1. For steel the elastic modulus $E = 210$ GPa. For a given specimen material strength $\sigma_f = 250$MPa. (**40 Points**)
   (a) Provide an estimate for theoretical strength σ_c based on atomistic potential.
   (b) What is the ratio σ_f/σ_c and what is the reason for discrepancy?
   (c) Obtain an estimate for the ratio $r = a/x_0$ for a = representative defect size in the material and x_0 = atomistic lattice length scale.
   (d) If $x_0 \approx 10^{-10}$ m obtain an estimate for surface energy γ_c.

2. For a specimen which can be characterized by a point force P and displacement u we have two measurements of (u_1, P_1) for crack length a_1 and (u_2, P_2) for crack length a_2. The stiffness is defined as $K = P/u$. Recalling that $G = -\frac{\partial \Pi}{\partial A} = -\frac{\partial \Pi}{\partial a B}$ for A crack surface, a crack length, and B crack (a) obtain an equation for G in terms of u and $\frac{\partial K}{\partial a}$, (b) Given that compliance $C = 1/K$ show that $G = \frac{P^2}{2B}\frac{\partial C}{\partial a}$. (**70 Points**)
   Hint: Note that $K = \tan(\theta)$, $G \approx -\frac{\Pi_2 - \Pi_1}{\Delta a B} = -\frac{U_{e2} - U_{e1} - W_{12}}{\Delta a B} = \frac{\text{Shaded area}}{\Delta a B}$. Finally, express shaded area using $\Delta\theta$.

   Remark: You do not need to show that G = $\lim_{\Delta a \to 0}$ shared area / $(B\Delta a)$. You need to show that the limit expression is equal to $G = \frac{P^2}{2B}\frac{\partial C}{\partial a}$ by using the small angle approximation for $\Delta\theta$.

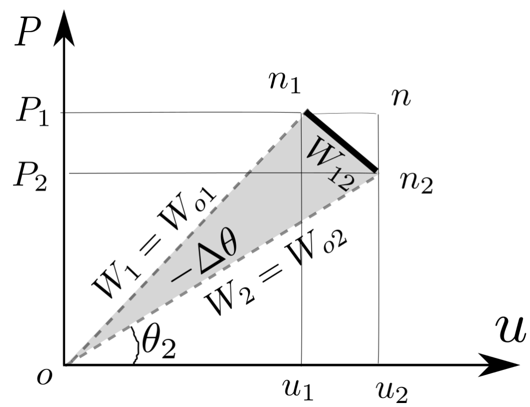

Figure 1: Force displacement relation for a point force system.

3. A composite laminate is made by bonding two long thin strips of steel with an adhesive epoxy, as shown in figure 2. A patch of the adhesive was intentionally left out in the central section in order to create a central crack of length $2a$ in the bilayer plate. The joined plates are pulled apart by equal and opposite tensile forces, P. From beam theory, the deflection of a double cantilever beam of length L (half the crack length) under load P is $\Delta = \frac{PL^3}{192EI} \Rightarrow$

$$\frac{\delta}{2} = \frac{P(2a)^3}{192EI} \quad \Rightarrow \quad \delta = \frac{Pa^3}{12EI}$$

where the crack opening δ is the displacement of load P. Since there are two crack front we have,

$$2G = \frac{P^2}{2B}\frac{\partial C}{\partial a}$$

for $C = \delta/P$. The parameters for the problem are: initial crack length $2a_0 = 60$ mm, $E = 200$ GPa, $H = 0.97$ mm and $B = 10.1$ mm.

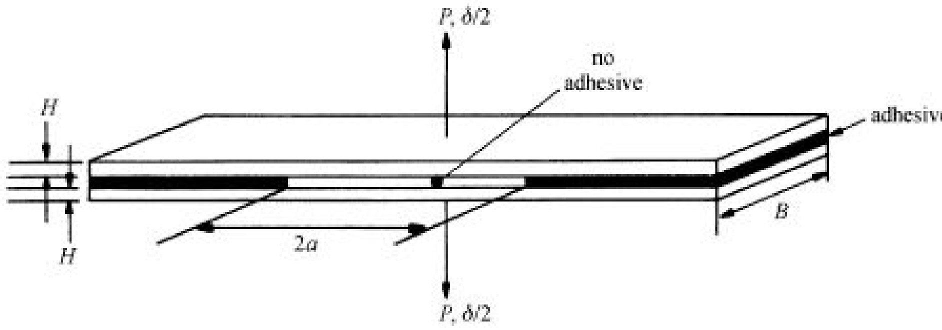

Figure 2: Geometry and loading of joined plates (image source: Suresh, Fatigue of materials)

Consider two different R (crack propagation resistance) equations:

$$R_1(a) = R_0 \tag{1a}$$

$$R_2(a) = \begin{cases} R_0 & a = a_0 \\ R_0 + \mathit{\Delta} R \left[1 - \left(\frac{a-a_0-l}{l}\right)^2\right] & a_0 < a < l + a_0 \\ R_0 + \mathit{\Delta} R & l + a_0 < a \end{cases} \tag{1b}$$

Basically R_1 corresponds to a constant resistance and R_2 is for a case that R increases from R_0 from $a = a_0$ to $R_0 + \mathit{\Delta} R$ at $a = a_0 + l$ by a parabolic equation that smoothly (zero slope at $a = a_0 + l$) transitions to constant $R = R_0 + \mathit{\Delta} R$ for larger crack lengths. The parameters $\mathit{\Delta} R$ and l are strengthening of crack resistance due to crack propagation and a characteristic length respectively. Let $R_0 = 300$ Pa.m, $\mathit{\Delta} R = 1200$ Pa.m and $l = a_0 = 30$ mm. Answer the following (**140 Points**):

(a) Plot R-Curve for R_1 and R_2 (Resistance R versus crack length a).

(b) Consider load control and displacement control methods to initiate and cause crack propagation. In load control we increase P until at P_{ini} crack starts propagating. If a critical value P_{cr} exists once $P = P_{\text{cr}}$ crack propagation becomes unstable, *i.e.*, propagates without increasing P. Same concepts apply for displacement control to δ_{ini} and δ_{cr} (if it exists). Add G curves for load control $P = 45, 105, 130$N and displacement control for $\delta = 1$ mm and 3mm to the same R plot that included R_1 and R_2 curves.

(c) For constant resistance given by R_1 obtain P^1_{ini}, P^1_{cr} (if for any a crack becomes unstable) and for load control δ^1_{ini} and δ^1_{cr} (if for any a displacement control method results in unstable crack propagation).

(d) For increasing resistance case R_2 obtain P^2_{ini}, P^2_{cr} and for load control δ^2_{ini} and δ^2_{cr}. Again, note that the critical values may not exist.

(e) Schematically add G curves for P^2_{ini}, δ^2_{ini}, P^2_{cr} (if exists), and δ^2_{cr} (if exists) to G, R plot from previous steps. Which of loadings eventually result in unstable crack propagation?

(f) For stable crack growth the relation between a, P, and δ is obtained by $G = R$ (crack force equal to crack resistance). For load control and displacement control solve for both P and δ as functions of a for two different cases of resistance R_1 and R_2 (2×2 solutions) and generate the following plots: 1) P (vertical axes) plotted versus δ (horizontal), 2) a versus P, and 3)(optional) a versus δ. The explicit form of these relations are not needed and only the plots need to be correct.

(g) Based on all the generated plots compare load control and displacement control and also constant R versus increasing R cases.