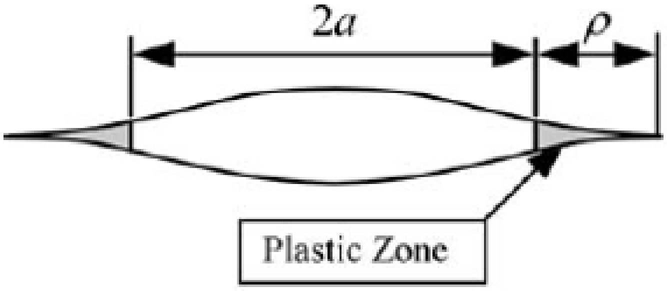
2016/10/13

Thursday, October 13, 2016
8:40 AM

Another use of plastic zone radius

- Effective crack length

Basically by knowing the plastic zone size one can modify the crack length

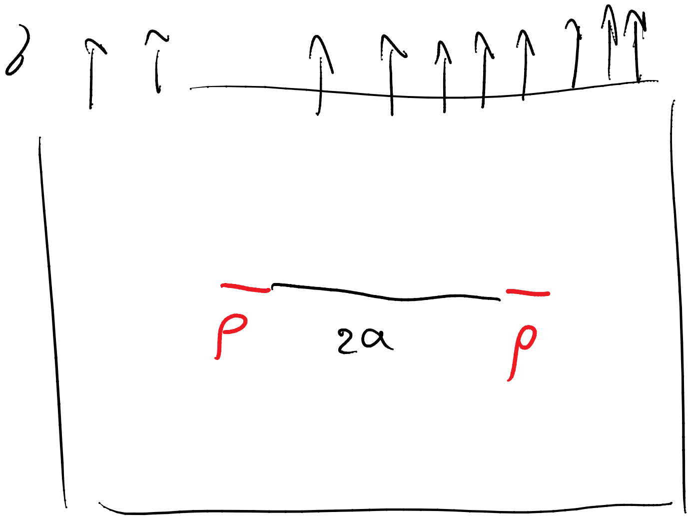


Initial crack length 2a
Two times of the plastic zone can be added to the crack length to find the effective crack length

Effective crack length = 2(a+ρ)

Example

Infinite domain



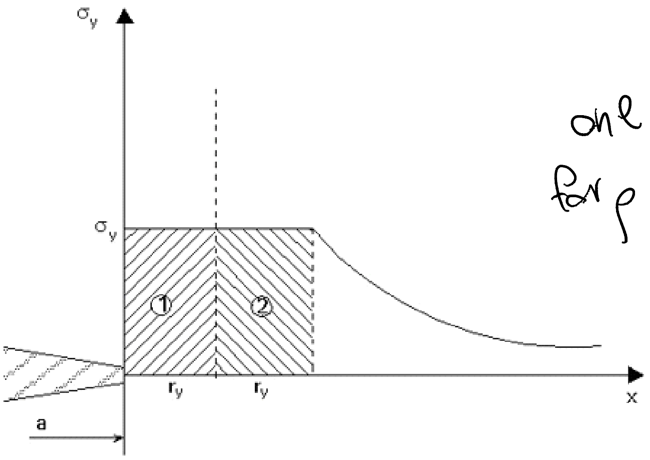
a function of ρ

$$K_{eff} = \sigma\sqrt{\pi(a+\rho)}$$

ρ _ ?   we can use any of the estimates

$\rho = ?$ we can use any of the estimates we found

plastic zon



(b) Real crack with plastic zone

one representation for ρ

$$\underset{\rho\,=}{\boxed{r_y}} = \frac{1}{2\pi}\left(\frac{K_{I\,\text{eff}}}{\sigma_{ys}}\right)^2$$

It's a function of Keff

$$K_{eff} = \sigma\sqrt{\pi(a+\rho)} = \sigma\sqrt{\pi\left(a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2\right)}$$

appears on both sides

$$K_{eff}^2 = \sigma^2\left(\pi a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2\right)$$

$$K_{eff}^2\left(1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2\right) = \sigma^2\pi a$$

$$\boxed{K_{eff} = \sigma\sqrt{\pi a}\left\{\frac{1}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}\right\}}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{correction factor}}$$

By including the plastic zone size as a part of effective crack K increases.

$$\text{correction factor} = \frac{1}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}$$

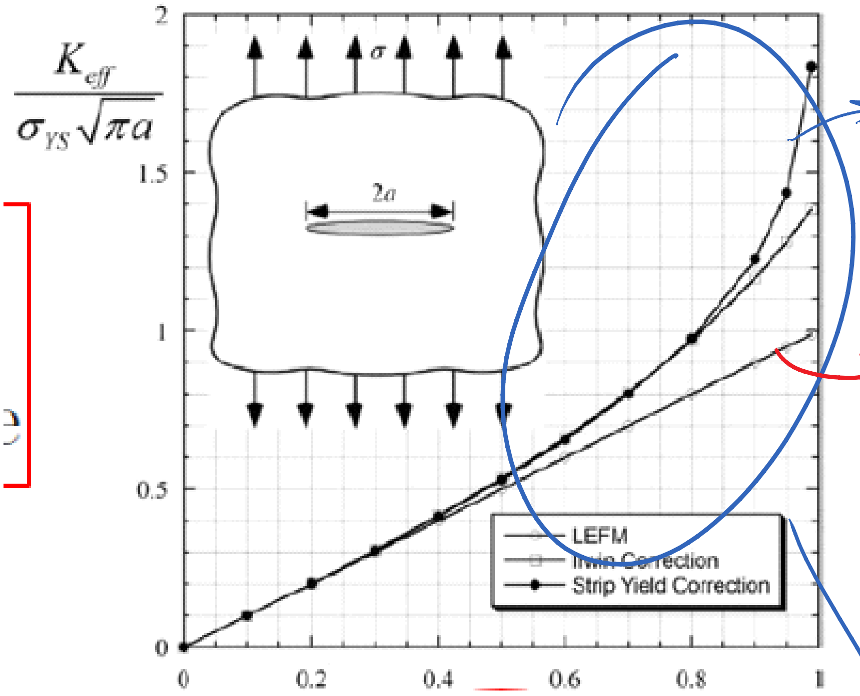
plastic zone size ↑

$$\frac{r_p}{r_s} \propto \left(\frac{\sigma}{\sigma_{ys}}\right)^2, \quad \text{Must have } \frac{r_p}{r_s} \ll 1$$

↓ K- dominant zone size

for SSY to be satisfied.

$$K_{eff} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{YS}}\right)^2}}$$



σ/σ_{ys}

Adjusted K

$= \sigma\sqrt{\pi a}\left(\frac{1}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}\right)$

LEFM solution

$K = \sigma\sqrt{\pi a}$

$\frac{\sigma}{\sigma_{ys}}$ large K

from LEFM $= \sigma\sqrt{\pi a}$ is not representative

Notes:

1. This is crude way to include the effect of plasticity in LEFM theory when loads increase. Basically, we effectively increase K by including plastic zone as a part of the crack. Clearly it is better to do a full nonlinear analysis when the load / yield stress σ/σ_{ys} is high.

2. SSY condition

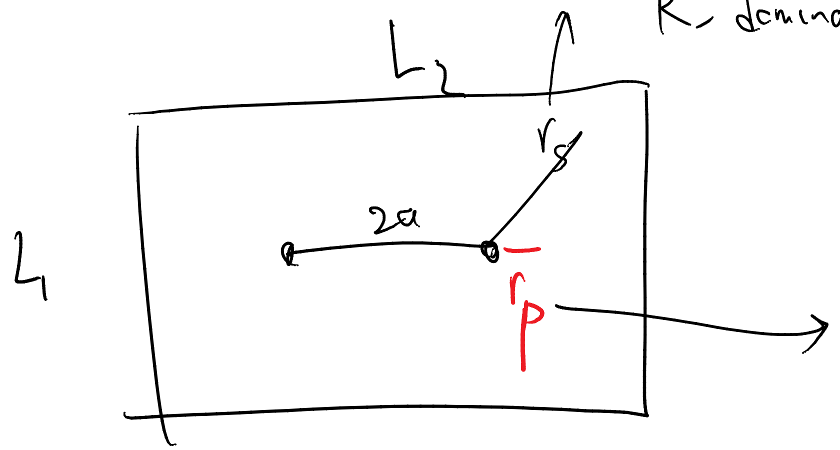
Nonlinear zone size is

MUCH SMALLER than

ALL relevant length scales of

the problem.

K dominant zone size



good representation of nonlinear zone size

$$r_p \ll \{L_1, L_2, a, r_s\}$$

$$\frac{r_p}{r_s} \propto \left(\frac{\sigma}{\sigma_{ys}}\right)^2$$

$\frac{\sigma}{\sigma_{ys}} \ll 1$ a necessary condition (but not sufficient) for LEFM.

3. In general finding effective crack length requires iteration

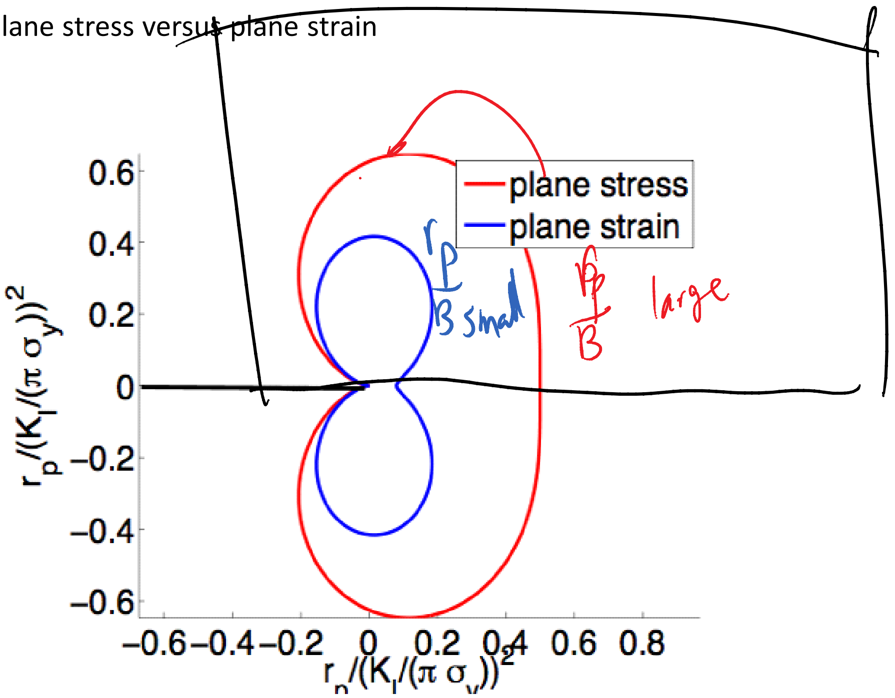
$$\left.\begin{aligned} K_{eff} &= f\Big(\overset{\text{effective crack length}}{\widetilde{a+\rho}}, W, L, \ldots\Big)\, \sigma\sqrt{\pi(a+\rho)} \\ \rho &= \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2 \end{aligned}\right\}$$

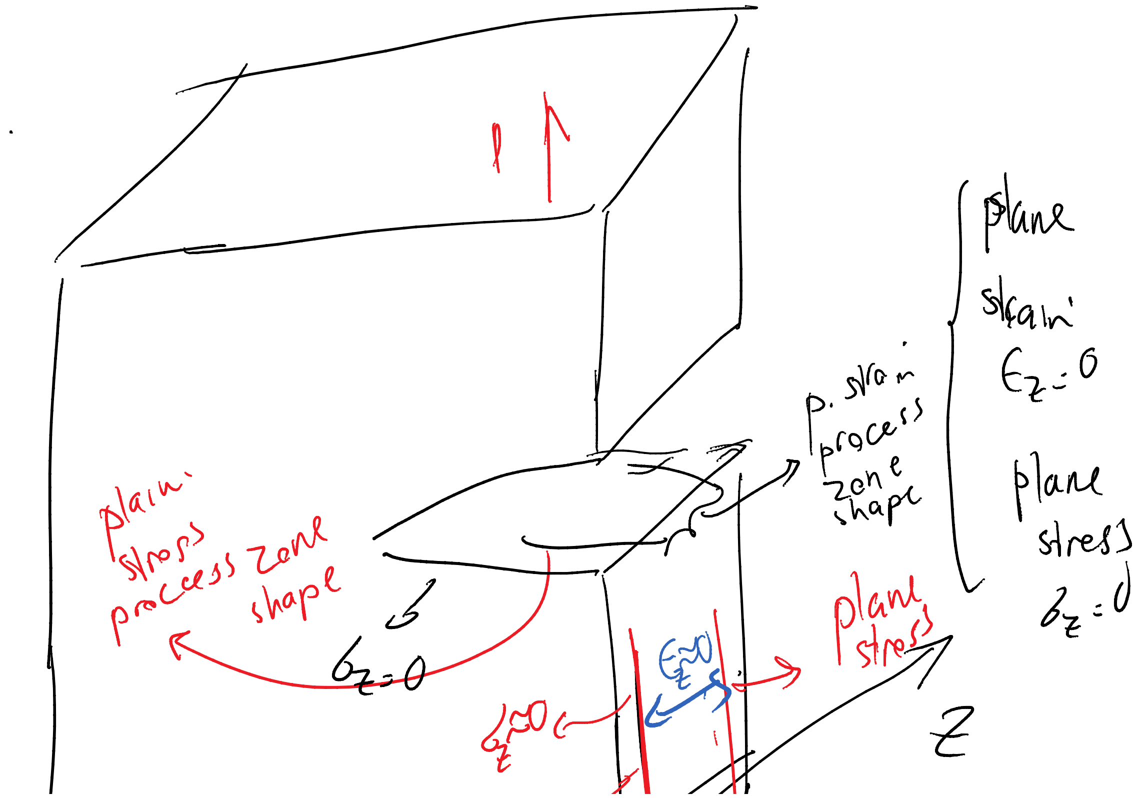
Consider a large central cracked plate subjected to a uniform stress of 130 MPa. The fracture toughness K_c=50MPa√m, the yield strength σ_{ys}=420MPa.

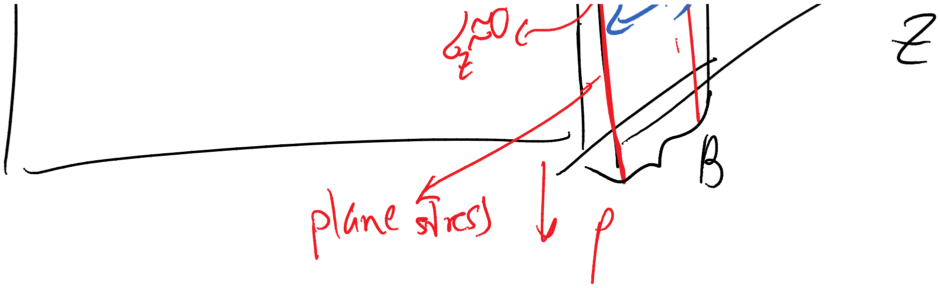
(a) What is the maximum allowable crack length?
(b) What is the maximum crack length if plastic correction is taken into account. Plane stress and Irwin's correction.

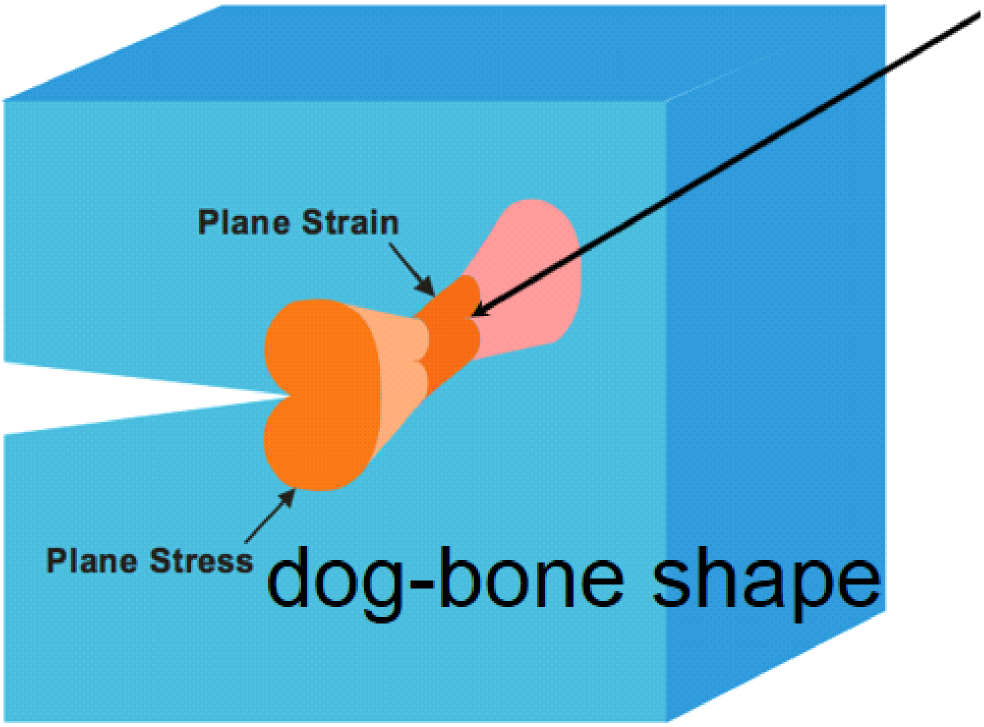
Going back to the 2D models for plasticity

Plane stress versus plane strain



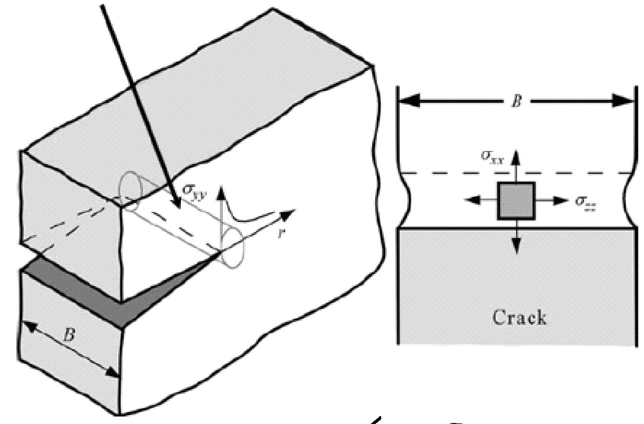






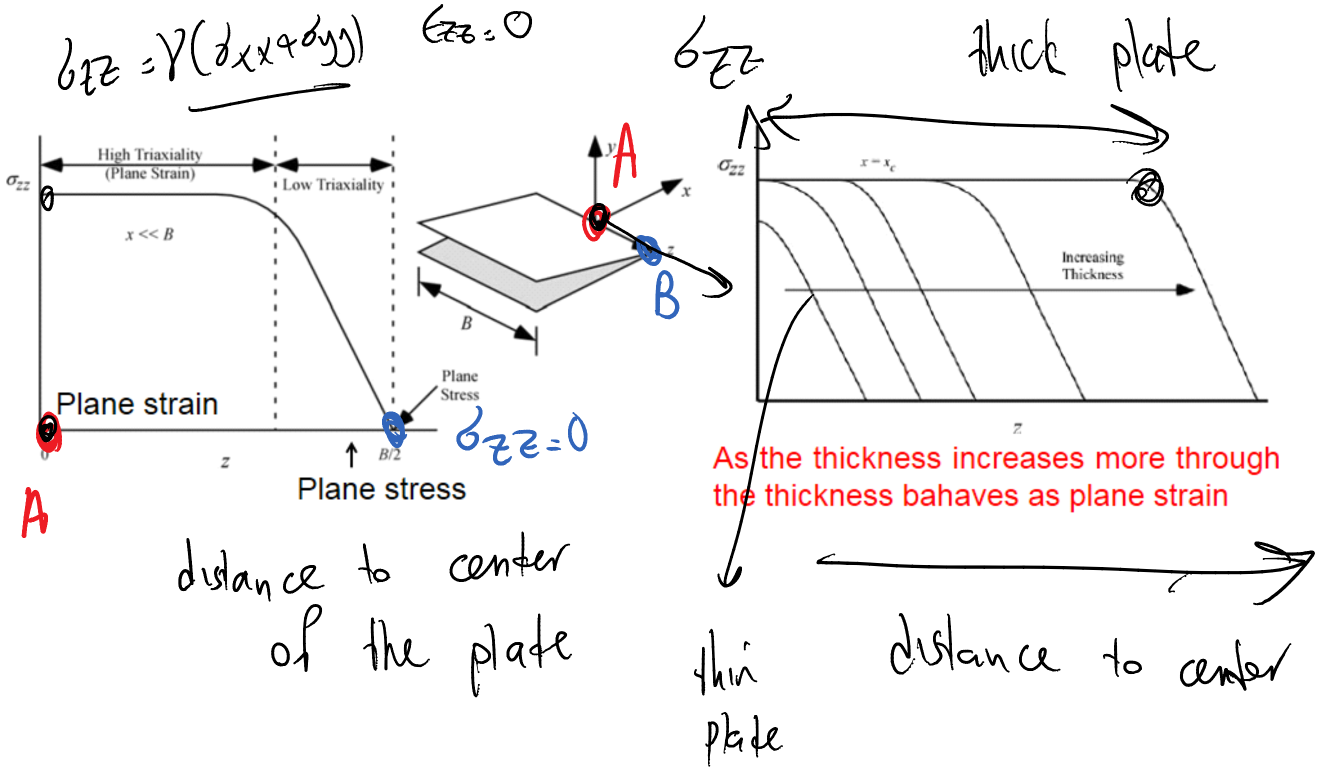
dog-bone shape

constrained by the surrounding material



$\underline{\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})}$ $\varepsilon_{zz} = 0$ σ_{zz} thick plate

$\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$ $\varepsilon_{zz}=0$

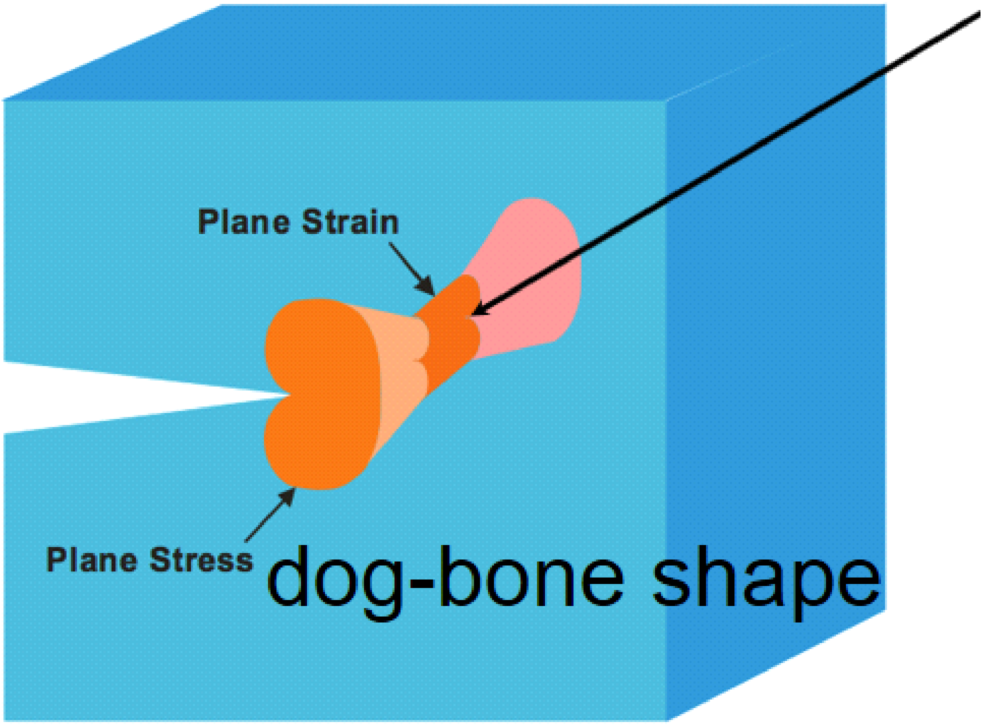


As the thickness increases more through the thickness bahaves as plane strain

Relation of plate thickness to fracture mechanics

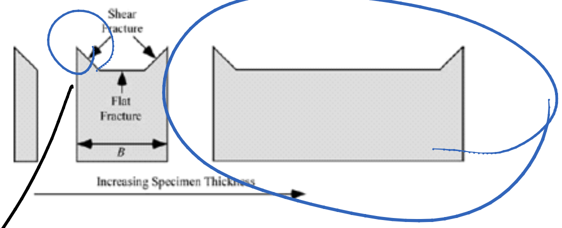
1. Dog bone shape process zone shape

1.



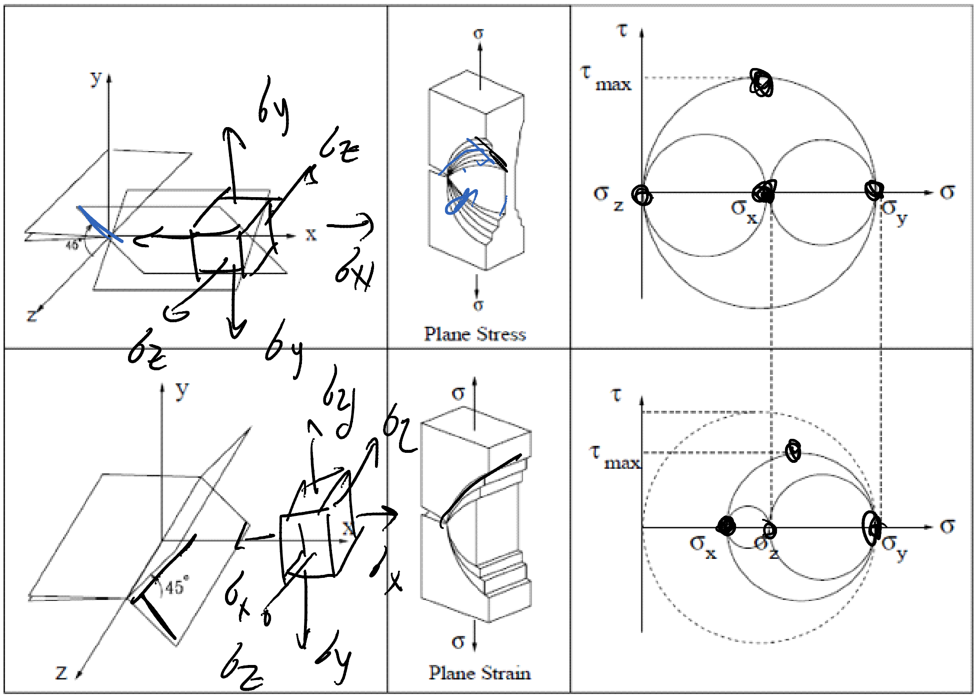
2. The shape of fractures surfaces

Higher percentage of plate thickness is in plane strain mode for thicker plates



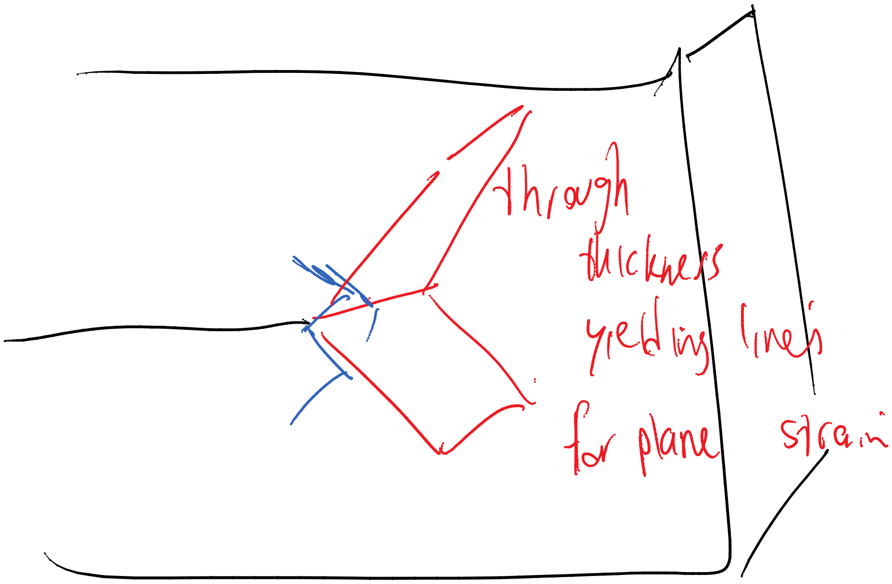
close to the surface material breaks at 45° degree

Plane stress/plane strain: Fracture loci

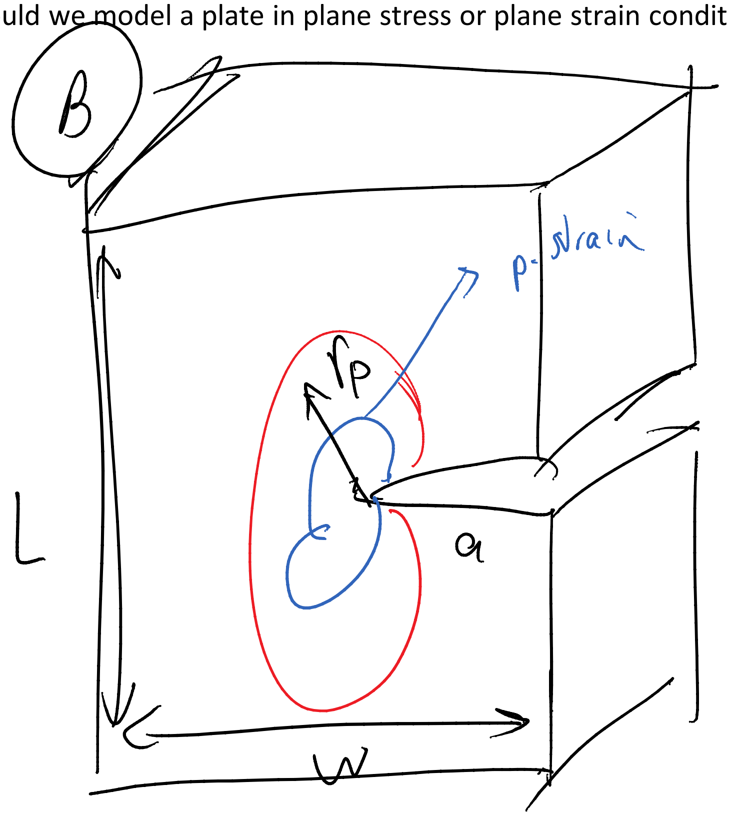


Loci of maximum shear stress for plane stress and strain

$$\boxed{\sigma_z = \nu(\sigma_x+\sigma_y)}$$



When should we model a plate in plane stress or plane strain condition for fracture mechanics?



$L, W \gg B$

to plate

2D model

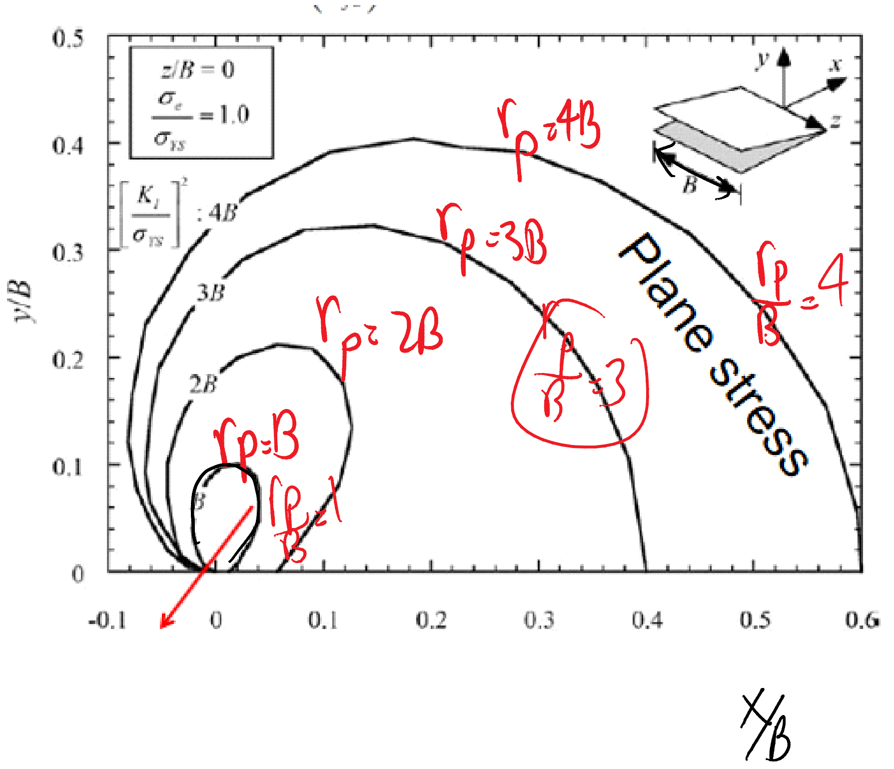
$$r_p \propto \left(\frac{K}{\sigma_y}\right)^2$$

r_p should be compared to B to decide whether the plate is in plane-strain or plane stress condition?

$$\frac{r_p}{B} \ll 1 \updownarrow$$

plane strain

because "B is very large"



Change of plastic loci to plane stress mode as "relative B decreases".
Nakamura & Park, ASME 1988

$$r_p = \left(\frac{K_I}{\sigma_y}\right)^2$$

a proof that as $\frac{r_p}{B} \nearrow$

(B decreases)

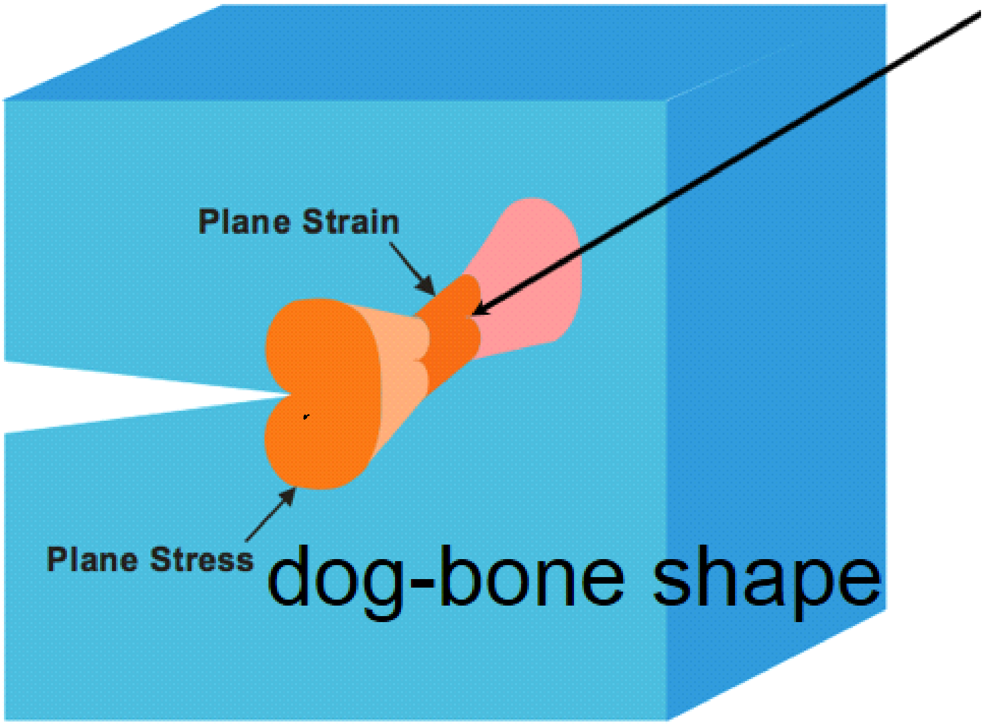
plane stress condition holds.

Reasons why we care about plane stress / plane strain condition in fracture mechanics

1. The shape of process zone (where yielding occurs)
2. Shape / loci of fracture surface
3. Critical stress intensity facture (fracture toughness)

$$K_I = \boxed{K_{IC}}$$

crack can propagate

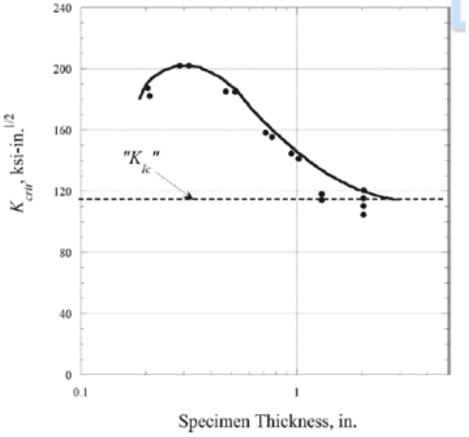


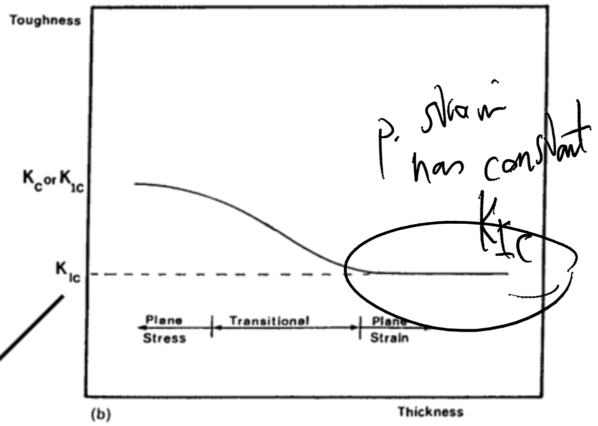
In plane stress condition we have larger process zone -> larger energy dissipation (per unit area of crack) -> larger fracture toughness

Plane stress/plane strain

Toughness vs. thickness

Plane stress/plane strain Toughness vs. thickness





p. strain has constant K_{Ic}

as $B \rightarrow$

K_{Ic} reaches a limiting value

Plane strain fracture toughness

lowest K (safe)

$$\text{(Irwin)}\ K_c = K_{Ic}\left(1+\frac{1.4}{B^2}\left[\frac{K_{Ic}}{\sigma_{ys}}\right]^4\right)^{1/2} \approx \left(\frac{r_p}{B}\right)^2$$

Note that $\frac{1}{B^2}\left[\frac{K}{\sigma_{ys}}\right]^4 \propto \left(\frac{r_p}{B}\right)^2$

adjusted K_c based on thickness

fracture toughness for p. STRAIN

Experiments find K_{Ic} (fracture toughness for plane strain) and the equation above helps us find Kc for other thicknesses.

Most Fracture mechanics experiments are done under plane strain condition.

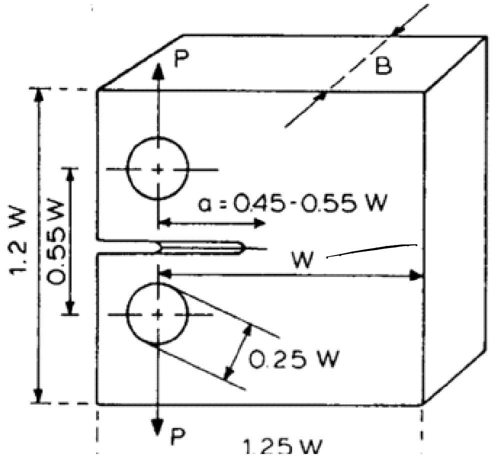
Fracture toughness tests

- Prediction of failure in real-world applications: need the value of fracture toughness
- Tests on cracked samples: PLANE STRAIN condition!!!

Compact Tension Test

$$K_I = \frac{P}{B\sqrt{W}} \frac{\left(2+\frac{a}{W}\right)\left[0.886+4.64\frac{a}{W}-13.32\left(\frac{a}{W}\right)^2+14.72(\left(\frac{a}{W}\right)^3-5.6(\frac{a}{W}\right.}{\left(1-\frac{a}{W}\right)^{3/2}}$$

$$a, B, (W-a) \geq 2.5\left(\frac{K_{Ic}}{\sigma_{YS}}\right)^2$$



ASTM (based on Irwin's model for plane strain condition:

$$\boxed{a, B, (W-a) \geq 2.5\underbrace{\left(\frac{K_{Ic}}{\sigma_Y}\right)^2}_{r_p}}$$

$B \geq 2.5 r_p$

to ensure plane strain condition for test.

Fracture toughness test

ASTM E399

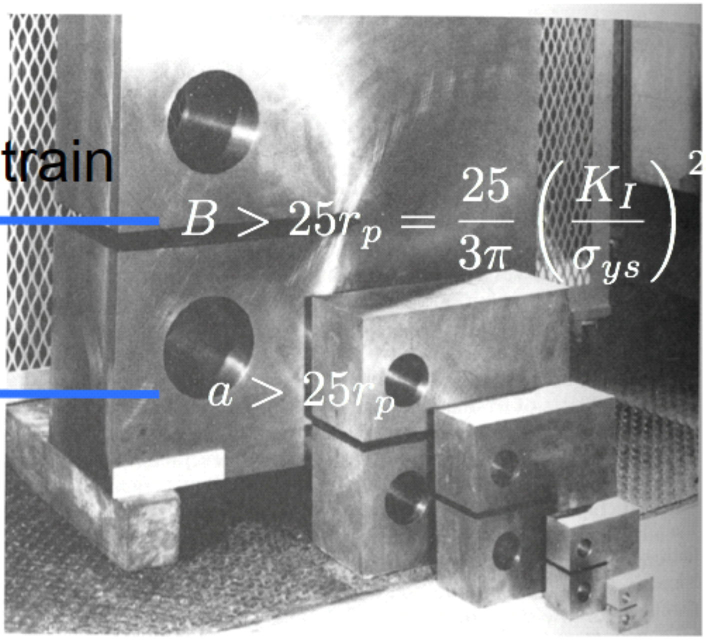
$$B \geq 2.5\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

$$a \geq 2.5\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

r_p

$$W \geq 2.5\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

plane strain



$$B > 25 r_p = \frac{25}{3\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

$$a > 25 r_p$$

Linear fracture mechanics is only useful when the plastic zone size is much smaller than the crack size

5.3. J Integral (Rice 1958)

	LEFM	Plastic FM (PFM) Nonlinear FM (NLFM)
Global (Energy)	$G \geq G_c (= R)$ crack can propagate	$J \geq J_c (= R)$ J Integral
local (stress)	$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta)$	$\sigma_{ij} = (J)^{\alpha} r^{(\beta)} f_{ij}(\theta)$

$G = \frac{1}{E'} K_I^2$

mode I

J integral is in fact energy release rate G in general (with nonlinear loading)

$G = J$

$J = ?$

- J integral in fracture

$$J = J_1 = \int_\Gamma \left(W n_1 - t_i \frac{\partial u_i}{\partial x_1} \right) d\Gamma \quad \left[\frac{\text{N}}{\text{m}} \right]$$

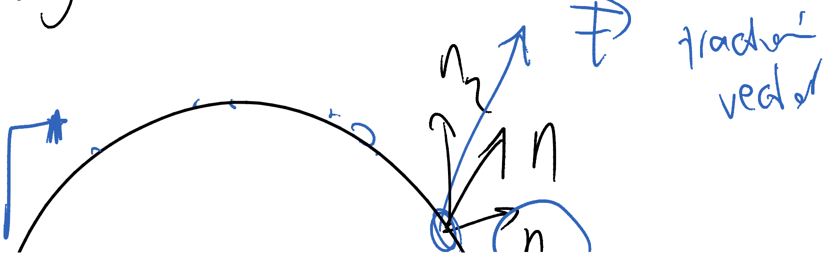
energy density

traction

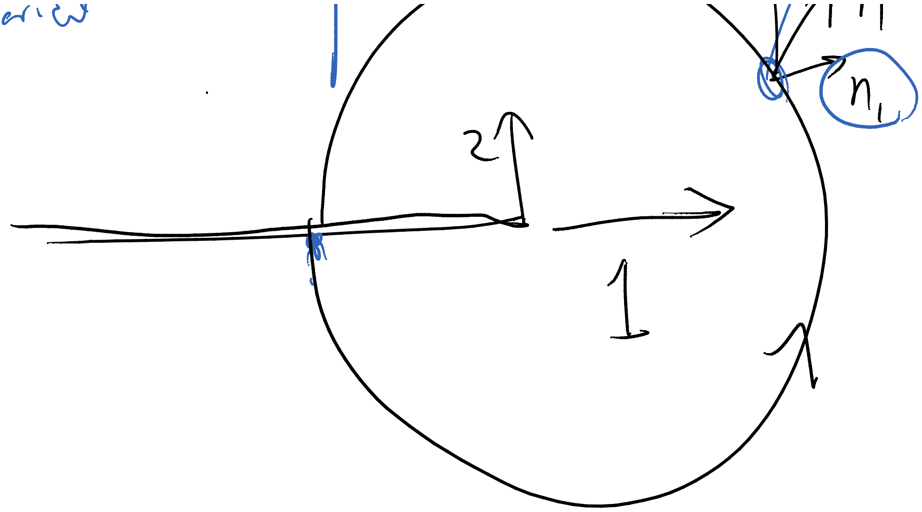
displacement

$W = \frac{1}{2} \sigma : \varepsilon$

for linear material



materiw



$G = J$

$K = \sqrt{GE'}$