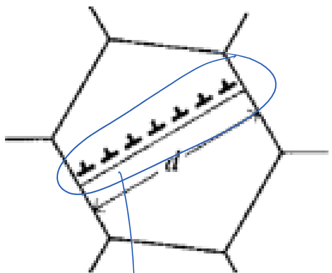
2018/08/29

Wednesday, August 29, 2018 11:35 AM

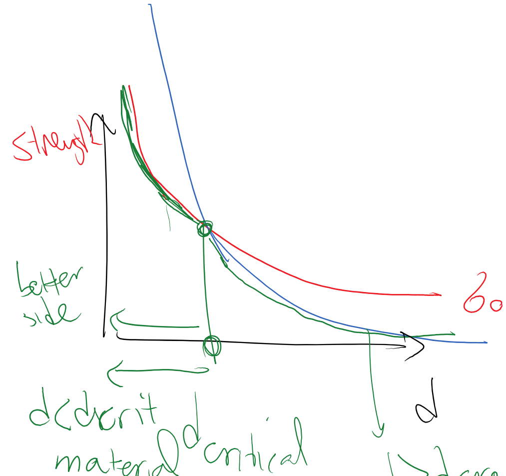
Ductile to brittle transition,
Effect of the grain size:



Hall-Petch effect

yield strength $\sigma_y = \sigma_o + \frac{k_y}{\sqrt{d}}$

yielding



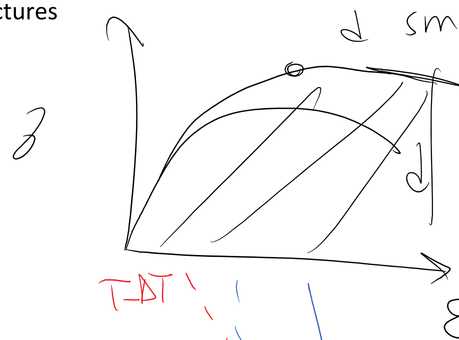
length scale of a crack ~ d

fracture strength $\sigma_{max} = \frac{K_c}{\sqrt{d}}$ → crack length

K_c → fracture toughness

For small enough grains material yields first and for large ones fractures

By decreasing the grain size two good things happen:
1. Strength increases
2. Toughness increases



strength & toughness ~ energy under the curve increase

Relating grain size to thermal sensitivity of toughness

What happens if T ↘

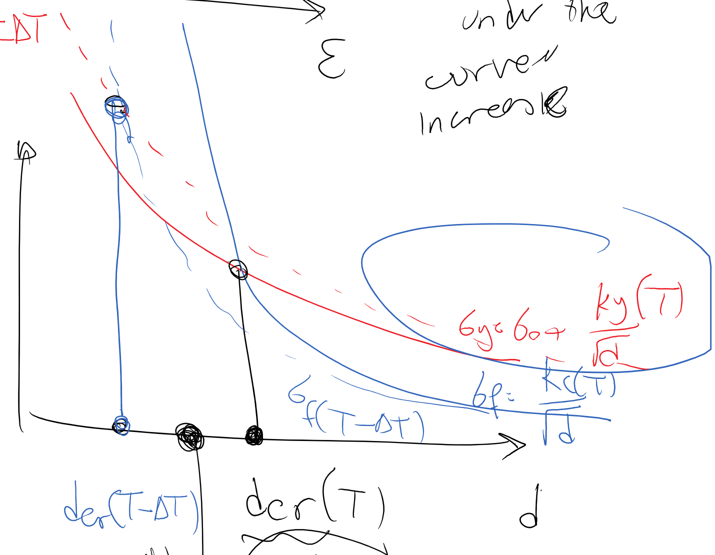
Shown by 5

$K_y \nearrow$ & $\sigma_y \nearrow$ because dislocation motion inhibited

$\sigma_f(T-\Delta T) = \frac{K_c(T-\Delta T)}{\sqrt{d}} \searrow$

by decreasing T

d_o grain size material was initially yielding first :)



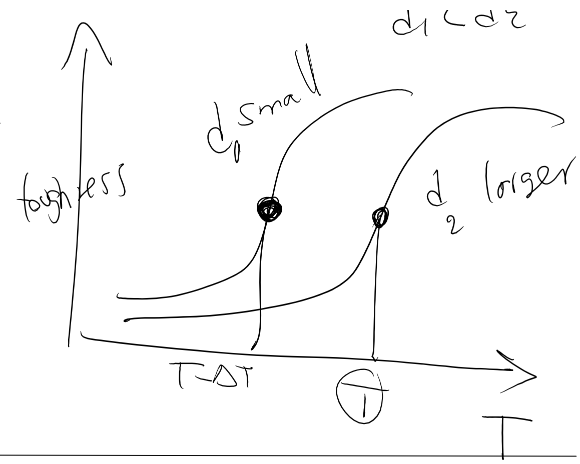
a sample grainsize $d_o < d_{cr}(T)$

$d_1 < d_2$

small

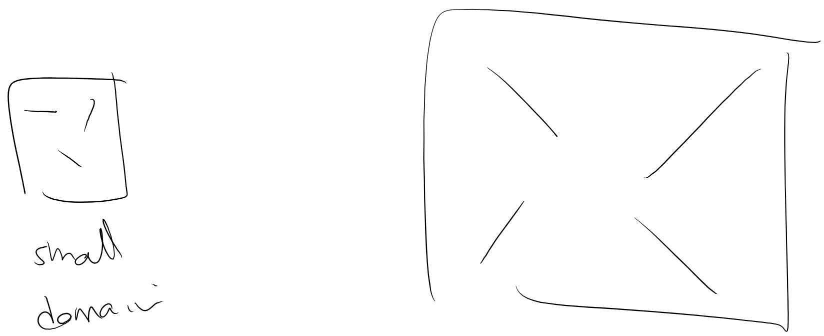
yielding first :)

Now it's in fracture zone :(



Larger grains -> material becomes less tough and less strong (lower strength)

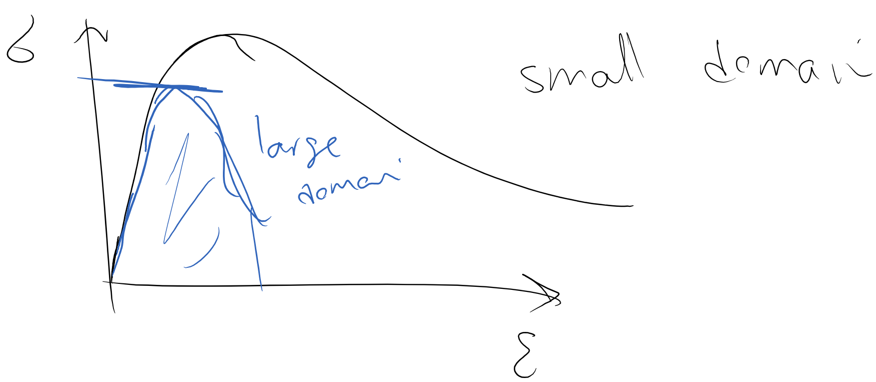
Size effect:
As make the domain larger, the strength (load capacity divided by "area") decreases

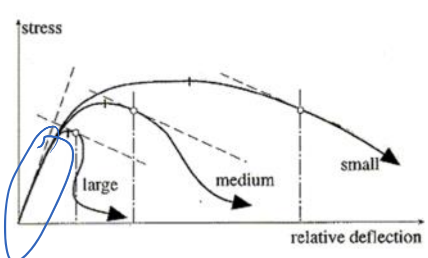


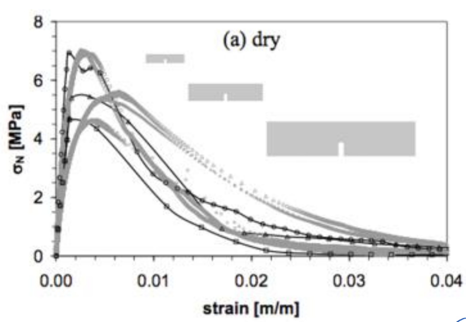
small domain
defects are small

defects get larger as well

strength goes down



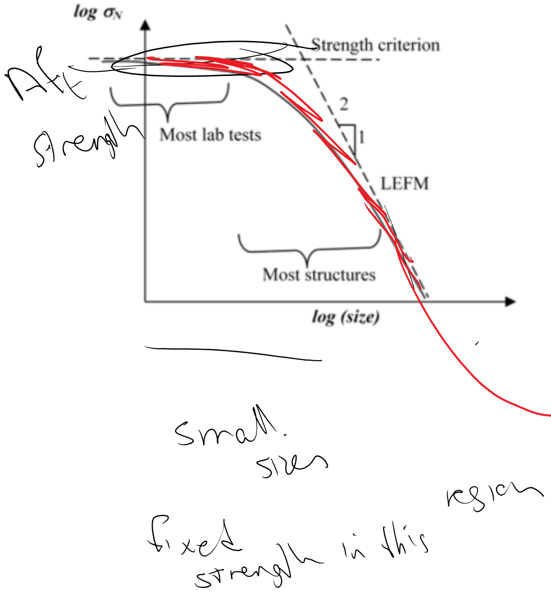


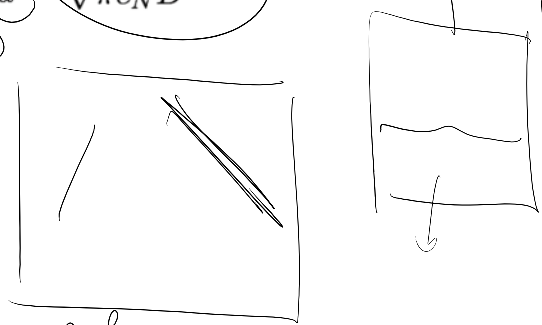


Elastic properties are almost insensitive to size

$$\sigma_N = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{K_{Ic}}{\sqrt{\pi c_N D}}$$

$a \propto D$ $\quad$ $\sigma_N \propto \frac{1}{\sqrt{D}}$





$$\sigma_N = \frac{Af_t}{\sqrt{1 + \frac{W}{B}}}$$

size

$\approx Af_t$

$\approx \frac{Af_t}{\sqrt{\frac{W}{B}}}$ → fracture lines

Bazant equation

Bazant's size effect law

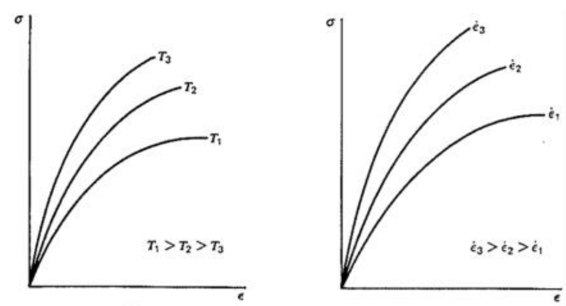
$$(\sigma_N)_u = Af_t\left(1 + \frac{W}{B}\right)^{-1/2} \qquad (14.8)$$

where

$(\sigma_N)_u$ = Nominal stress at failure of a structure of specific shape and loading condition.
W = Characteristic length of the structure.
A, B = Positive constants that depend on the fracture properties of the material and on the shape of the structure, but not on the size of the structure.
f_t = Tensile strength of the material introduced for dimensional purposes.

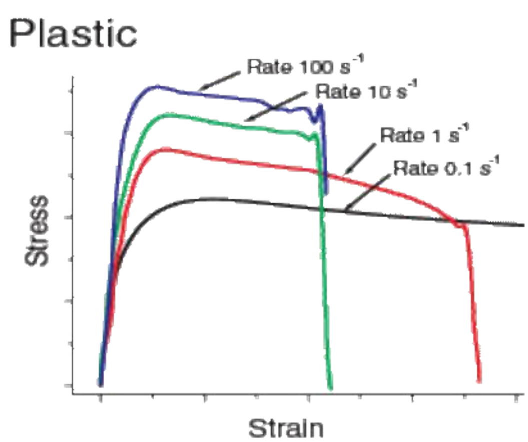
Every specimen becomes more brittle (lower fracture toughness) and even lower strength as larger specimens are considered, but this is mostly a brittle material (e.g. glasses, concrete, etc.) issue as unlike ductile materials (metal, ..) they don't have energy reserve and load balancing offered by plasticity.

7. Rate effect (how fast the load is applied)
   By applying the load fast, we don't let dislocations to contribute much and often strength goes up



Strain rate ↗ similar to T ↘

56



Whether or not toughness (energy) goes up or not is not clear

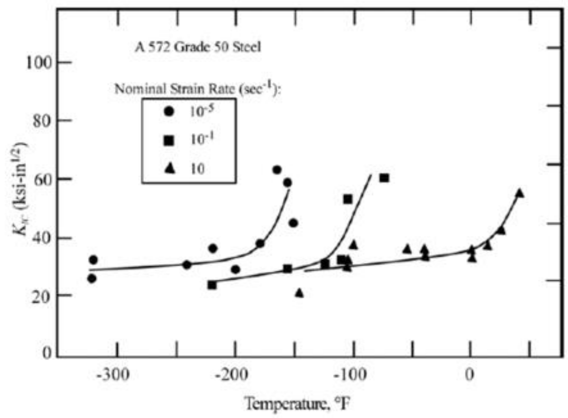
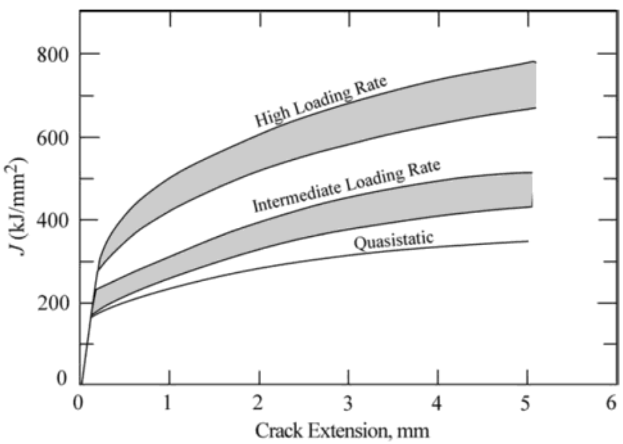
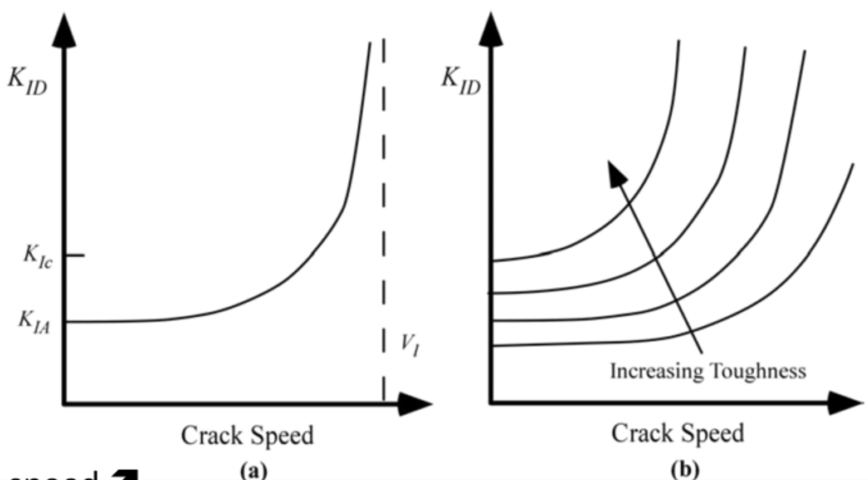


FIGURE 4.5 Effect of loading rate on the cleavage fracture toughness of a structural steel. Taken from Barsom, J.M., "Development of the AASHTO Fracture Toughness Requirements for Bridge Steels." *Engineering Fracture Mechanics*, Vol. 7, 1975, pp. 605–618.

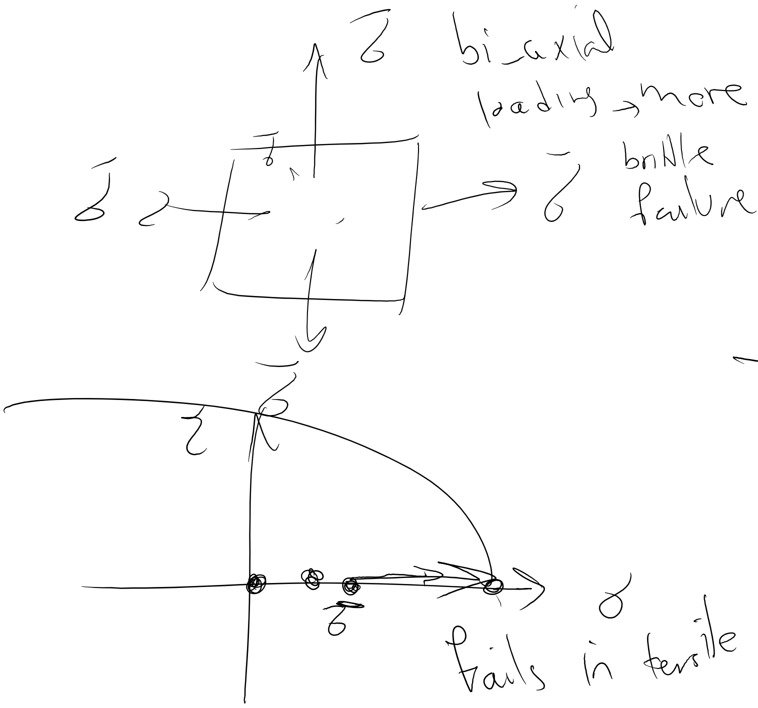


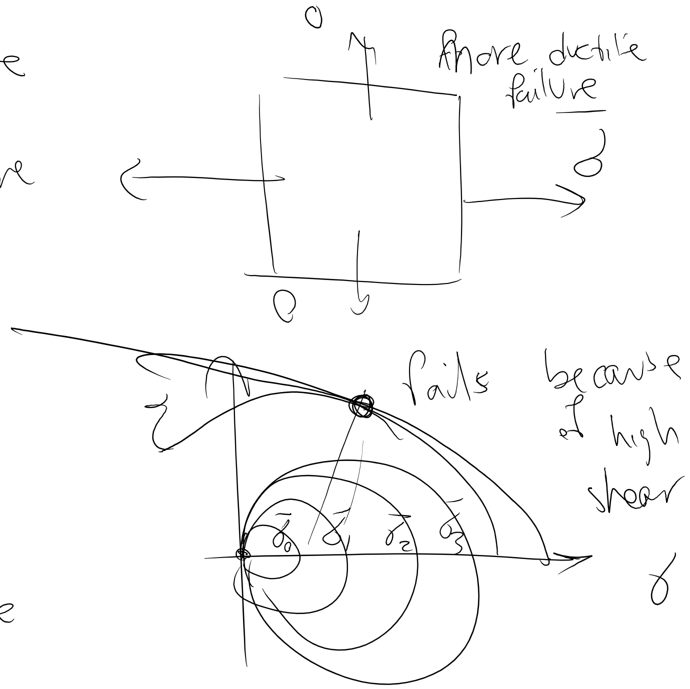


- Strain speed ➚
- K_{ID} ➚ (Insensitive at low speeds, quick increase approaching V_l)
- Increasing toughness makes K_{ID} more sensitive and grow faster

$$K_{ID} = \frac{K_{IA}}{1-\left(\frac{V}{V_l}\right)^m}$$

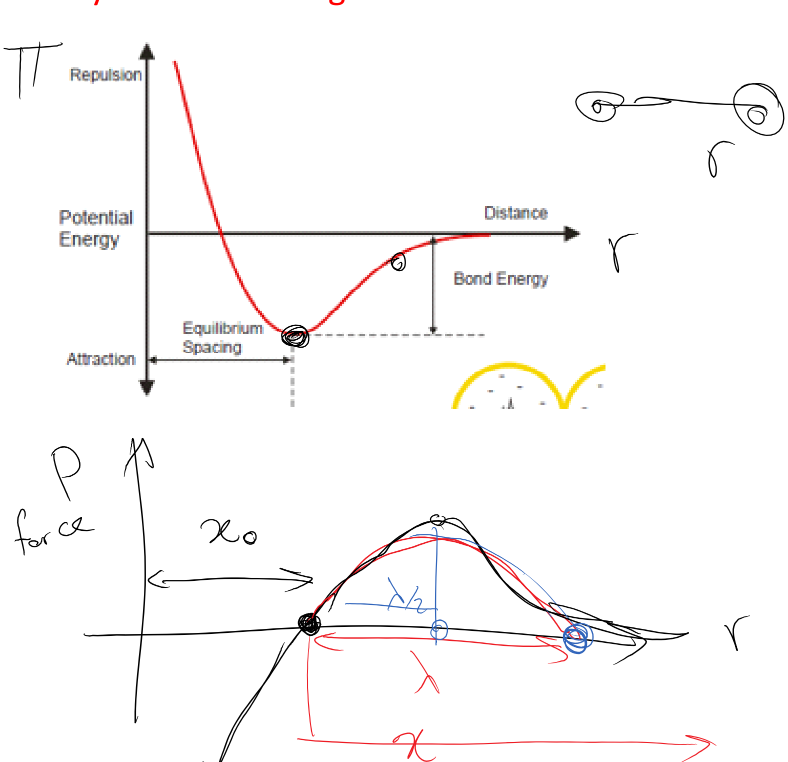
8. Triaxial stress and confinement





This can also be seen as larger specimens fail more in brittle mode (more triaxial stress state)

Why material strength is much lower than what theory predicts?!

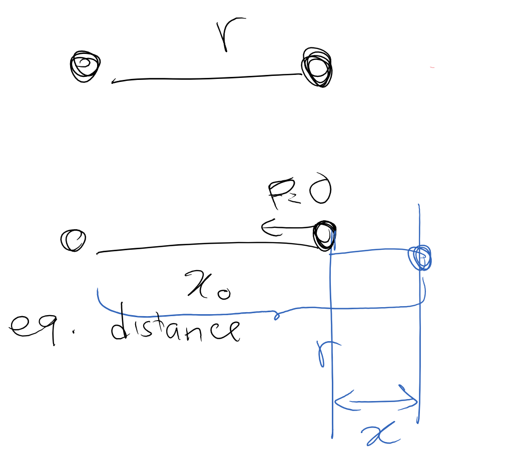


$$\vec{P} = -\frac{\partial \Pi}{\partial r}$$

1D

$$P = \frac{d\Pi}{dr}$$

1D



$$P = P_c \sin\left(\frac{\pi x}{\lambda}\right)$$

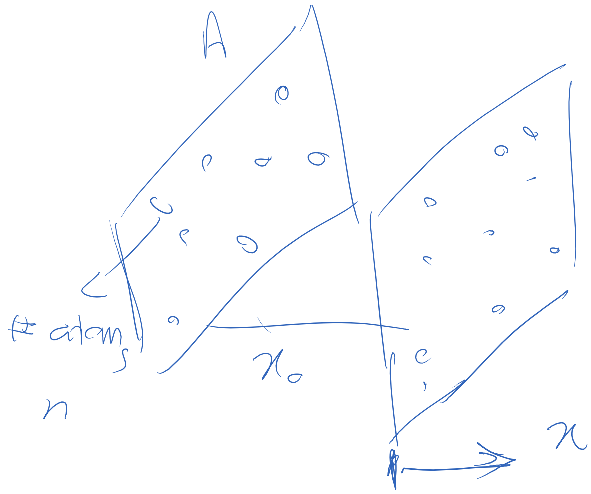
$$F = nP$$

$F = nP$

σ = stress $= \dfrac{F}{A}$

$$\sigma = \frac{P_c n}{A} \sin\left(\frac{\pi x}{\lambda}\right)$$

$$= \underbrace{P_c \left(\frac{n}{A}\right)}_{\sigma_c} \sin\left(\frac{\pi x}{\lambda}\right)$$



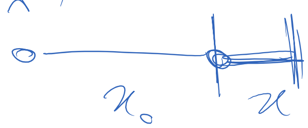
displacement from eq. position

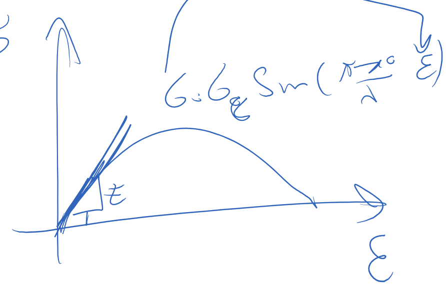
$\Longrightarrow$

$$\sigma = \sigma_c \sin\left(\frac{\pi x}{\lambda}\right)$$

$$\varepsilon = \frac{x}{x_0} \begin{array}{l} \rightarrow \text{change in length} \\ \rightarrow \text{initial length} \end{array}$$

$$\sigma = \sigma_c \sin\left(\frac{\pi \varepsilon x_0}{\lambda}\right)$$





$$E = \left.\frac{d\sigma}{d\varepsilon}\right|_{\varepsilon = 0}$$

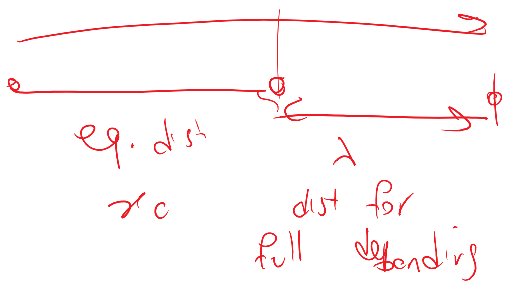
$\longrightarrow$

$$\boxed{E = \sigma_c \frac{\pi x_0}{\lambda}}$$

$\longrightarrow$

$$\boxed{\sigma_c \approx \frac{E}{\pi} \frac{\lambda}{x_0}}$$

strength



$$\frac{\lambda}{x_0} \approx O(1) \qquad \frac{\lambda}{x_0} \approx 1 \rightarrow \sigma_c \approx \frac{E}{\pi}$$

steel $\quad E \approx 200\ \text{GPa}$ $\qquad \boxed{\sigma_c \approx E}$

ultimate strength — $\sigma_u \approx 300 - 800\ \text{MPa}$