2016/11/22

Tuesday, November 22, 2016
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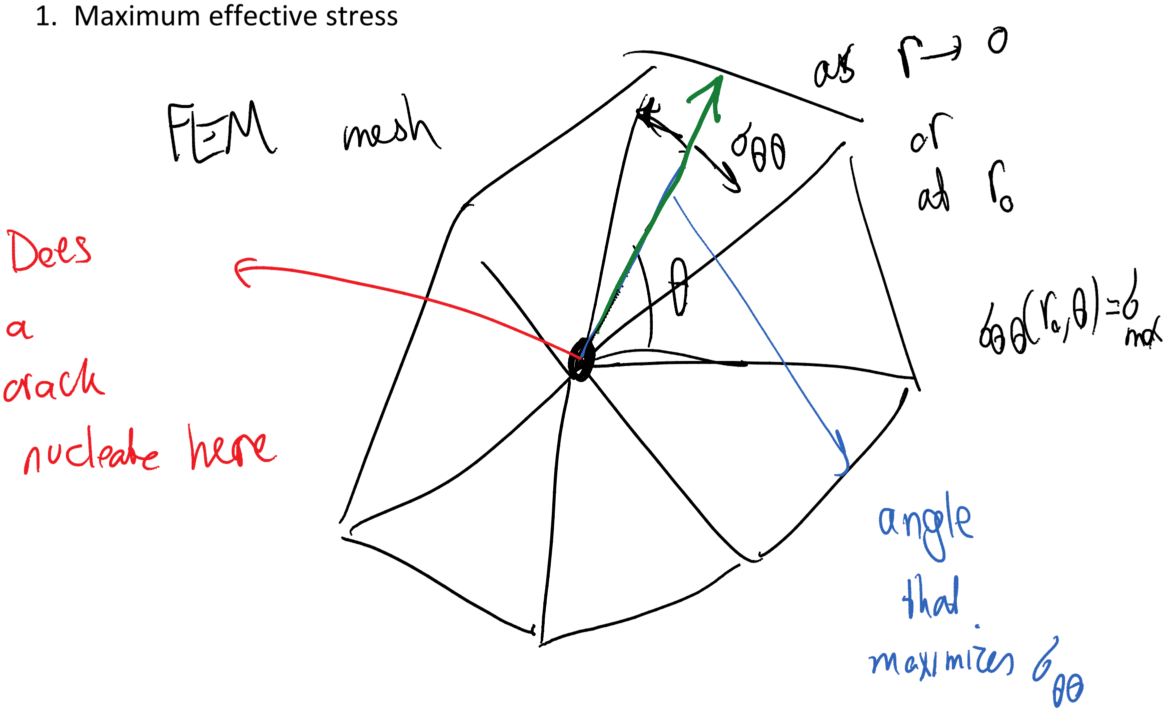
Crack propagation:
We talked about it before

Crack nucleation criterion

Different models:

1. Maximum effective stress

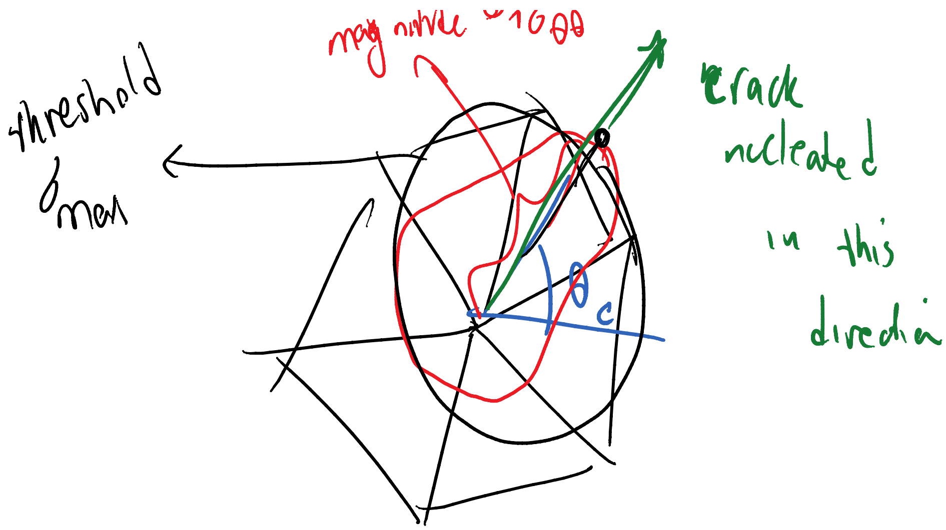


if the stress $\sigma_{\theta\theta}(r_0, \theta_{max})$ is the maximum of $\sigma_{\theta\theta}$ for $r=r_0$ & $\theta \in [0,2\pi]$ & this stress is higher than a threshold a crack can nucleate in θ_c direction

magnitude of $\sigma_{\theta\theta}$

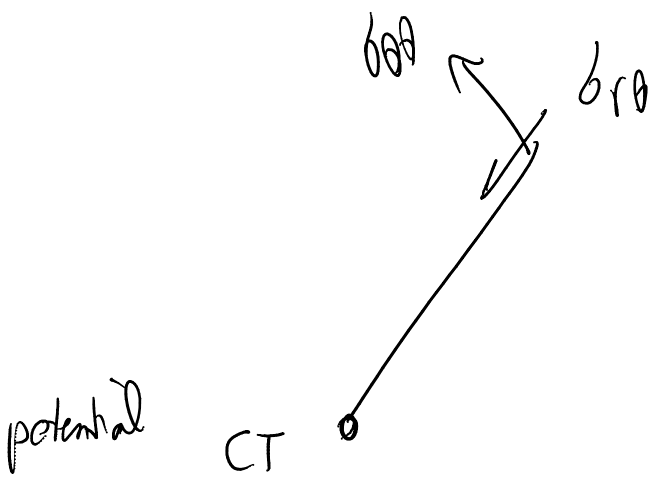
threshold

crack



We can generalize this

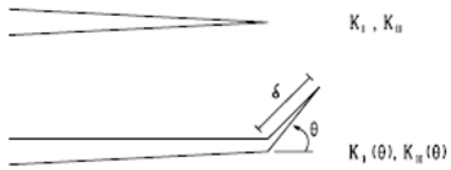
$$\sigma_{\theta\theta} \longrightarrow \sigma_{eff} = \sqrt{\langle\sigma_{\theta\theta}\rangle_+^2 + \beta^2 \sigma_{r\theta}^2}$$



This nucleation model goes really well with maximum circumferential (or effective) stress crack propagation model

How about the case when crack propagation is based on maximum energy release rate? Remember the following crack propagation condition:

Maximum Energy Release Rate



Stress intensity factors for **kinked crack extension**:
Hussain, Pu and Underwood (Hussain et al. 1974)

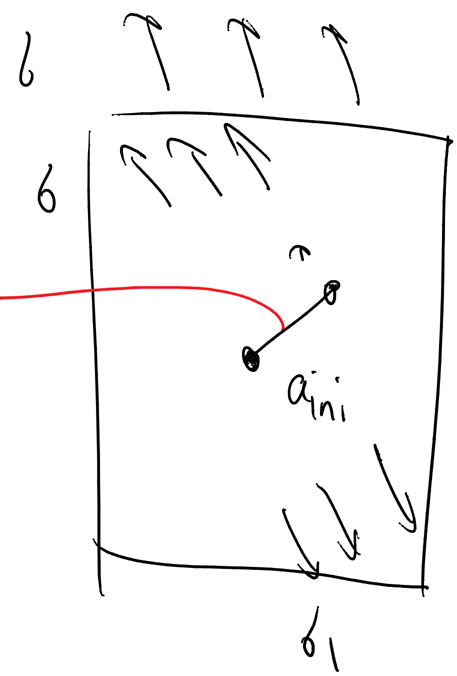
$$\left\{\begin{array}{c} K_I(\theta) \\ K_{II}(\theta) \end{array}\right\} = \left(\frac{4}{3+\cos^2\theta}\right)\left(\frac{1-\frac{\theta}{\pi}}{1+\frac{\theta}{\pi}}\right)^{\frac{\theta}{2\pi}}\left\{\begin{array}{c} K_I\cos\theta + \frac{3}{2}K_{II}\sin\theta \\ K_{II}\cos\theta - \frac{1}{2}K_I\sin\theta \end{array}\right\}$$

$$G(\theta) = \frac{1}{E'}\left(K_I^2(\theta) + K_{II}^2(\theta)\right)$$

$$G(\theta) = \frac{4}{E'}\left(\frac{1}{3+\cos^2\theta}\right)^2\left(\frac{1-\frac{\theta}{\pi}}{1+\frac{\theta}{\pi}}\right)^{\frac{\theta}{\pi}} [(1+3\cos^2\theta)K_I^2 + 8\sin\theta\cos\theta K_I K_{II} + (9-5\cos^2\theta)K_{II}^2]$$

What is an appropriate crack nucleation model?

existing defects that are not explicitly discretized



$$K_I = \sigma_1\sqrt{\pi a_{ini}}$$

local max principal stress

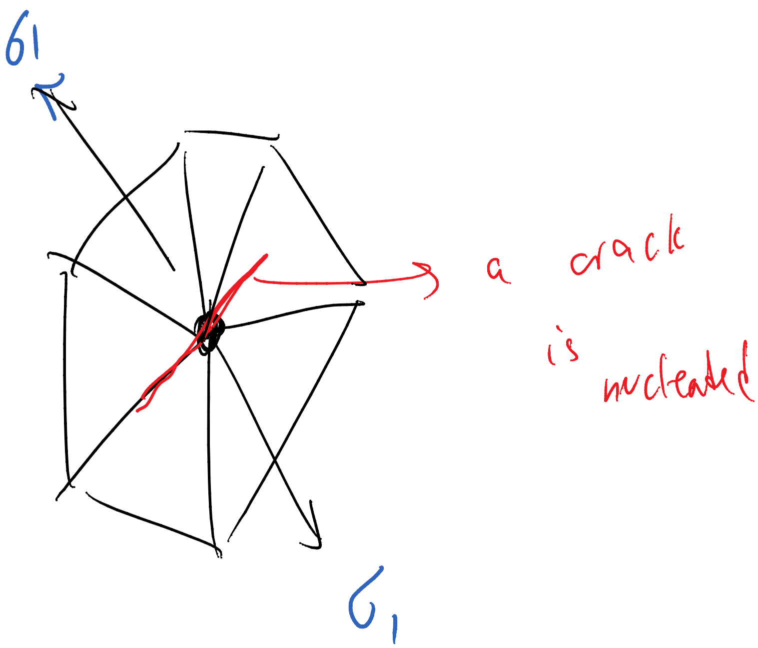
$$G = \frac{K_I^2}{E'} = \frac{\sigma_1^2 \, \pi a_{ini}}{E'} = G_c$$

G_c → fracture resistance

$$\boxed{\sigma_1 = \sqrt{\frac{G_c E'}{\pi a_{ini}}}} \quad \sigma_{crit}$$

$$\sigma_1 > \sigma_{crit} \implies \text{a crack is nucleated}$$

Normal to σ_1



What is a_{ini}?

* A user-specified flaw size in material that is not explitly modeled in FEM mesh.

* can also assign a_{ini} as a random variable $\longrightarrow$ stochastic fracture modeling

8. Fatigue

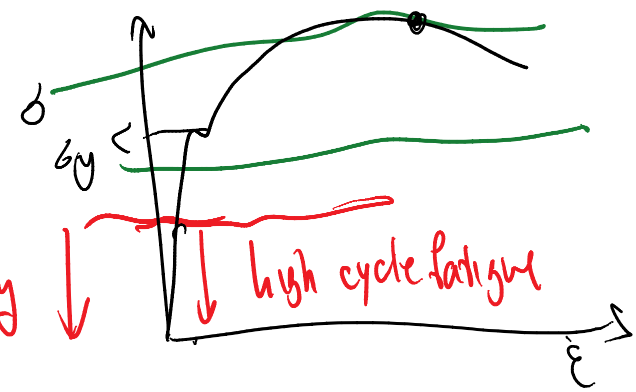
Fatigue Regimes

high cycle

Table 7.1 Classification of fatigue damage

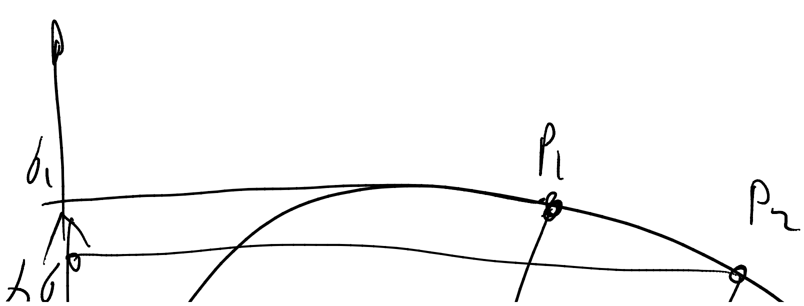
Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$
Very high cycle fatigue	$> 10^7$	$< \sigma_F$	≈ 0	≈ 0
High cycle fatigue	10^5 to 10^6	$< \sigma_Y$	≈ 0	≈ 0
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

low cycle

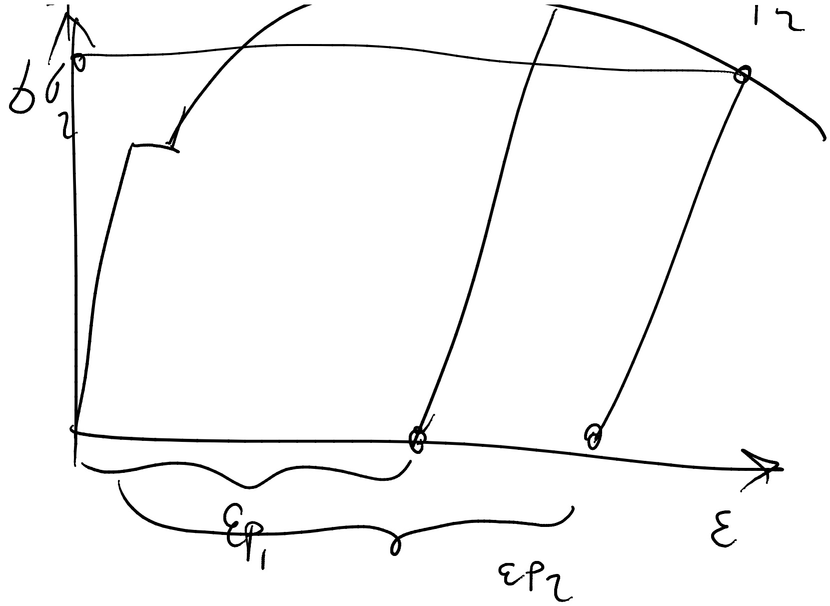


low cycle fatigue stresses are high

work with stress.
" " strain



Strain is used for low cycle fatigue because when we have large plastic deformation strain is a much better measure

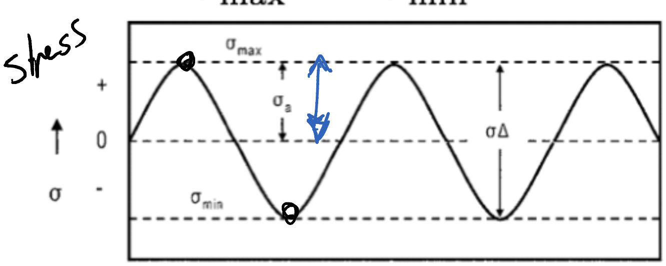


We only consider high cycle fatigue ->

- Very high cycle and high cycle fatigue:
  - Stresses are well below yield/ultimate strength.
  - There is almost no plastic deformation (in terms of strain and energy ratios)
  - Fatigue models based on LEFM theory (e.g. SIF K) are applicable.
  - Stress-life approaches are used (stress-centered criteria)

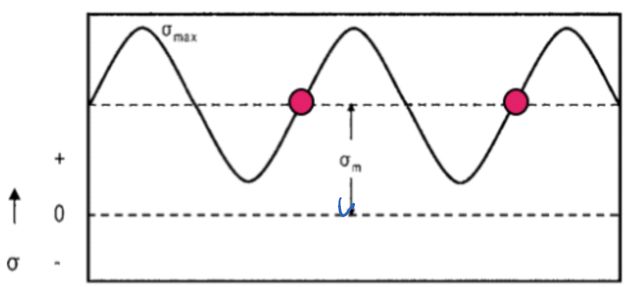
Definitions:

$$\sigma_{\max} = -\sigma_{\min}$$



Fully Reversed Loading

time



Tension-Tension with Applied Stress

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

$$\sigma_a = 0.5(\sigma_{\max} - \sigma_{\min})$$

half of change in stress

$$\sigma_m = 0.5(\sigma_{\max} + \sigma_{\min})$$

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad \text{load ratio}$$

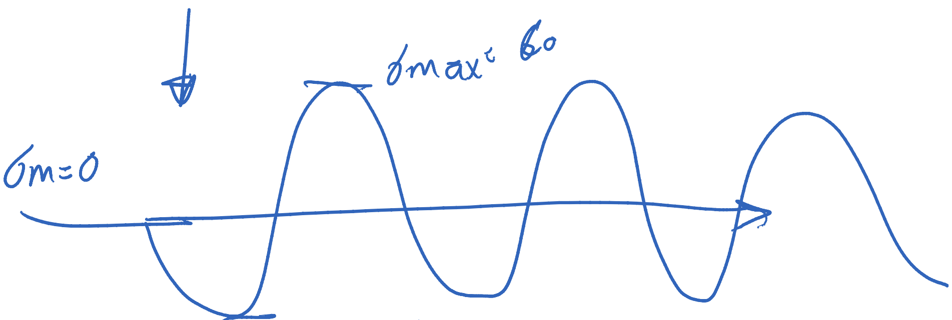
$R = \frac{\sigma_{\min}}{\sigma_{\max}} > 0$

if stresses decrease $R \to -\infty$

$\sigma_{\max} \to 0 \quad R \to -\infty$

$\sigma_{\max} = \sigma_0$

$\sigma_m = 0$

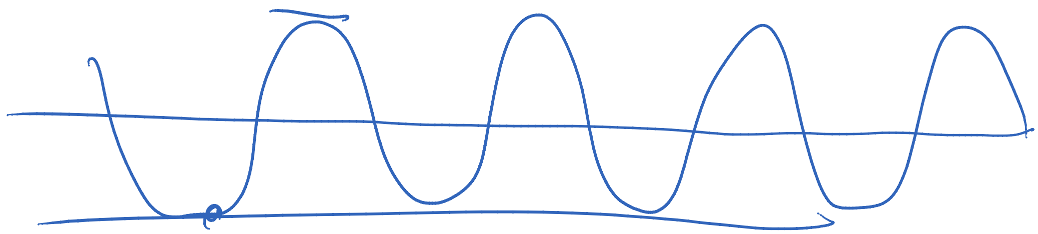


$\sigma_{\min} = -\sigma_0$

$R = \frac{-\sigma_0}{\sigma_0} = -1$

$\sigma_{\max} = 2\sigma_0$

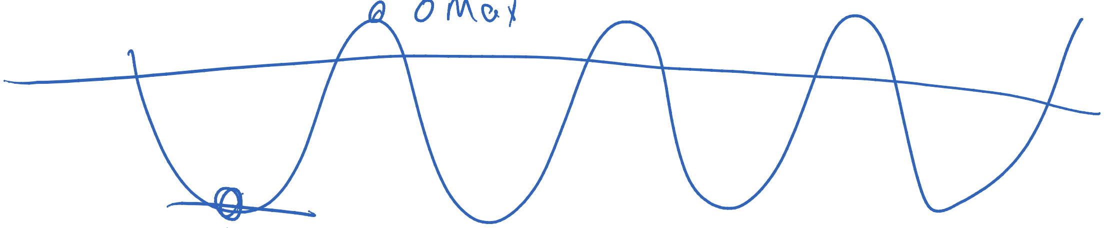
$R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{0}{2\sigma_0} = 0$



$\sigma_m = \sigma_0$

$\sigma_{\min} = 0$

$\sigma_{\max}$



σ_m average fixed

$\sigma_{\min}$

let $\sigma_{\max} \nearrow \infty$

$R = \frac{\sigma_{\min}}{\sigma_{\max}} \to 1$

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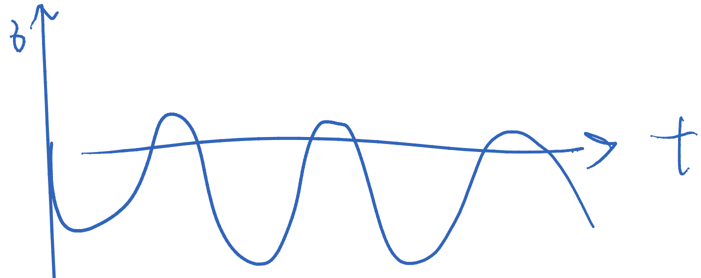
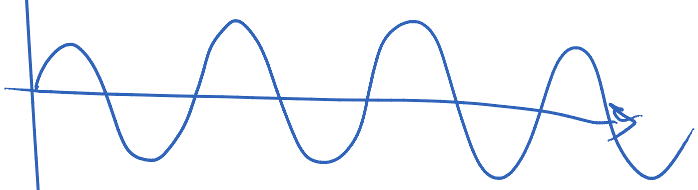
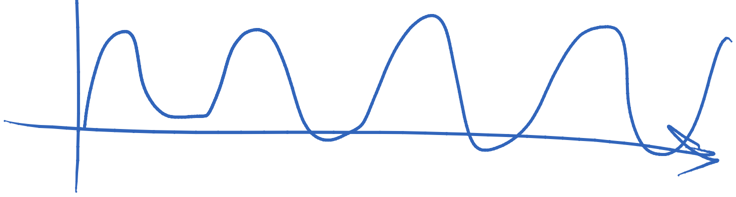


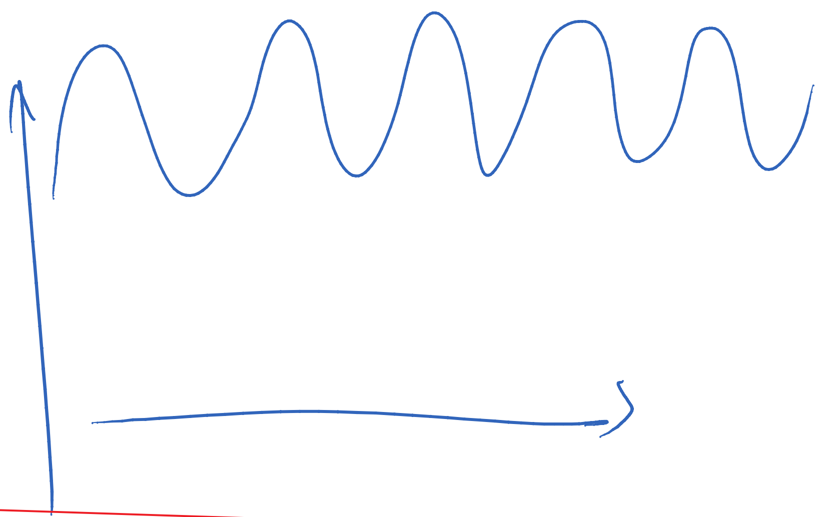
$R \to -\infty$



$R = -1$

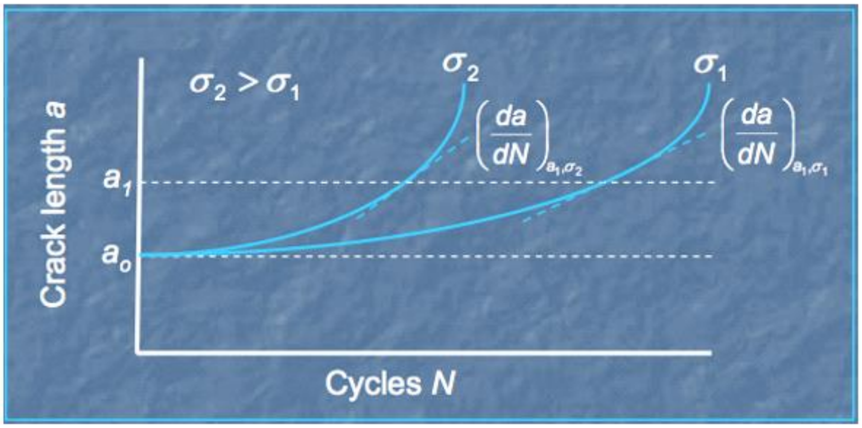


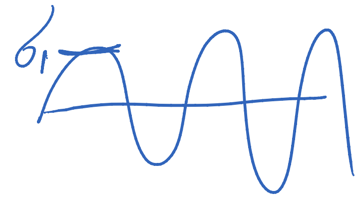
$R = 0$



$\sigma_{max} \to \infty$

$R \to 1$

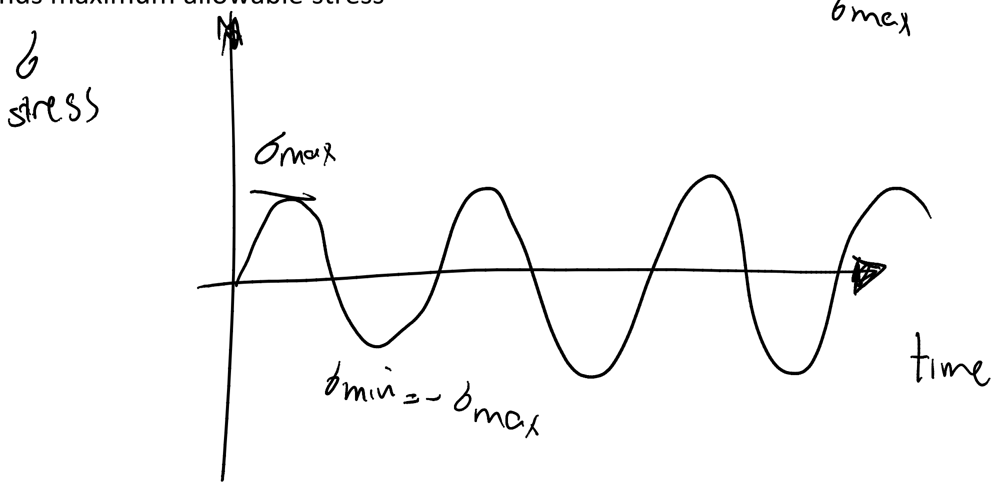


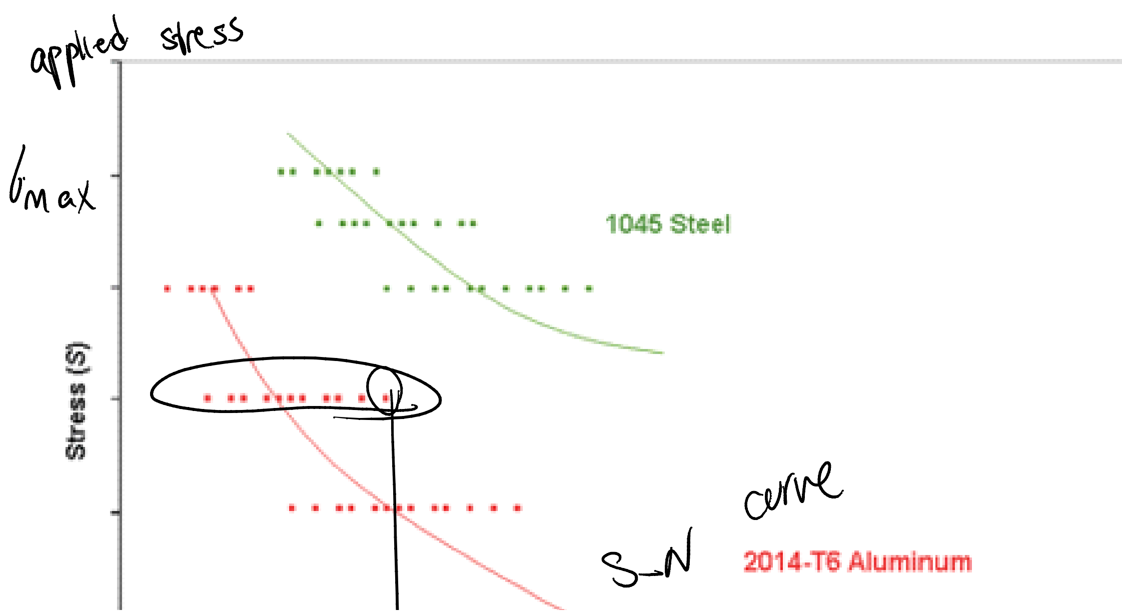


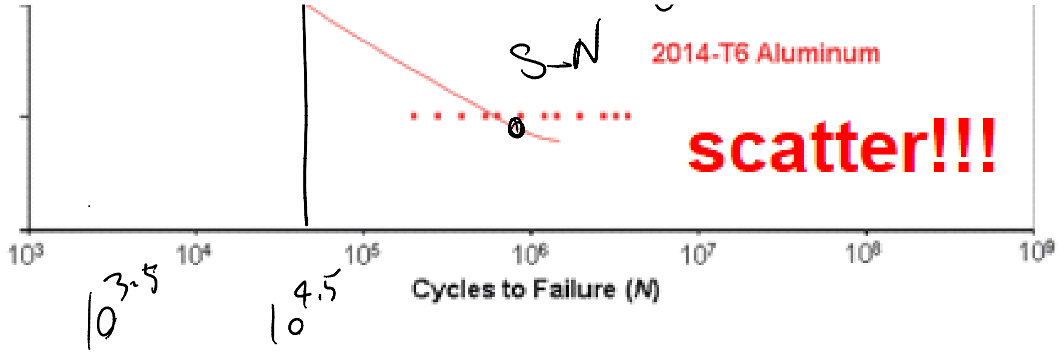
Two types of approaches for fatigue crack growth:

$$R = \frac{\sigma_{min}}{\sigma_{max}} = -1$$

1. Older approach: Finds maximum allowable stress





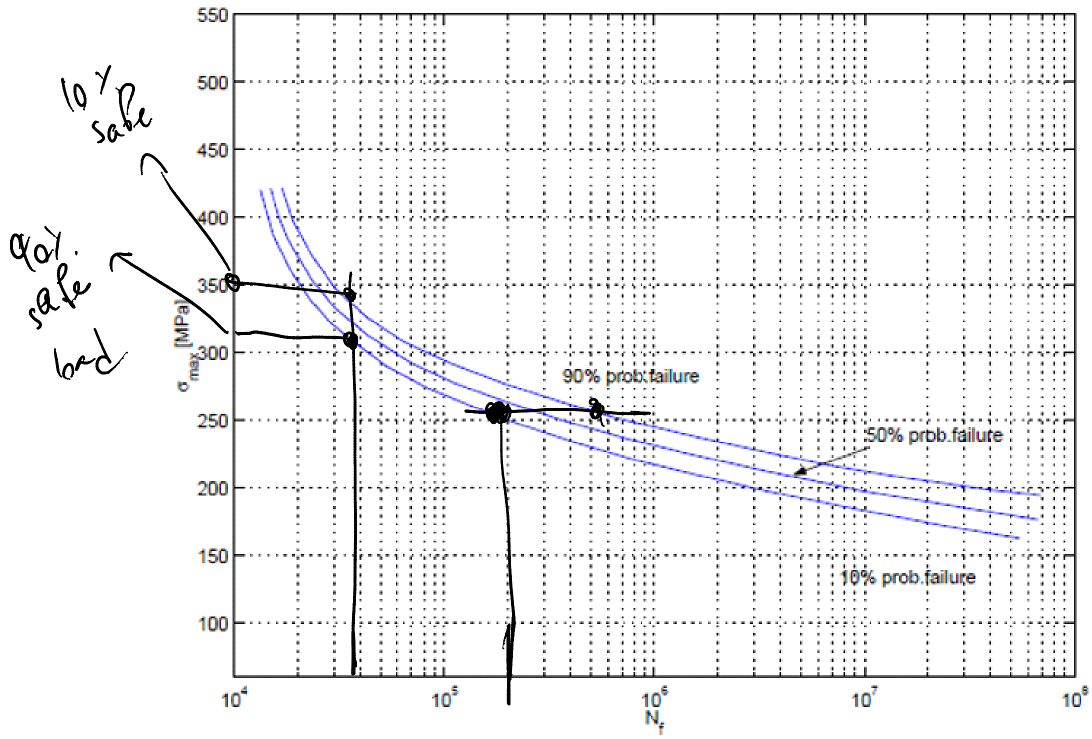


about an order of magnitude difference in the # cycles that material breaks under cyclic loading

How can we handle the scatter in data?

We add P(probability) to the S-N curves:

S-N-P curve: scatter effects

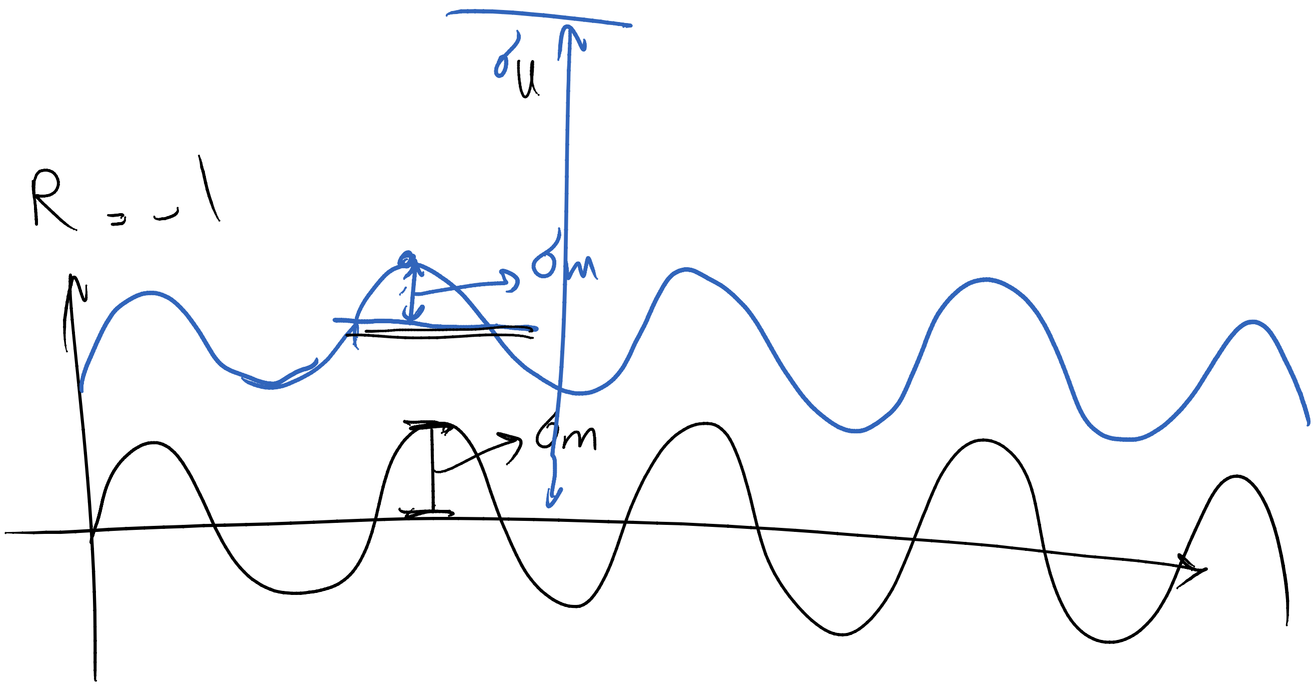


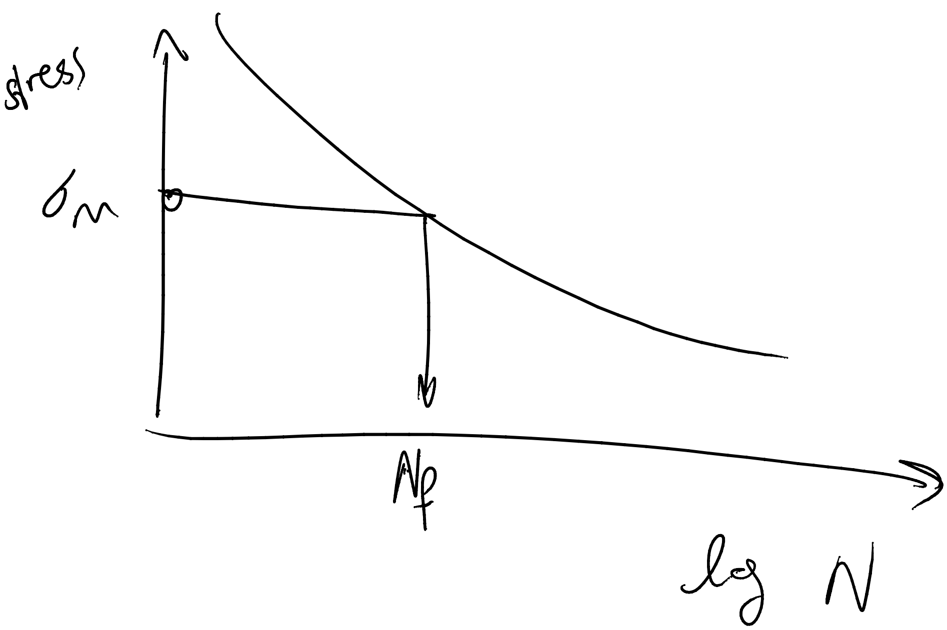
P-S-N-curves

One correction to S-N plot when the loading is not centered at zero

S-N curves are for

R = -1





$\sigma_{average}$

correction factor

Approach 2:

$$\sigma_a = \sigma_{f0}\left[1-\left(\frac{\sigma_m}{\;}\right)^{\gamma}\right]$$

Approach 2:
Correction-factor formulas

$$\sigma_a = \sigma_{f0}\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^{\gamma}\right]$$

$\sigma_{allowable}$

$\sigma_{allowable}$ for $R=-1$
symmetric case

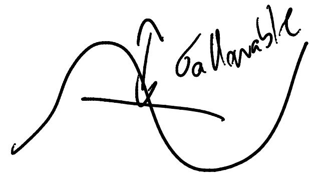
$\sigma_{ultimate}$

Correction factor $= \left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^{\gamma}\right]$

for $\sigma_m = \sigma_u$, " " $= \left[1-(1)^{\gamma}\right] = 0$

$\sigma_m \longrightarrow \sigma_u$

$\sigma_{allowable} \Rightarrow 0$



Gerber (1874) $\quad \dfrac{\sigma_a^*}{\sigma_a} = 1 - \left(\dfrac{\sigma_m}{\sigma_u}\right)^2$

Goodman (1899) $\quad \dfrac{\sigma_a^*}{\sigma_a} = 1 - \dfrac{\sigma_m}{\sigma_u}$

Soderberg (1939) $\quad \dfrac{\sigma_a^*}{\sigma_a} = 1 - \dfrac{\sigma_m}{\sigma_{y0}}$

$$\sigma_a = \sigma_{f0}\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^r\right]$$

where σ_a is the amplitude of allowable stress (alternating stress).

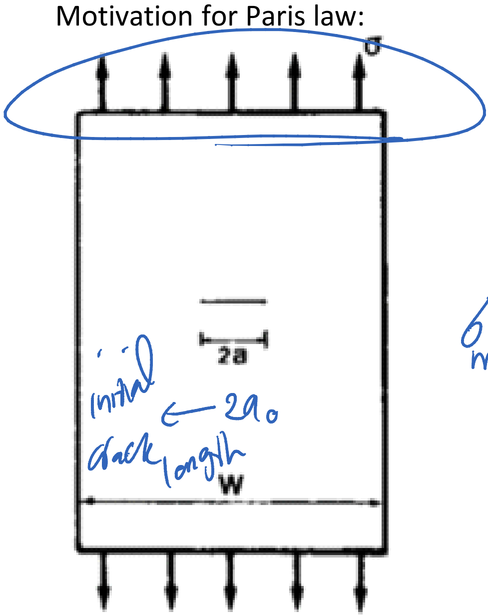
σ_{f0} is the stress at fatigue fracture when the material under zero mean stress cycled loading

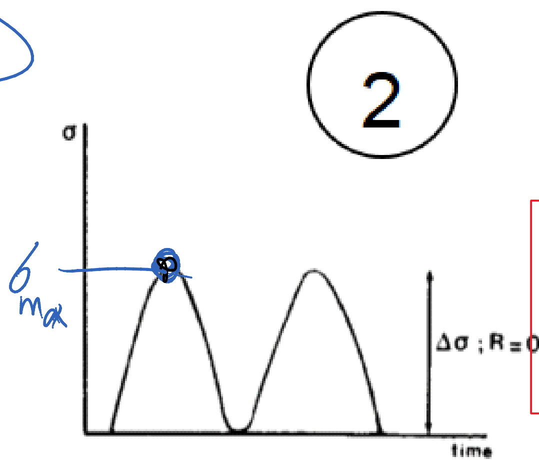
σ_m is the mean stress of the actual loading.

σ_u is the tensile strength of the material.

r = 1 is called Goodman line which is close to the results of notched specimens.
r = 2 is the Gerber parabola which better represents ductile metals.

Motivation for Paris law:





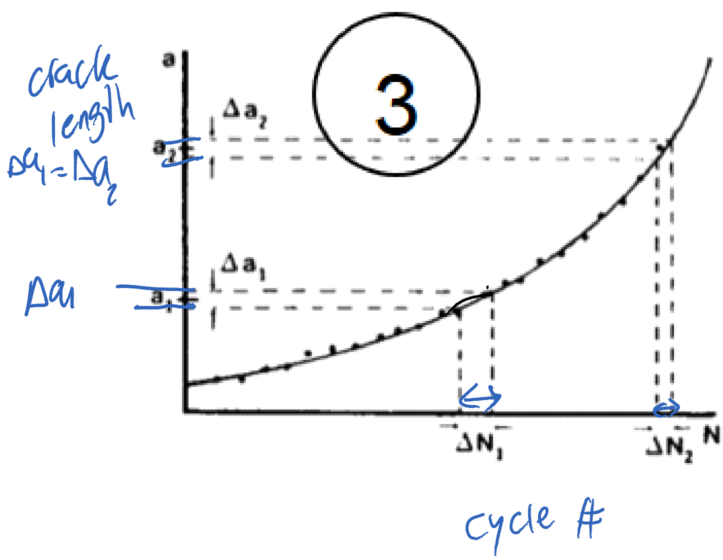
$$K = \sigma\sqrt{\pi a}$$

$$\boxed{K_{max} = \underbrace{\sigma_{max}}_{\text{constant}}\underbrace{\sqrt{\pi a}}_{\text{increases}}}$$

$K_{max} \uparrow$ as $a \uparrow$

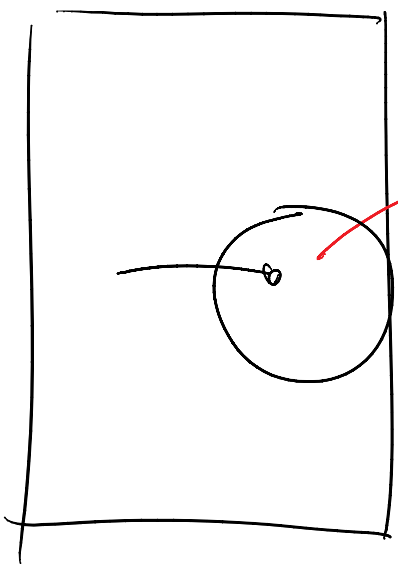
$\Longrightarrow$ we know

$\left.\begin{array}{l}\frac{da}{dN} \uparrow \text{ as } a\uparrow \\ K_{max} \uparrow \text{ as } a\uparrow\end{array}\right\} \rightarrow$



$\Delta N_2 < \Delta N_1$

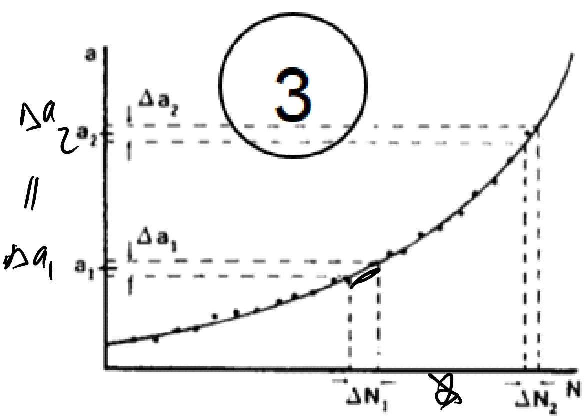
Maybe K_{max} determines crack growth rather than ($\Delta\sigma$, or σ_{max})



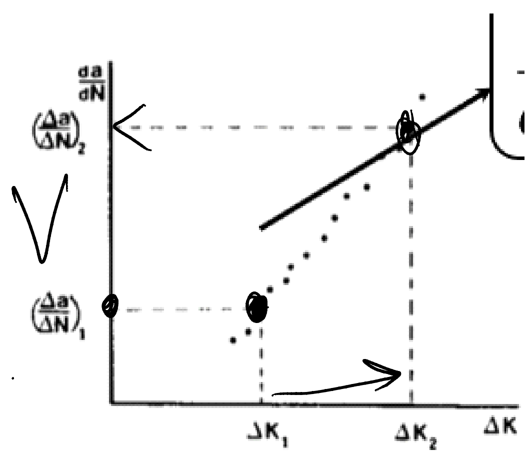
what determines local stress field here?

$$\sigma^{ij} = \frac{K}{\sqrt{2\pi r}} f^{ij}(\theta)$$

K → logically K should determine fatigue crack growth!



$\Delta N_1 > \Delta N_2$



$\Delta K_1 = \Delta\sigma\sqrt{\pi a_1} < \Delta K_2 = \Delta\sigma\sqrt{\pi a_2}$

Paris law:

Postulated $\frac{da}{dN} = f(\Delta K)$

Specific form of f:

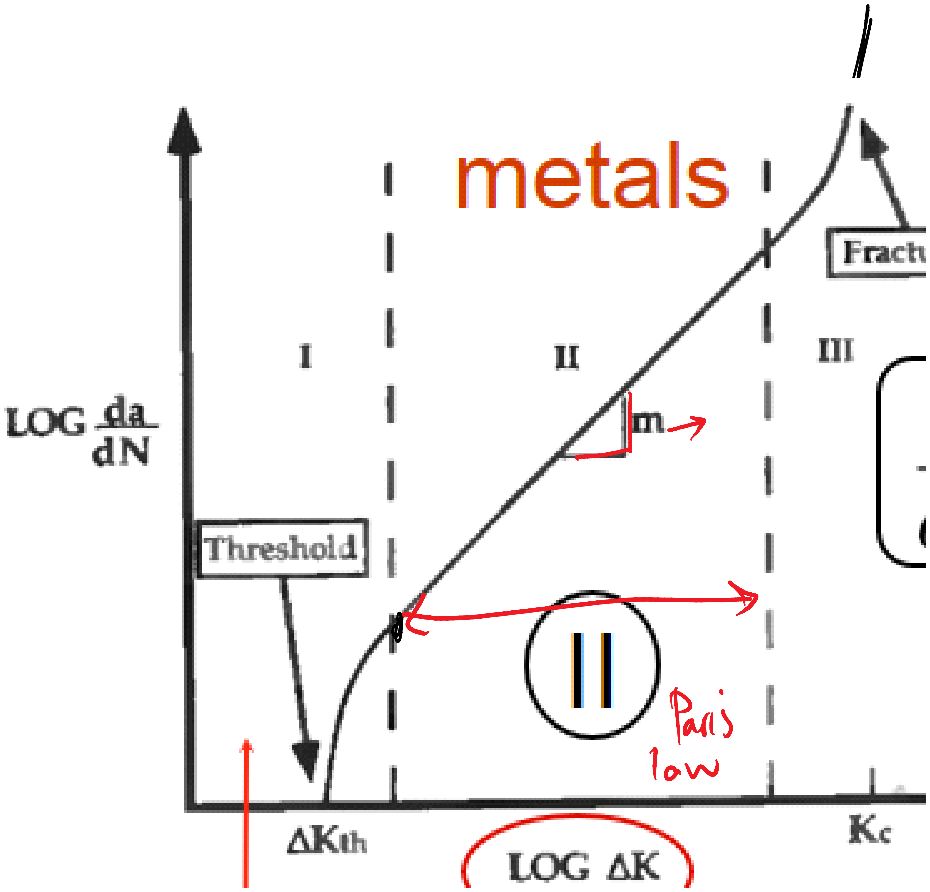
$$\boxed{\frac{da}{dN} = C(\Delta K)^m, \; \Delta K = K_{\max} - K_{\min}}$$

In log-log scale

$$\log \frac{da}{dN} = \log C(\Delta K)^m =$$

$$\underbrace{\log \frac{da}{dN}}_{y \text{ axis}} = \log C + m \underbrace{\log \Delta K}_{x \text{ axis}}$$

the slope in log-log plot



$\Delta K < \Delta K_{th} \longrightarrow$ no fatigue crack propagation

I slow fatigue crack propagation

II medium " " "

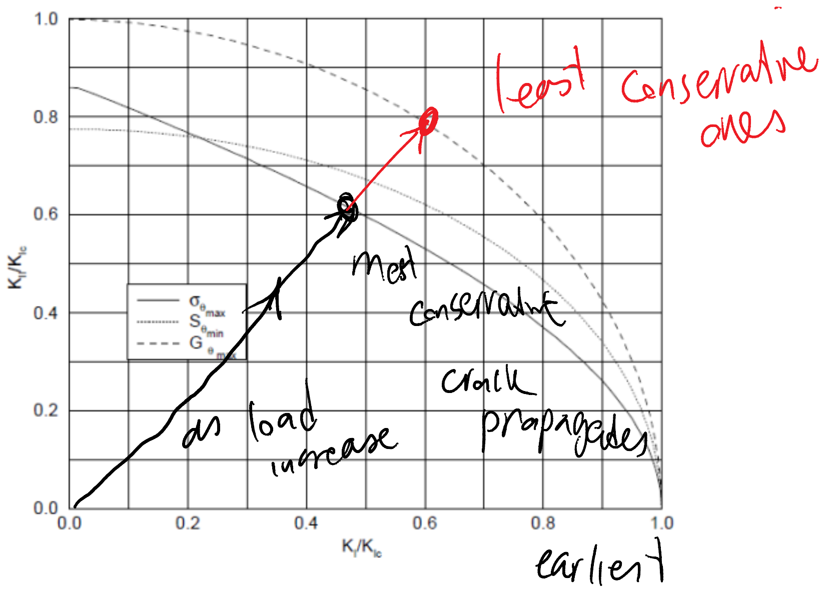
Paris law holds

III fast - crack propagation

III

fast - crack propagation

HW3:



$K_I = K_{II}$