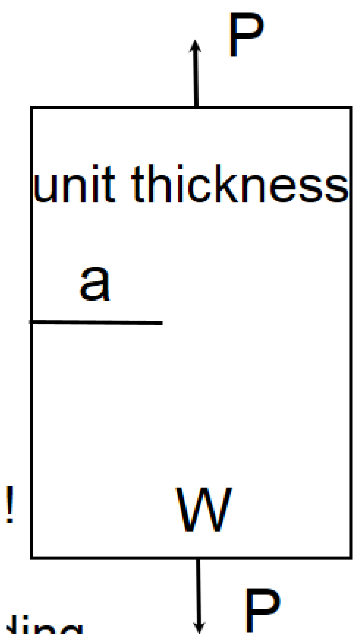
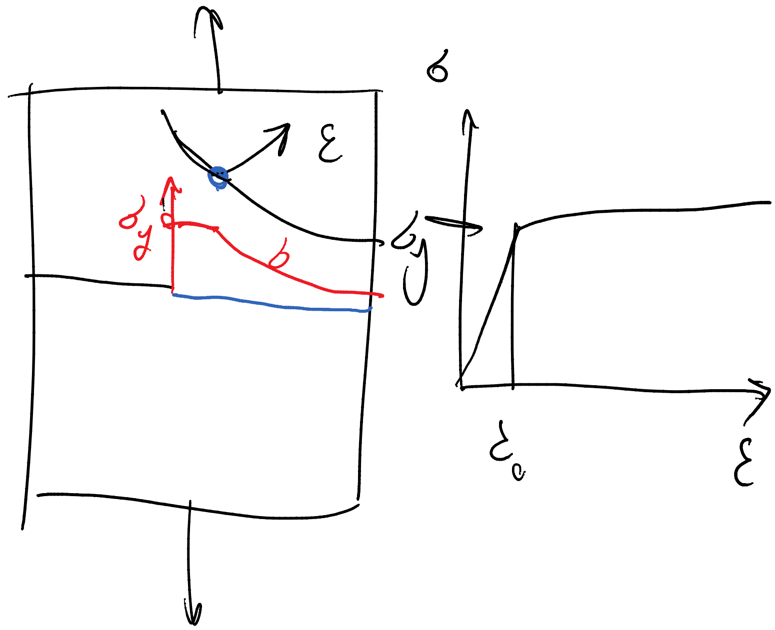
2016/10/25

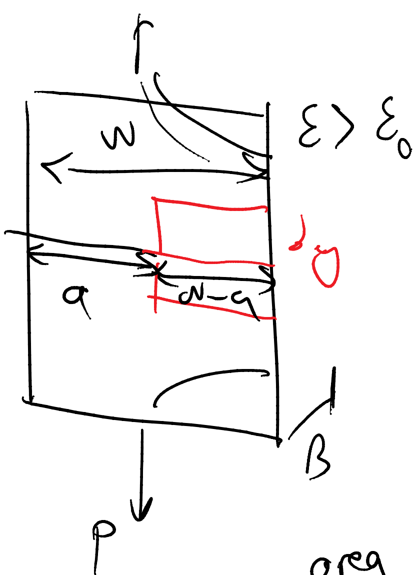
Tuesday, October 25, 2016
8:39 AM

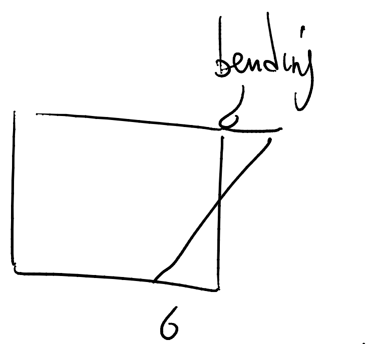
Fracture vs. Plastic collapse



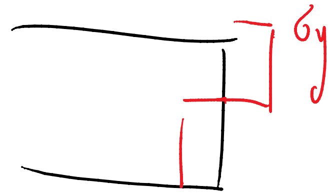


Ultimate load analysis





ultimate moment



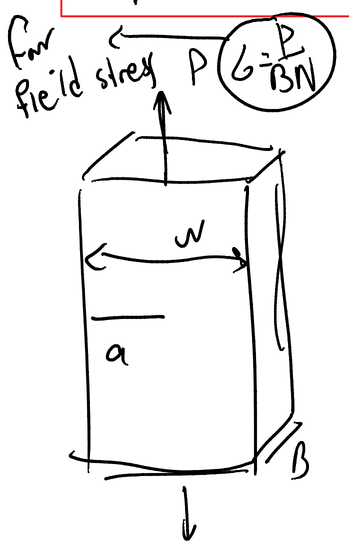
area

$$P = \{W - a | B\}\ \sigma_{ys}$$

$$\sigma_{ys}: \begin{cases} \sigma_y & \text{plane stress} \\ \dfrac{\sigma_y}{1-2\nu} & \text{plane strain} \end{cases}$$

$$\boxed{P_u = (W-a)\, B\, \sigma_{ys}}$$

Plastic ultimate load



$$K = \sigma\sqrt{\pi a}\, f\left(\frac{a}{W}\right)$$

σ_f fracture ultimate stress is obtained by

$$K = K_c \quad : \quad K_c = \sigma_c\sqrt{\pi a}\, f\left(\frac{a}{W}\right)$$

$$\sigma_f = \frac{K_c}{\sqrt{\pi a}\, f\left(\frac{a}{W}\right)}$$

fracture load $\leftarrow P_f = \sigma_f\,(B\,W) \Rightarrow$

$$\boxed{P_f = \frac{K_c B W}{\sqrt{\pi}\, f\left(\frac{a}{W}\right)} \frac{1}{\sqrt{a}}}$$

For brittle materials Kc low sigma_y is high -> ac is low and they are more prone to fracture

Ductile material Kc high / sigma_y low -> ac is high they often fail under plastic yield

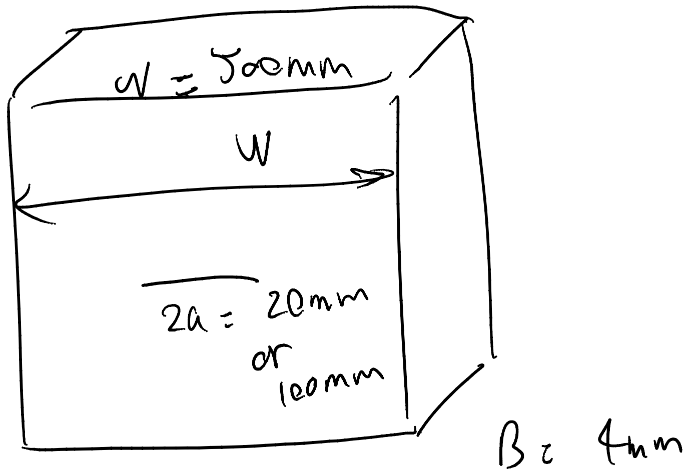
Example

Example

Example 4.11 Estimate the failure load under uniaxial tension for a centre-cracked panel of aluminium alloy of width W=500 mm, and thickness B=4 mm, for the following values of crack length $2a$ = 20 mm and $2a$ = 100 mm. Yield stress σ_y=350 MPa and fracture toughness K_{Ic}=70 MPa$\sqrt{m}$

$\sigma_y = 350\,\text{MPa}$

$K_c = 70\,\text{MPa}\sqrt{m}$



For $a = 20\text{mm}$ does it yield first or fracture?

plastic load

$$P_u = \sigma_y \underbrace{(\qquad)}_{\text{load bearing area}} = \sigma_y\left(B(W-2a)\right)$$

$$= \underbrace{(350e6)}_{Pa}\left\{\left(\underbrace{500}_{W} - \underbrace{20}_{2a}\right)\times 1e\text{-}3\right\}^{m}\left\{\underbrace{4e\text{-}3}_{B}\right\}^{m}$$

$$\rightarrow \boxed{F_u = 672\text{ kN}}$$

Fracture load $\quad K = \sigma\sqrt{\pi a}\, f(a,W)$

1st step $\sigma_c \quad K_c = \sigma_c\sqrt{\pi a}\, f(a,W)$

$$\Rightarrow \quad \sigma_c = \frac{K_c}{\sqrt{\pi a}\; f(a,W)} \qquad f(a/W) = \sqrt{\sec\left(\frac{a\pi}{W}\right)}$$

$2a = 20$ mm

$$= \frac{70e6 \ \text{Pa}\sqrt{m}}{\sqrt{\pi(10e\text{-}3)m}\ \sqrt{\sec\left(\frac{\pi \times 10}{500}\right)}} \Rightarrow$$

Fracture ultimate stress $\boxed{\sigma_c = 394\text{MPa}}$

load for fracture :

$$F_c = \sigma_c \underbrace{(B \ W)}_{\text{entire area}} = 394e6^{\,Pa} \underbrace{(4e\text{-}3)}_{B}{}^{m} \underbrace{(500e\text{-}3m)}_{W}$$

$\Rightarrow \boxed{F_c = 788 \text{ kN}}$ remember $\boxed{F_{q} = 672\text{kPa}}$ plastic

For $2a = 20$mm ($a = 10$mm) the specimen fails under plastic yielding

How about $2a = 100mm$ $(a = 50mm)$

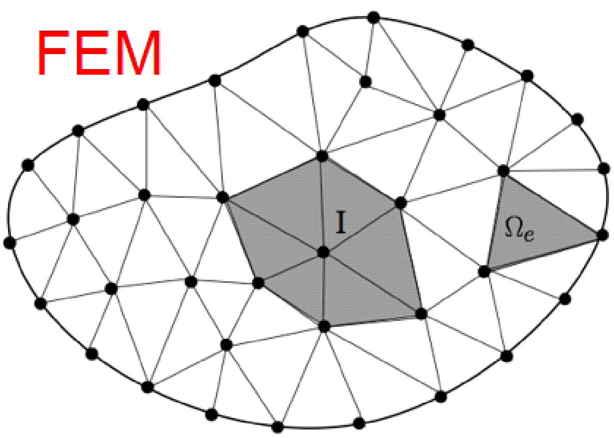
Do the same calculation:

$$\boxed{\begin{array}{l} \text{plastic yielding} \leftarrow F_u = 560 \text{ kN} \\ \text{fracture} \leftarrow F_c = 335 \text{ kN} \bigstar \end{array}} \rightarrow \text{it fractures first}$$

We prefer the material to fail by yielding rather than fracture because the large displacements give us warnings to fix the problem

6.1 Fracture mechanics in Finite Element Methods (FEM)

We focus on FEM as a means to obtain solutions



Basis of solving solid mechanics problem

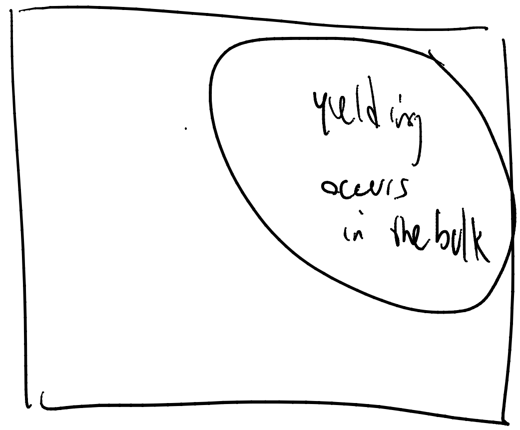
Models for failure / fracture of materials:

Models for failure / fracture of materials:

2D failure models

- yielding

- Bulk damage models



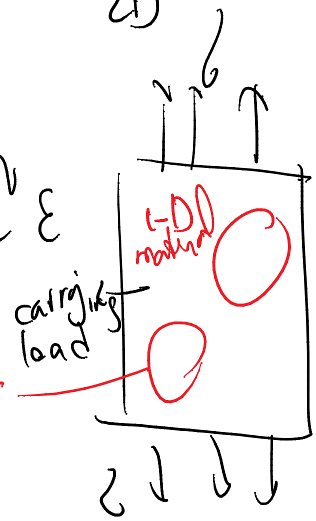
2D

Linear elastic material $\sigma = C\varepsilon$

D damage parameter $\sigma = (1 - D)\, C\varepsilon$

$$D = \frac{\text{volume voids}}{\text{total volume}}$$

voids

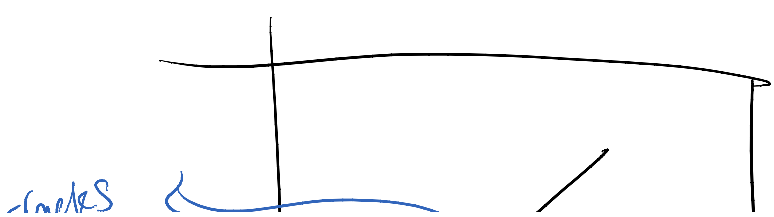


$D = 1 \qquad \sigma = 0$

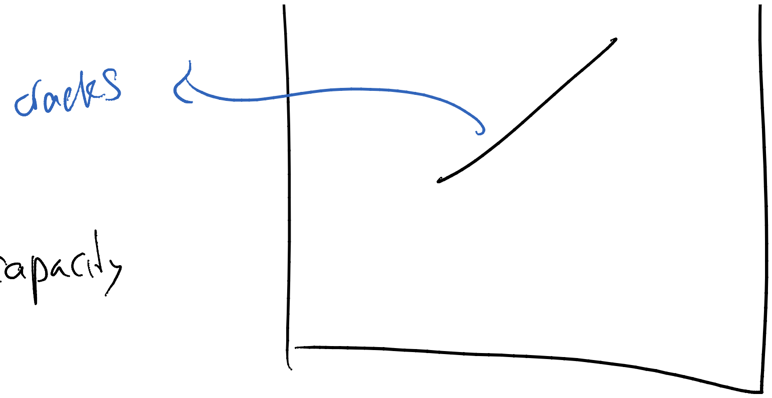
$D = 0 \qquad \sigma = C\varepsilon$

$0 < D < 1 \qquad \sigma = \underbrace{(1-D)\, C}_{\text{partial damage}}\, \varepsilon$

$\boxed{\text{1D models}}$



cracks reduce
the load bearing capacity
of structure



The last two models are **continuum**:
Case 1: material degradation occurs in the bulk (either by plastic deformation or by using a damage model)
Case 2: Cracks are responsible for reducing load bearing capacity of structure

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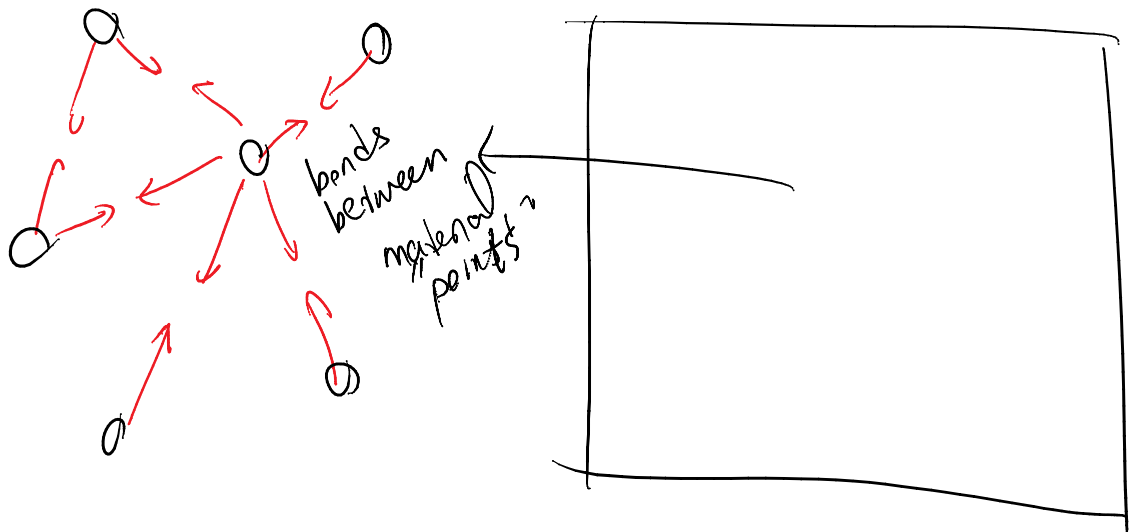
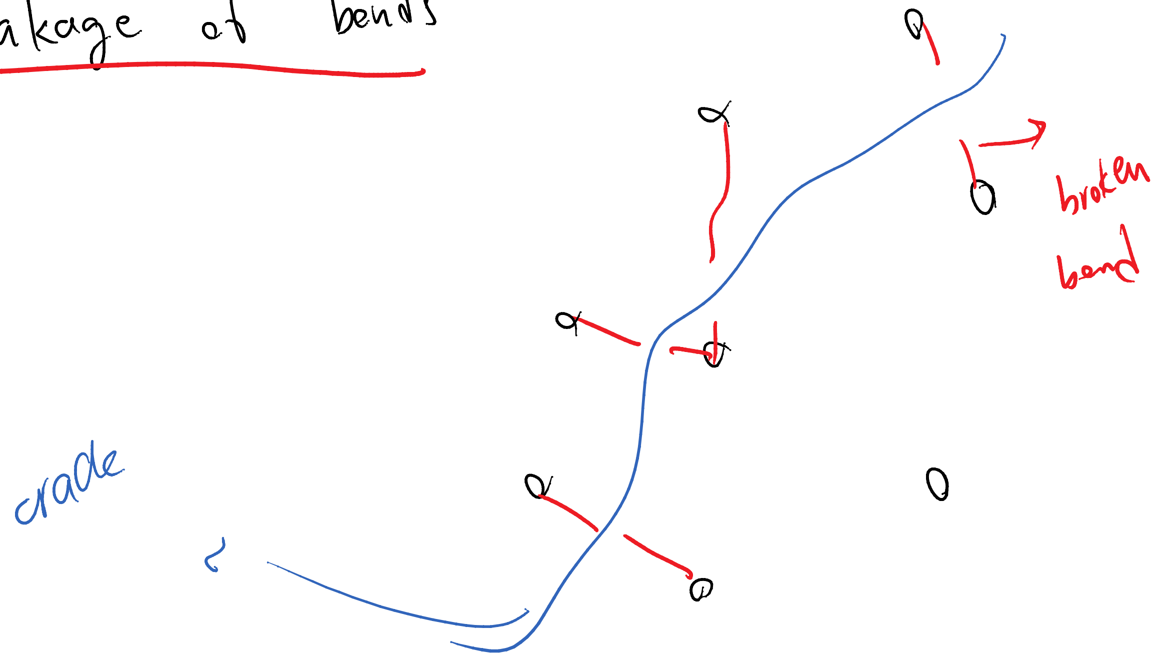
As opposed to continuum models we can have **discrete representation of material**



- **Discrete element method** -> material is idealized as a set of points interacting with each other
- **Peridynamics**: A very similar idea

In these models failure is represented by breakage of bonds



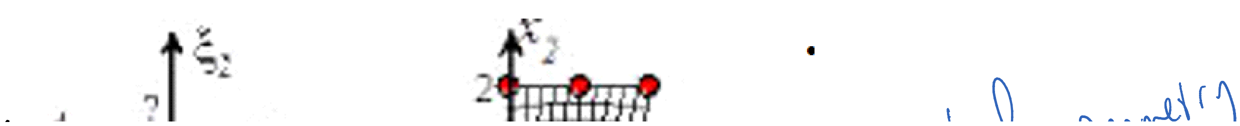
Summary:

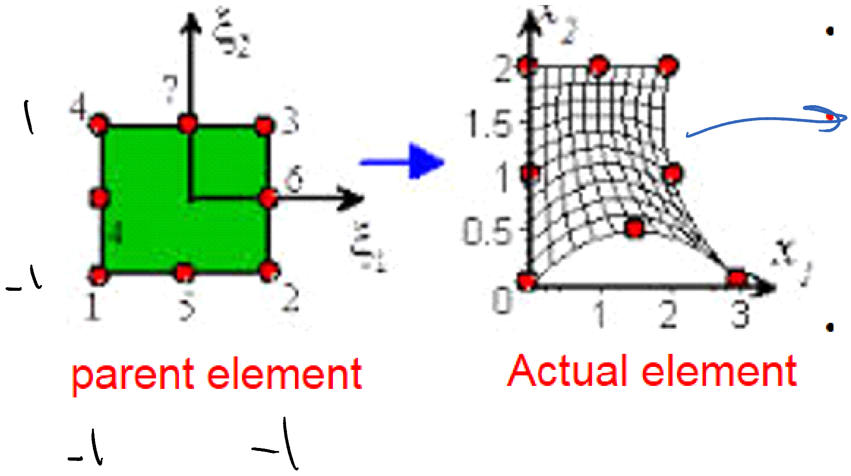
Popular failure models for material:

1. Material is modeled as continuum:
    a. Continuum degradation models (Plasticity / Bulk damage model, etc.) ☆ 1.a
    b. Lower dimensional defects in the domain (e.g. cracks)
2. Discrete models where the continuum is modeled as an ensemble of points interacting by mutual forces between them (Discrete element method, Peridynamics, etc.) ☆ 2

We continue with 1.b (continuum domains with cracks) and use FEM to analyze them

Isoparametric Elements





actual geometry of the element

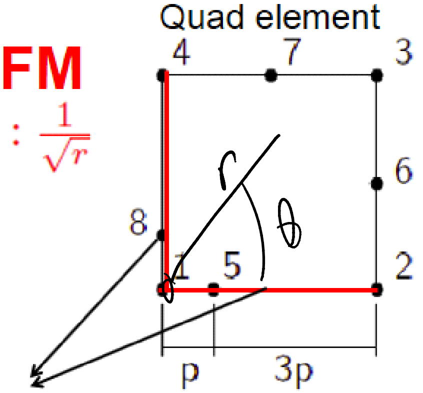
Isoparametric elements are very important in fracture mechanics
Why?

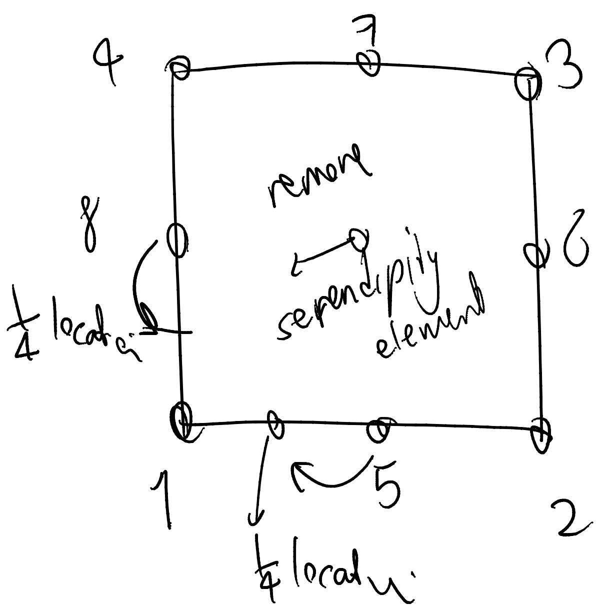
Isoparametric singular elements

- **LEFM**

$\epsilon, \sigma : \frac{1}{\sqrt{r}}$

Quarter point
Quad element





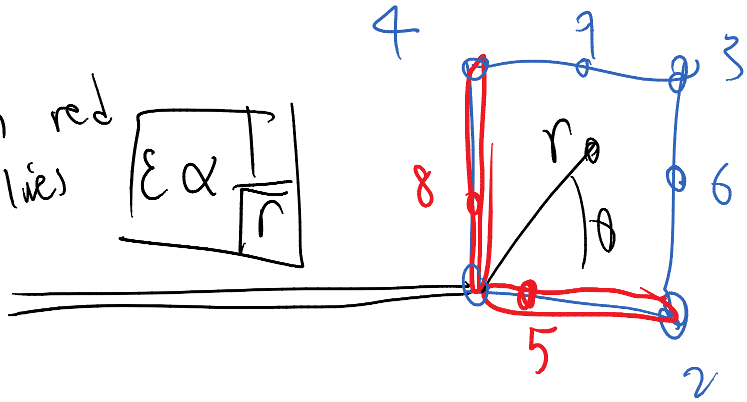
singular form $\frac{1}{\sqrt{r}}$ only along these lines
NOT recommended

$\varepsilon \propto \frac{1}{\sqrt{r}}$ on the red lines

So if point 1 is a crack tip

at least on those lines we recover strain singularities needed

on red lines $\boxed{\varepsilon \propto \frac{1}{\sqrt{r}}}$



LEFM

$$\varepsilon(r,\theta) = \frac{1}{\sqrt{r}}\,\tilde{\varepsilon}_{ij}(\theta)$$

At least on the red lines FEM is capable of capturing LEFM exact solution

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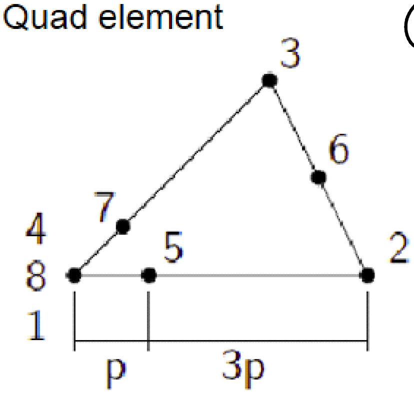
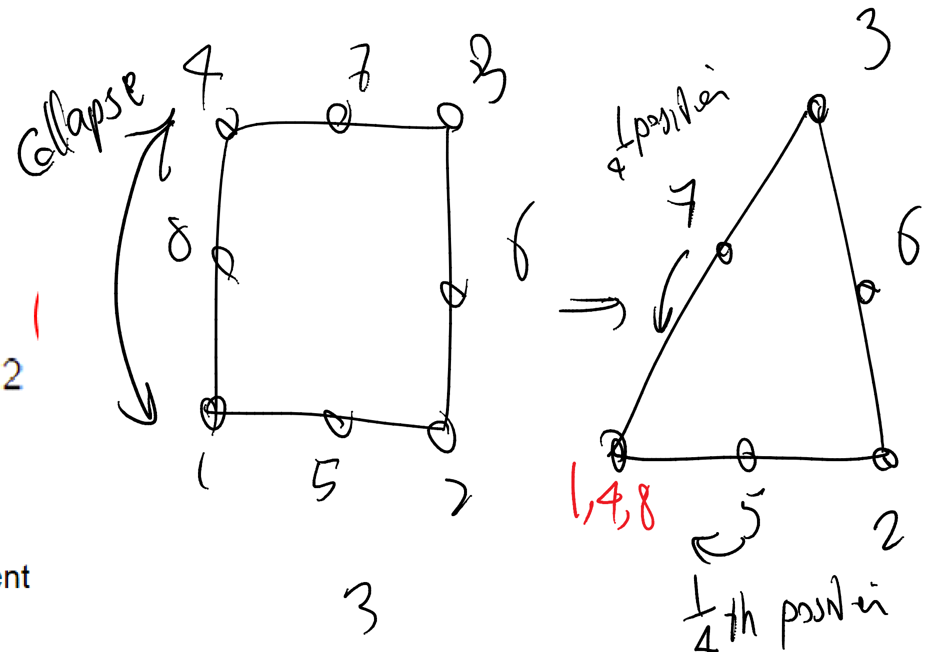
Improve this element by having one with singularity inside the element

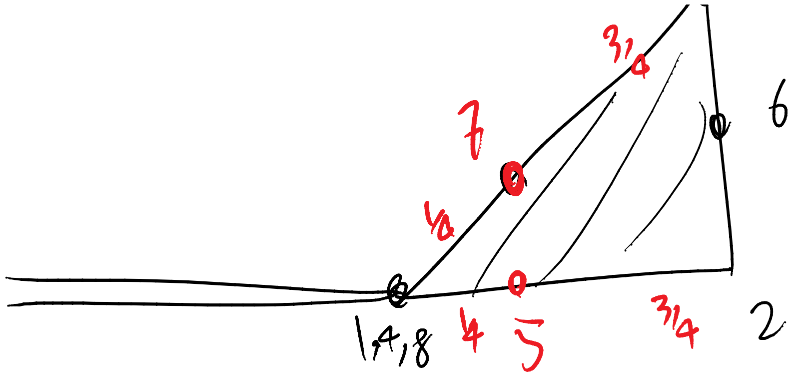
Quarter point **collapsed** Quad element



Improvement:

- $\frac{1}{\sqrt{r}}$ from inside all element





$\varepsilon \propto \frac{1}{\sqrt{r}}$ ✓

Problem

- Solution inaccuracy and sensitivity when opposite edge 3-6-2 is curved

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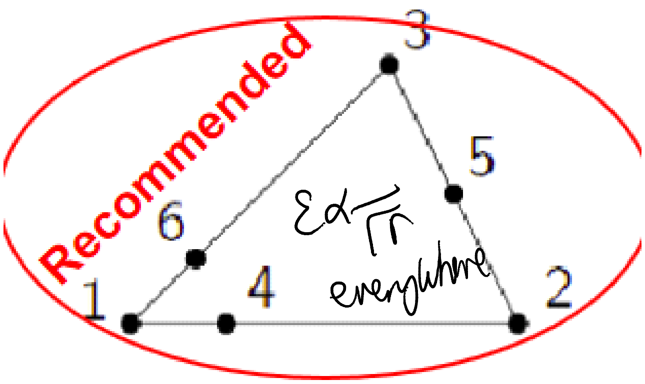
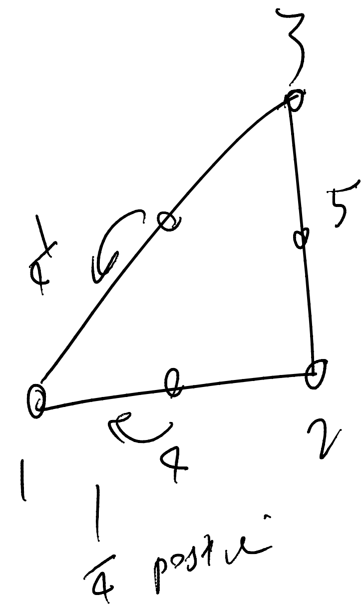
The best element for LEFM

Quarter point
Tri element

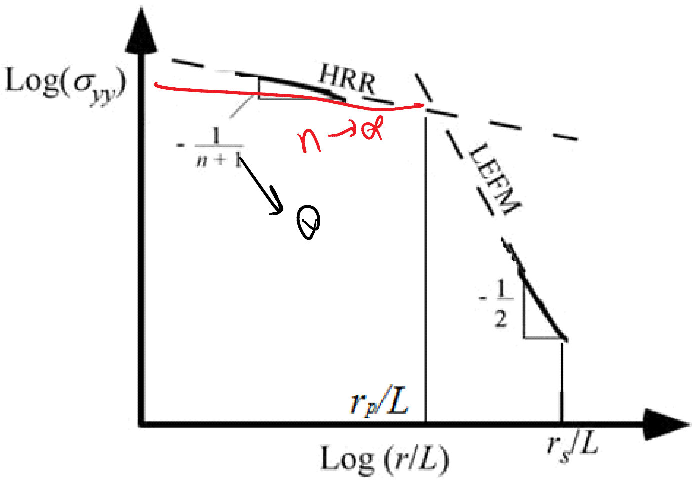


Improvement:

- Better accuracy and less mesh sensitivity

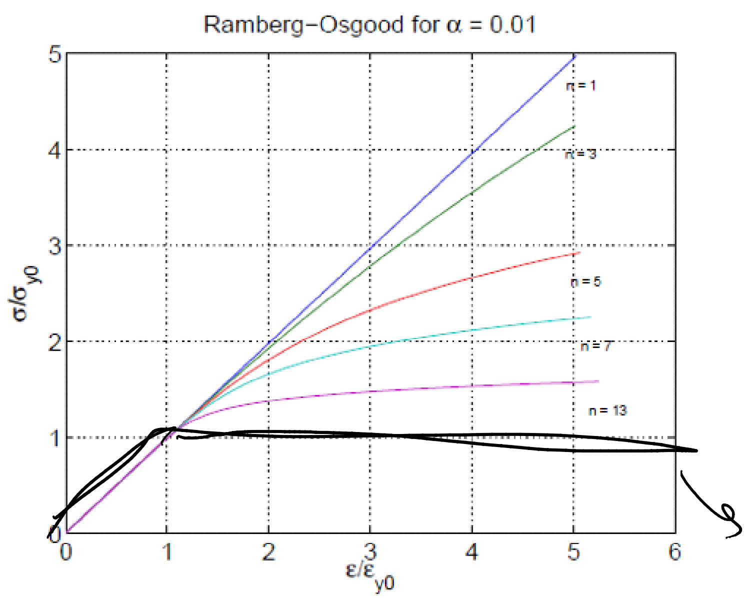


Elastic perfectly plastic



$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n,\theta)$$

$$\epsilon_{ij} = \frac{\alpha \sigma_0}{E} \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\epsilon}_{ij}(n,\theta)$$



$n \nearrow \infty$

elastic perfectly plastic

$n \to \infty$

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n,\theta)$$

$\frac{1}{n+1} \to 0$

$$\epsilon_{ij} = \frac{\alpha \sigma_0}{E} \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\epsilon}_{ij}(n,\theta)$$

$\frac{n}{n+1} \to 1$

$n \to \infty$

$$
\begin{cases}
\varepsilon(r) = \dfrac{1}{r} \\
\sigma(r) \to \dfrac{1}{r^{0}}
\end{cases}
$$

for $n \to \infty$

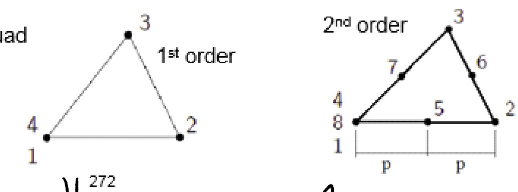
e.g. elastic perfectly plastic response

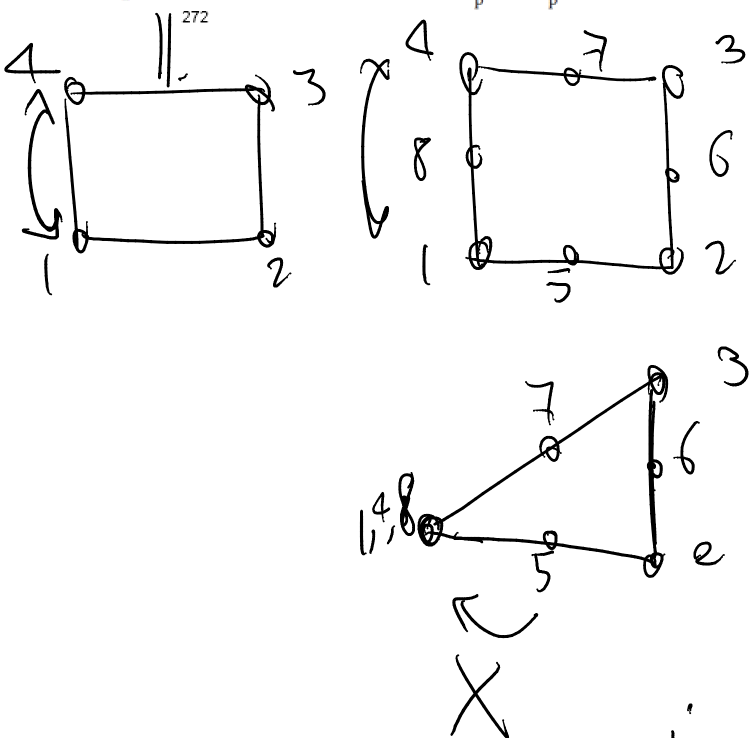
LEFM $\varepsilon(r), \sigma(r) \propto \dfrac{1}{\sqrt{r}}$

- **Elastic-perfectly plastic**

$\epsilon : \frac{1}{r}$

Collapsed Quad elements





leave it at the center