2018/09/10

Monday, September 10, 2018 11:30 AM

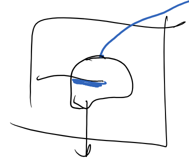
R and Pi (G) curves:

Crack resistance vs. energy release rate:

$$\underbrace{\dot{W}}_{\text{external work rate / external power}} = \underbrace{\dot{U}_e}_{\text{internal energy rate}} + \dot{U}_p + \dot{\Gamma} + \dot{U}_k$$

$\dot{U}_k$: kinetic energy power

$\dot{\Gamma}$: power consumed by creating new fracture surface



$\dot{U}_p$: power consumed by plastic deformation mostly around a crack tip

$$\frac{dx}{dA} = \frac{\partial a}{\partial t} \Big/ \frac{\partial A}{\partial t}$$

$A$: surface of crack

$$\dot{(\ )} \rightarrow \frac{d}{dA}$$

Resistance to crack growth

$$\boxed{-\frac{d}{dA}\underbrace{(U_e - W)}_{\text{potential energy } \Pi} = \frac{d(U_p + \Gamma)}{dA} + \frac{dU_k}{dA}}$$

$$\underbrace{-\frac{d\Pi}{dA}} = R + \frac{dU_k}{dA}$$

$G = -\frac{d\Pi}{dA}$ = energy release rate

$G$: determined by external loading, ... ($\Pi = U_e - W$)

how much energy becomes available if crack is advance by unit area

$R = \frac{d\Gamma + U_p}{dA}$

$R$: material property

Resistance: how much energy is needed to create a unit surface of crack

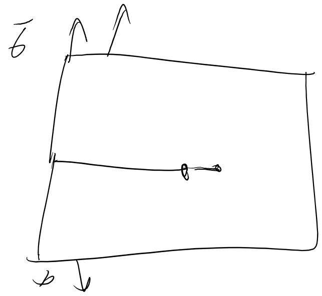
Under quasi-static conditions $U_k = 0$

$G \quad d\Pi \quad R$

$$G = -\frac{d\Pi}{dA} = R$$

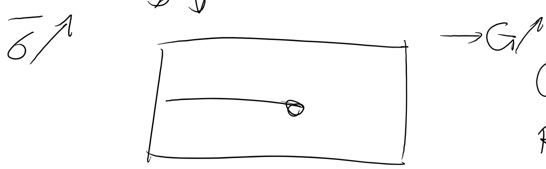
$G$ → energy release rate

$R$ → resistance




$G = 2$ $\quad$ $G < R$

$R = 3$ $\quad$ crack cannot advance

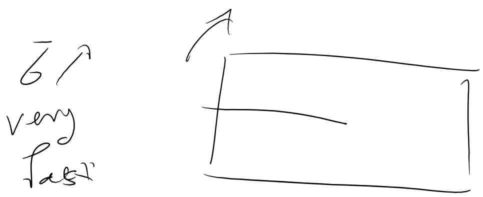



$G = 3$

$R = 3$

$G = R$

crack can advance




$G > R$ $\quad$ crack advances

$$G = R + \frac{dU_k}{dA}$$

additional energy turns into kinetic energy

# Energy release rate

Irwin 1956 $\boxed{G \equiv -\frac{d\Pi}{dA}}$ a.k.a Crack extension force
Crack driving force

G: Energy released during fracture per unit of newly created fracture surface area

$\boxed{G = 2w_f}$

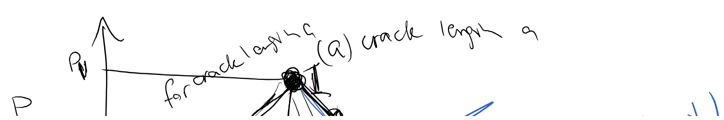
the resistance of the material that must be overcome for crack growth

energy available for crack growth (crack driving force)

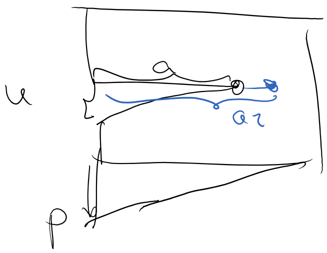
Energy release rate failure criterion $\boxed{G \geq G_c}$ → Resistance of material

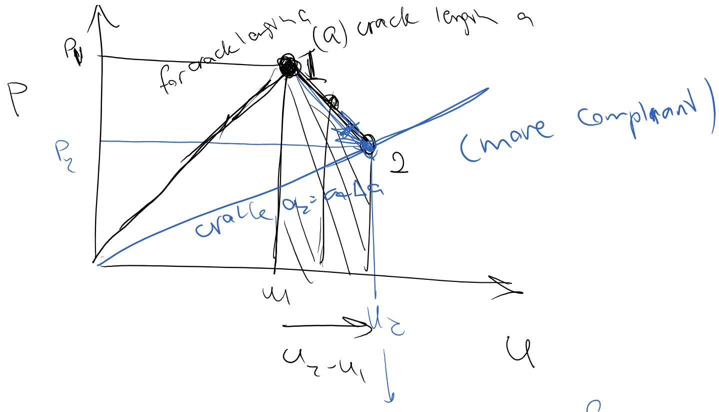
fracture energy, considered to be a material property (independent of the applied loads and the geometry of the body).

How do we compute G?



How do we compute G?




$$\Delta W = \int_{\text{state 1}}^{\text{state 2}} P \underbrace{du}_{\text{displacement}} =$$

$$\int_{u_1}^{u_2} P\,du \approx (u_2 - u_1)\left(\frac{P_1 + P_2}{2}\right)$$

$$G = -\frac{d\Pi}{dA} \qquad \Pi = U_e - W$$

our goal is to evaluate G:

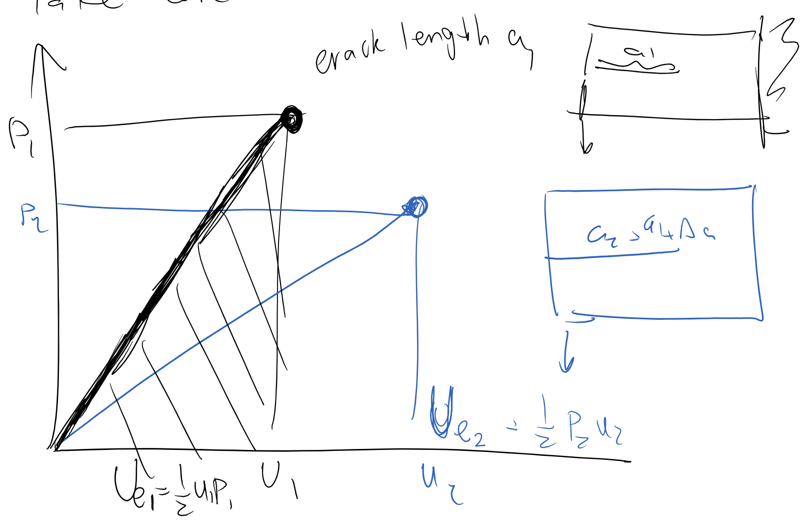
Already know how to take care of W in the def. of $\Pi$

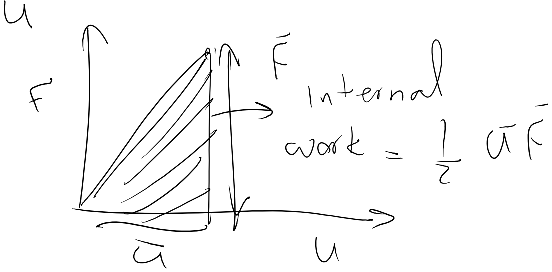
How about $U_e$

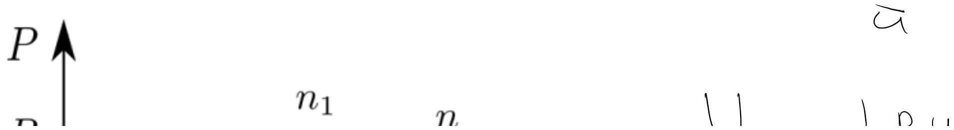

$$\text{energy} = \int_0^{\bar{u}} F(u)\,du$$

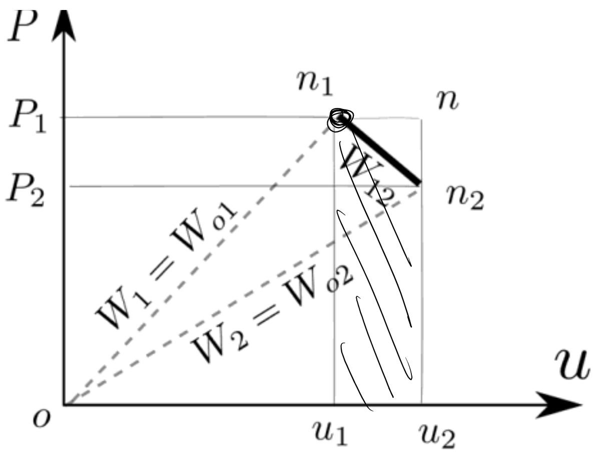
$$= \int_0^{\bar{u}} (ku)\,du =$$

$$\frac{1}{2} k\bar{u}^2 = \frac{1}{2}(k\bar{u})\bar{u} = \frac{1}{2}\bar{F}\bar{u}$$






$$U_{e_1} = \frac{1}{2} P_1 u_1$$

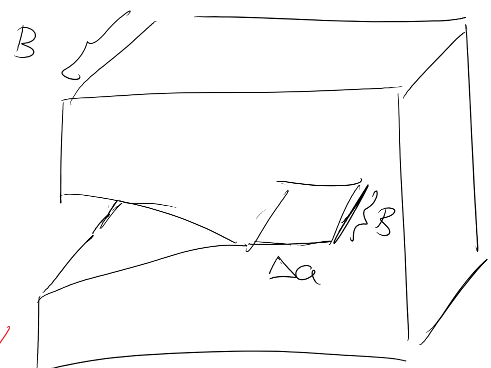
$$U_{e_2} = \frac{1}{2} P_2 u_2$$

$$\Delta W = \int_{u_1}^{u_2} P\,du \approx \left(\frac{P_1+P_2}{2}\right)(u_2-u_1)$$

$$\Delta U_e = U_{e_2} - U_{e_1} = \frac{1}{2} P_2 u_2 - \frac{1}{2} P_1 u_1$$

$$\Pi = U_e - W \longrightarrow \Delta\Pi = \Delta U_e - \Delta W = \frac{1}{2} P_2 u_2 - \frac{1}{2} P_1 u_1 - \left(\frac{P_1+P_2}{2}\right)(u_2-u_1)$$

$$G = -\frac{d\Pi}{dA} \approx -\frac{\Delta\Pi}{\Delta A}$$



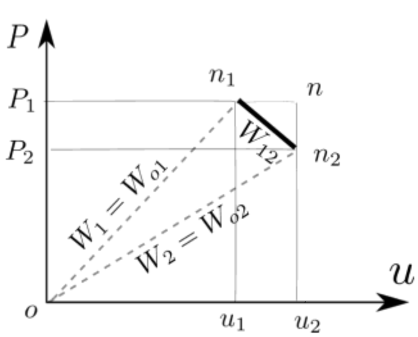

$\Delta A = \underbrace{B\Delta a}_{\text{change in crack area}}$

$$\rightarrow G = \frac{1}{B\Delta a}\left(\frac{1}{2} P_2 u_2 - \frac{1}{2} P_1 u_1 - \left(\frac{P_1+P_2}{2}\right)(u_2-u_1)\right)$$

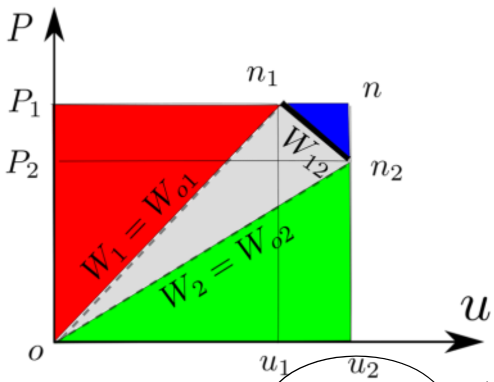
$\Delta a \rightarrow 0$

$$G = \lim_{\Delta a \to 0} \frac{1}{B\Delta a}\left(\cancel{\frac{1}{2} P_2 u_2} - \cancel{\frac{1}{2} P_1 u_1} - \frac{1}{2} P_1 u_2 + \cancel{\frac{1}{2} P_1 u_1} - \cancel{\frac{1}{2} P_2 u_2} + \frac{1}{2} P_2 u_1\right)$$

$$\boxed{G = \lim_{\Delta a \to 0} \frac{1}{B\Delta a}\left(\underbrace{-\frac{1}{2} P_2 u_1 + \frac{1}{2} P_1 u_2}_{\text{very useful for prob 2 in HW1}}\right)}$$




Geometric interpretation:



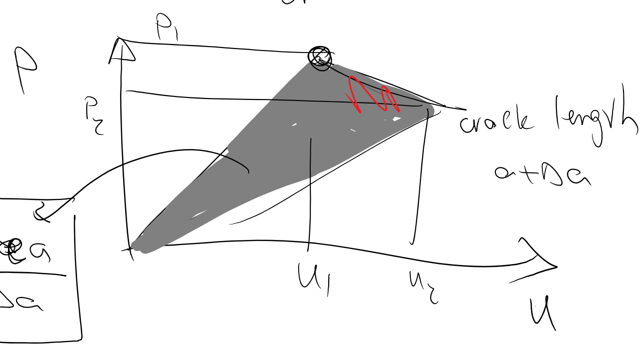

$$G = \frac{1}{B\Delta a}\left(P_1 u_2 - \frac{P_1 u_1}{2} - \frac{P_2 u_2}{2} - \frac{(P_1 - P_2)(u_2 - u_1)}{2}\right)$$
$$= \frac{\text{Grey area}}{B\Delta a}$$

$$G = -\frac{\Delta \Pi}{\Delta A} = -\frac{\Delta V_e + \Delta W}{\Delta A} =$$

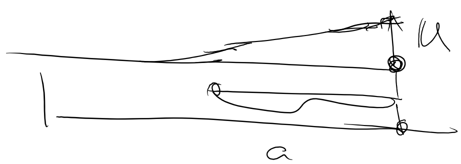
$$\left(\frac{1}{2}u_1 P_1 - \frac{1}{2}u_2 P_2 + \left(\frac{P_1 + P_2}{2}\right)(u_2 - u_1)\right)$$

maniplate it

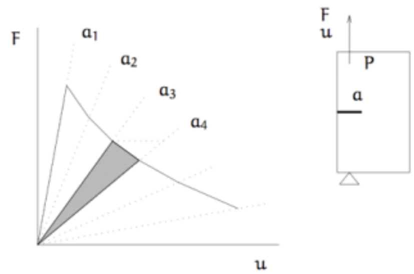
crack length a




$$\boxed{G = \frac{\text{Area}}{B\Delta a}}$$

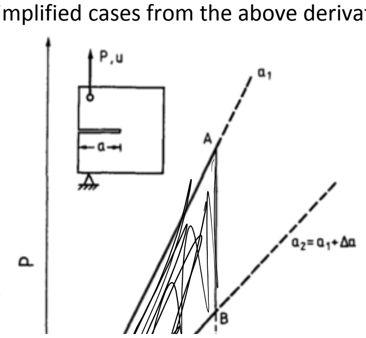


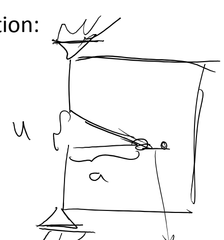

# G from experiments




$$\boxed{G(a_3) = \frac{1}{B}\frac{\text{shaded area}}{a_4 - a_3}}$$

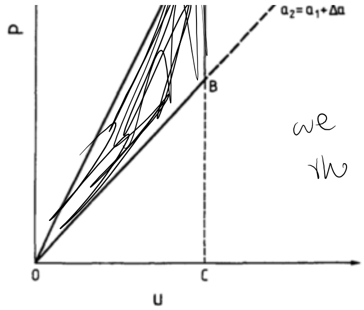
Simplified cases from the above derivation:

1. 


by cutting
the material

1.




we expect the load to drop

by cutting the material force the crack to grow

$\Delta W = 0$ area under P_u between state A, B

$U_{e1} = \frac{1}{2} P_A u$

$U_{e2} = \frac{1}{2} P_B u$

2. Fixed grip

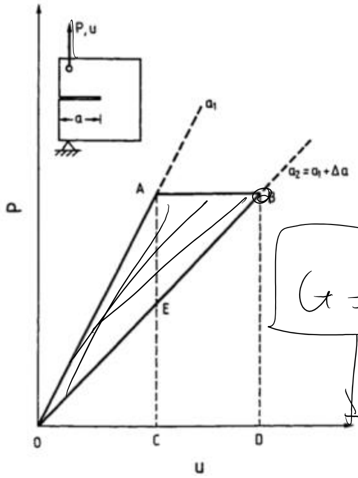
$$-\Delta\Pi = -\Delta U_e + \Delta W = -\frac{1}{2} P_B u + \frac{1}{2} P_A u + 0$$

$$\rightarrow G = \lim_{\Delta a \to 0} \frac{-\Delta\Pi}{A\,\Delta a} = \frac{\frac{1}{2}(P_A - P_B)u}{B\Delta a}$$

shaded area

fixed grip $\boxed{G = \lim_{\Delta a \to 0} \frac{\frac{1}{2}(P_A - P_B)u}{B\,\Delta a}}$

2. Dead load




load if fixed

$\Delta W = (u_B - u_A) P$

$\Delta U_e = \frac{1}{2}(u_B - u_A) P$

$$\boxed{G = \frac{-\Delta\Pi}{B\Delta a} = \frac{\frac{1}{2}\overbrace{(u_B - u_A)P}^{\text{shaded area}}}{B\Delta a}}$$

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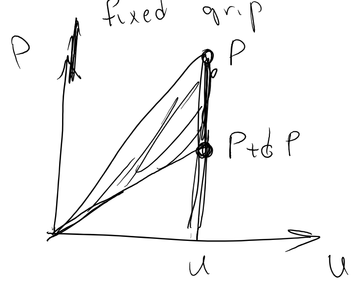
How can we come up with a useful expression for G if we have P-u relation for a given loading configuration?

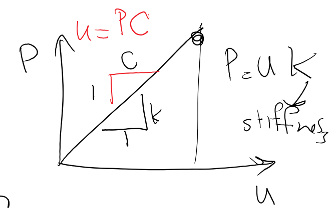
P(u) relation $\longrightarrow$ G

fixed grip




$\boxed{C = \frac{u}{P}}$ $= \frac{1}{K}$

Compliance




$P = \frac{u}{C}$, $\underline{u = PC}$

shaded area $\frac{1}{2}(-\Delta P)u$

(2) dead load

$$G = \lim_{\Delta a \to 0} \frac{\text{shaded area}}{B\,\Delta a} = \lim_{\Delta a \to 0} \frac{\frac{1}{2}(-\Delta P)\,u}{B\,\Delta a} =$$

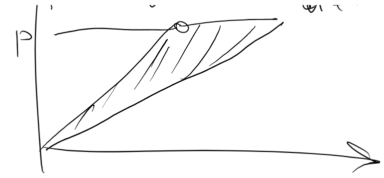
$$= \frac{-u}{2B}\frac{dP}{da} = \frac{-u}{2B}\frac{d\left(\frac{u}{C}\right)}{da} = \frac{-u}{2B}\cdot u\frac{d\frac{1}{C}}{da}$$

Constant in fixed grip

$$= \frac{-u^2}{2B}\underbrace{\left(\frac{-dC}{da}\frac{1}{C^2}\right)}_{\frac{d\,1/C}{da}} = \frac{1}{2B}\left(\frac{u}{C}\right)^2\frac{dC}{da}$$

$$\boxed{G = \frac{P^2}{2B}\frac{dC}{da}}$$

fixed grip



$$G = \lim_{\Delta a \to 0} \frac{\text{shaded area}}{B\,\Delta a}$$

$$= \lim_{\Delta a \to 0} \frac{\Delta u\,P/2}{B\,\Delta a} = \frac{P}{2B}\frac{du}{da}$$

dead load is constant

$$= \frac{P}{2B}\frac{d\,PC}{da} =$$

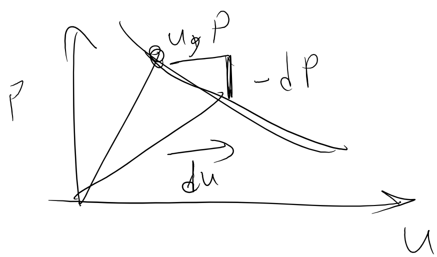
P is constant

$$\boxed{G = \frac{P^2}{2B}\frac{dC}{da}}$$

dead load

in HW1, prob 2 you'll show that

$$G = \frac{P^2}{2B}\frac{dC}{da} \quad \text{for any loading}$$



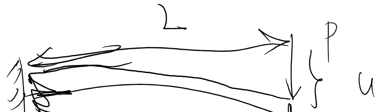

Example on the use of this equation

$$u = P\,\frac{1}{3}\frac{L^3}{EI}$$

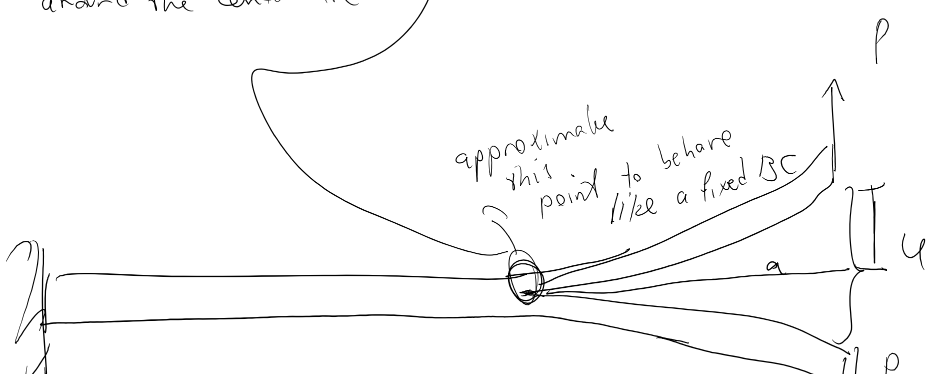
length of the beam

elastic modulus

moment around the center line




$$\frac{u}{2} = P\left(\frac{1}{3}\frac{a^3}{EI}\right)$$




$$\boxed{u = P\left(\frac{2}{3}\,\frac{a^3}{EI}\right)}$$

$P = K u$ stiffness

$u = C P$ compliance

$$\boxed{C = \frac{2a^3}{3EI} \quad \text{compliance}}$$

$$G = \frac{P^2}{2B}\,\frac{dC}{da} = \frac{P^2}{2B}\;3\times\frac{2}{3}\;\frac{a^2}{EI}$$

$$\longrightarrow \quad \boxed{G = \frac{P^2 a^2}{BEI}} \quad \text{(I)}$$