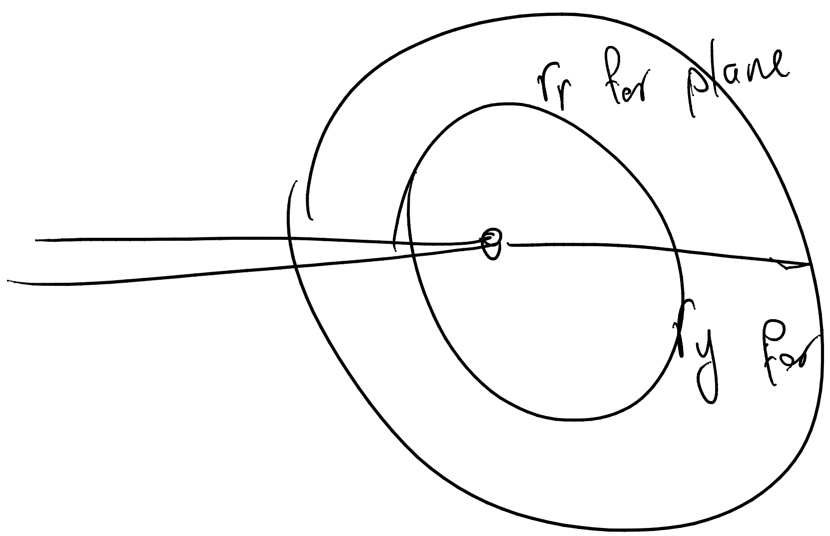
$\sigma_{yy} = \sigma_{ys}$ (yield occurs)

$$\boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2}$$



(σ_y plane stress

$$\sigma_{y\,s.e} = \begin{cases} \sigma_y & \text{plane stress} \\ \dfrac{\sigma_y}{1-2\nu} & \text{plane strain} \end{cases}$$



r_r for plane strain is $(1-2\nu)^2$ times smaller

r_y for plane stress

Von Mises Yield Criterion

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]^{1/2} = \sigma_y$$

Plane stress

$$\sigma_1 = \sigma_2 = \sigma_{yy}, \quad \sigma_3 = 0$$

$$\sigma_e = \sigma_1 = \sigma_y \quad \Rightarrow$$

$$\sigma_{ys} = \sigma_y \quad \Rightarrow$$

$$\boxed{r_p = \frac{K_I^2}{\pi\sigma_y^2}}$$

$$\boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2}$$

Plane strain

$$\sigma_1 = \sigma_2 = \sigma_{yy}$$

$$\sigma_3 = \nu(\sigma_{xx}+\sigma_{yy}) = 2\nu\sigma_{yy}$$

$$\sigma_e = (1-2\nu)\sigma_1 = \sigma_y \quad \Rightarrow$$

$$\sigma_{ys} = \frac{\sigma_y}{1-2\nu} \quad \Rightarrow$$

$$r_p = \frac{K_I^2}{\pi\sigma_{ys}^2} = (1-2\nu)^2\frac{K_I^2}{\pi\sigma_y^2} \quad \Rightarrow$$

$$\boxed{r_p \approx \frac{K_I^2}{3\pi\sigma_y^2}}$$

$(1 – 2 \times 0.2)^2 = 0.36$

for $\nu = .2$

Plastic zone size being larger for plane stress in fact is a good thing because the fracture response is more ductile!