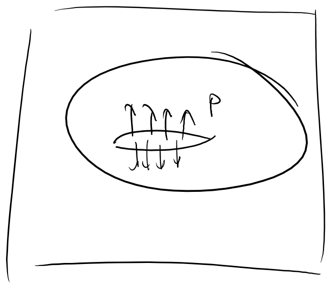
2018/09/24

Monday, September 24, 2018 9:34 AM

stress free



compressive load on crack surfaces

"Hydraulic fracturing"

... any fluid flow pressure

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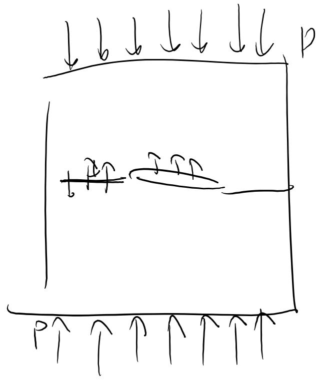
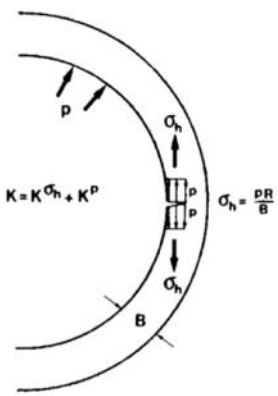
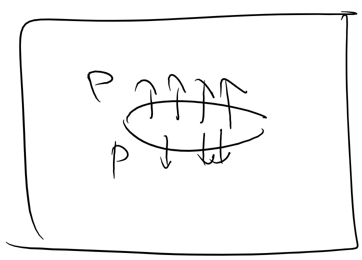


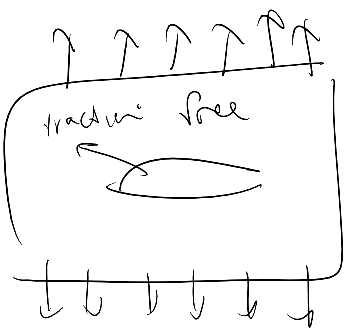
everywhere we have constant compressive stress



we know how to get the K of this problem

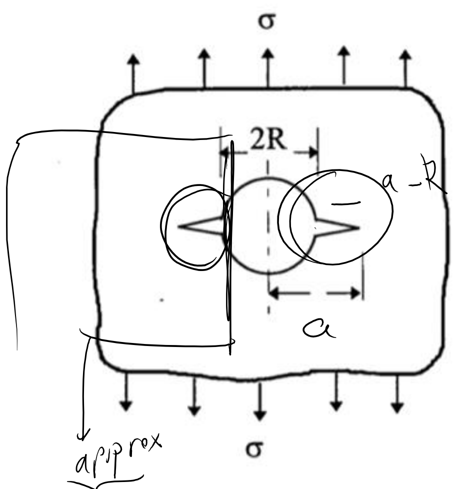
$K = 0$ (uniform stress)

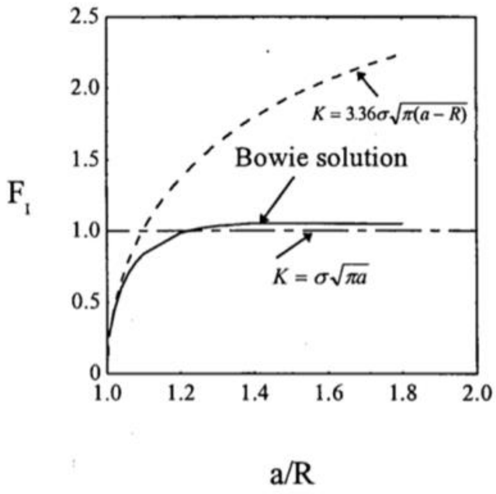




$K = K$

get it from tables if needed





$$K = f(\text{geometry})\,\sigma_{max}\sqrt{\pi\,\underbrace{(\text{crack length})}_{a-R}}$$

endge notch crack in ∞ domain

3 6

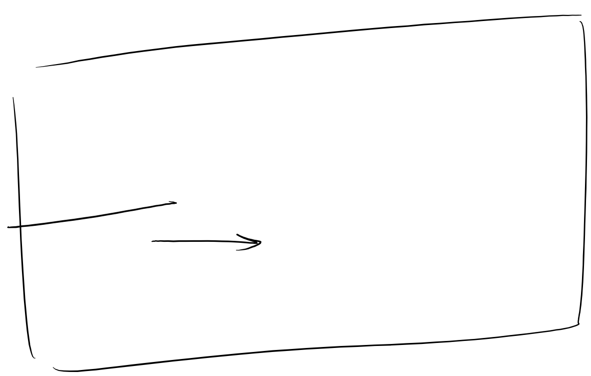
stress next to a circle σ_{yy}

$f = 1.12$

$$\boxed{K = 3.36\,\sigma\sqrt{\pi(a-R)} \quad \text{approximate Bowie's solution}}$$

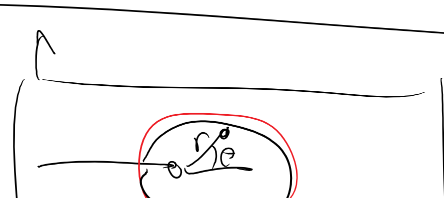
G = energy release rate

energy released per unit area advance of the crack

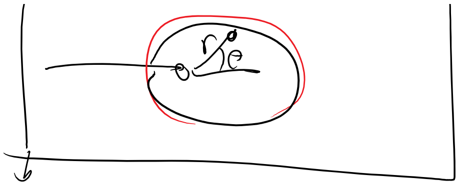


It's global in nature

$$\sigma_{ij}^{I} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{I}(\theta)$$



$$\sigma_{ij} = \frac{\quad}{\sqrt{2\pi r}} f_{ij}(\theta)$$



SIF: LOCAL $r \ll a$

Stress-based (stress solution)

Any relation between G & K?

$$[G] = \frac{[\text{energy}]}{[\text{area}]} = \frac{[\underset{\text{force}}{F}][L]}{[L]^2} = \boxed{[\underset{\text{stress}}{\sigma}] \cdot [L] = [G]}$$

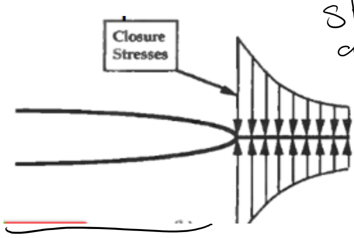
$$\boxed{[K] = [\sigma][L]^{1/2}}$$

$$K = \sigma\sqrt{\pi a}$$

$$\boxed{[K]^2 = [G][\sigma]}$$

$$\boxed{G = \frac{K^2}{\text{Stress/stiffness}}}$$

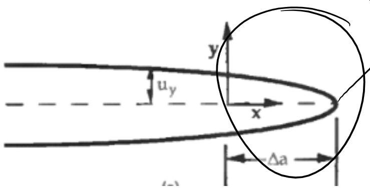
dimensionally makes sense!



stresses ahead of the crack

→ the work done by these stress = G energy release rate * area

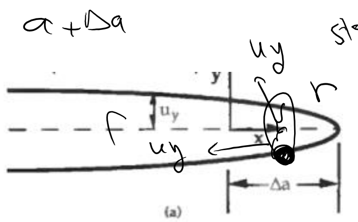
a



stresses become zero & the two faces open

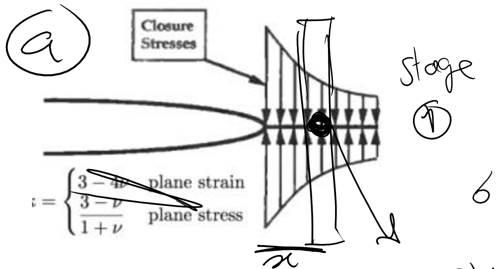
$$G = \lim \frac{\text{Energy released}}{\text{Area}} = \frac{\text{energy released}}{\Delta a\, B}$$

$$G = \lim \frac{}{\text{Area}} = \frac{}{\Delta a \underbrace{B}_{\text{thicknes}}}$$

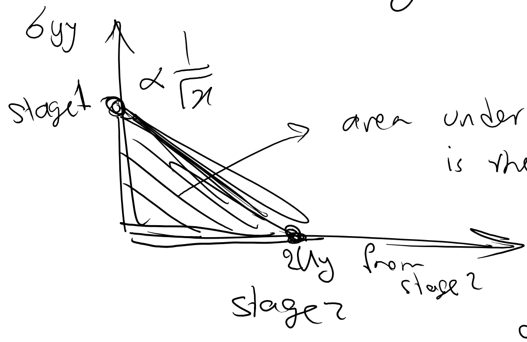


crack opening at the same location:

$$2\,u_y(x)$$



$$\text{Force} = \sigma_{yy}(x)\,\underbrace{(dx\,B)}_{\text{diff. area}}$$

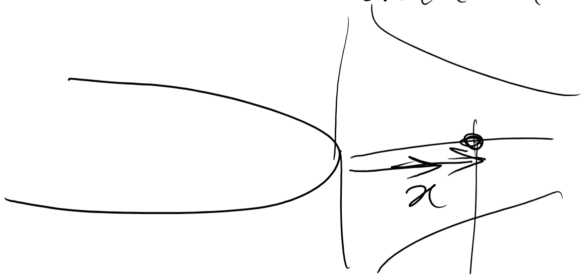


$$\text{diff work} = \Big(\underbrace{\sigma_{yy}(x)(dx)\,B}_{\text{force}}\Big)\Big(\underbrace{2\,u_y(x)}_{\text{displacemt}}\Big)\Big(\frac{1}{2}\Big)$$

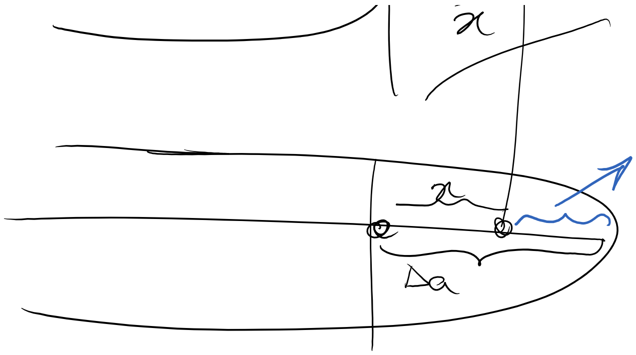
$$\boxed{\text{Work done } \Delta U = \int_0^{\Delta a} \sigma_{yy}(x)\,B\,u_y(x)}$$

$$G = \lim_{\Delta a\to 0} \frac{\Delta U}{B\,\Delta a} \Rightarrow \boxed{G = \lim_{\Delta a\to 0} \frac{1}{\Delta a}\int_0^{\Delta a} \sigma_{yy}(x)\,u_y(x)\,dx}$$

crack advance area



$$\boxed{\sigma_{yy}(x) = \frac{K_I(a)}{\sqrt{\pi x}}}$$



$\sqrt{\pi x}$

$r = \Delta a - x$ distance behind the new crack tip position

$$U_y(r) = \frac{K(a+\Delta a)}{2\mu}\sqrt{\frac{r}{2\pi}}(\kappa+1)$$

$$\kappa = \begin{cases} 3-4\nu & \text{plane strain} \\ \frac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$$

$r = \Delta a - x$

$$\boxed{U_y(x) = \frac{K_I(a+\Delta a)}{2\mu}\sqrt{\frac{\Delta a - x}{2\pi}}(\kappa+1)} \quad (ii)$$

plug σ_y (i) & U_y (ii) into (*) to get

$$G = \lim_{\Delta a \to 0}\int_0^{\Delta a}\sigma_{yy}(x)\,U_y(x)\,dx = \frac{K_I(a)\,K_I(a+\Delta a)}{4\pi\mu\,\Delta a}\int_0^{\Delta a}\sqrt{\frac{\Delta a - x}{x}}\,dx$$

Kolosov factor

$$\rightarrow \quad \boxed{G = \frac{(\kappa+1)K_I^2}{8\mu}}$$

shear modulus $= \frac{E}{2(1+\nu)}$

$$\boxed{G \propto \frac{K^2}{E}}$$ important observation

Simplifying this equation:

K-G relationship (cont.)

Mode I

$$G_I = \begin{cases} \frac{K_I^2}{E} & \text{plane stress} \\ (1-v^2)\frac{K_I^2}{E} & \text{plane strain} \end{cases}$$

$$G_I = \frac{K_I^2}{E'} \qquad E' = \begin{cases} E & \text{plane stress} \\ \frac{E}{1-\nu^2} & \text{p.} \end{cases}$$

$$G_I = \begin{cases} \frac{K_I^2}{E} & \text{plane stress} \\ (1-v^2)\frac{K_I^2}{E} & \text{plane strain} \end{cases}$$

G_I $\frac{2}{E'}$ $E' = \frac{E}{1-\nu^2}$ p. strain

$$G = \frac{K_{II}^2}{E'}$$

mode II problem

energy release rate for mixed mode

$$\boxed{G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}}$$

μ → shear modulus

Some uses of this eqn:

$G(a) = R(a)$ → crack can grow

if R is constant (often Γ is used for this fracture toughness)

$G(a) = \Gamma$ (fracture toughness))

if the problem is pure mode I

$$\frac{K_I^2(a)}{E'} = \Gamma \rightarrow$$

$$\boxed{K_I(a) = \sqrt{\Gamma E'}}$$ fracture toughness

$$\boxed{K_I(a) = \sqrt{\Gamma E'}}$$ fracture toughness

$$K_I = K_{IC}$$ critical

$$K_{IC} = \sqrt{\Gamma E'}$$

crack growth condition can be written in two different ways

$$\underbrace{G = \Gamma}_{\text{more general}} \qquad \text{OR} \qquad K_I = K_{Ic}$$

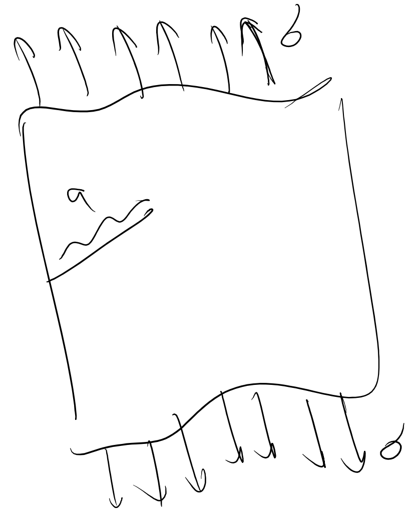
pure mode I

Uses of fracture toughness in design:

K as a failure criterion

Failure criterion $\boxed{K = K_c}$ $\quad f(a/W)\sigma\sqrt{\pi a} = K_c$

geometric correction factor



three main things

- a
- σ
- K_c

in most design / health monitoring practices 2 are given 3rd is needed.

(1) Material is known (K_c ✓) — we have something in service for example

loading is known (σ ✓)

geometry given

geometry given

$$\boxed{f\left(\frac{a_c}{W}\right)\sigma\sqrt{\pi a_c} = K_c}$$

Can be difficult to solve because $f\left(\frac{a_c}{W}\right)$ is nonlinear

→ So our inspection system must be able to detect a_c

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(2) a is known
material is known K_c
geometry is known
→ Find σ

$$\boxed{f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a} = K_c}$$

a is either known (existing crack)
OR is the tolerance of inspection tool

(3) σ, a, geometry are known → look up for a material

$$f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a} = K_{IC}$$

→ in this type of design we look for a certain material that addresses our need

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Linear Elastic Fracture Mechanics (LEFM) cannot be used for all problems. How do we know when it's reasonable to use it and w hat are more accurate theories for expressing the fields around a crack tip?

5. Elastoplastic fracture mechanics

5.1 Introduction to plasticity

5.2. Plastic zone models

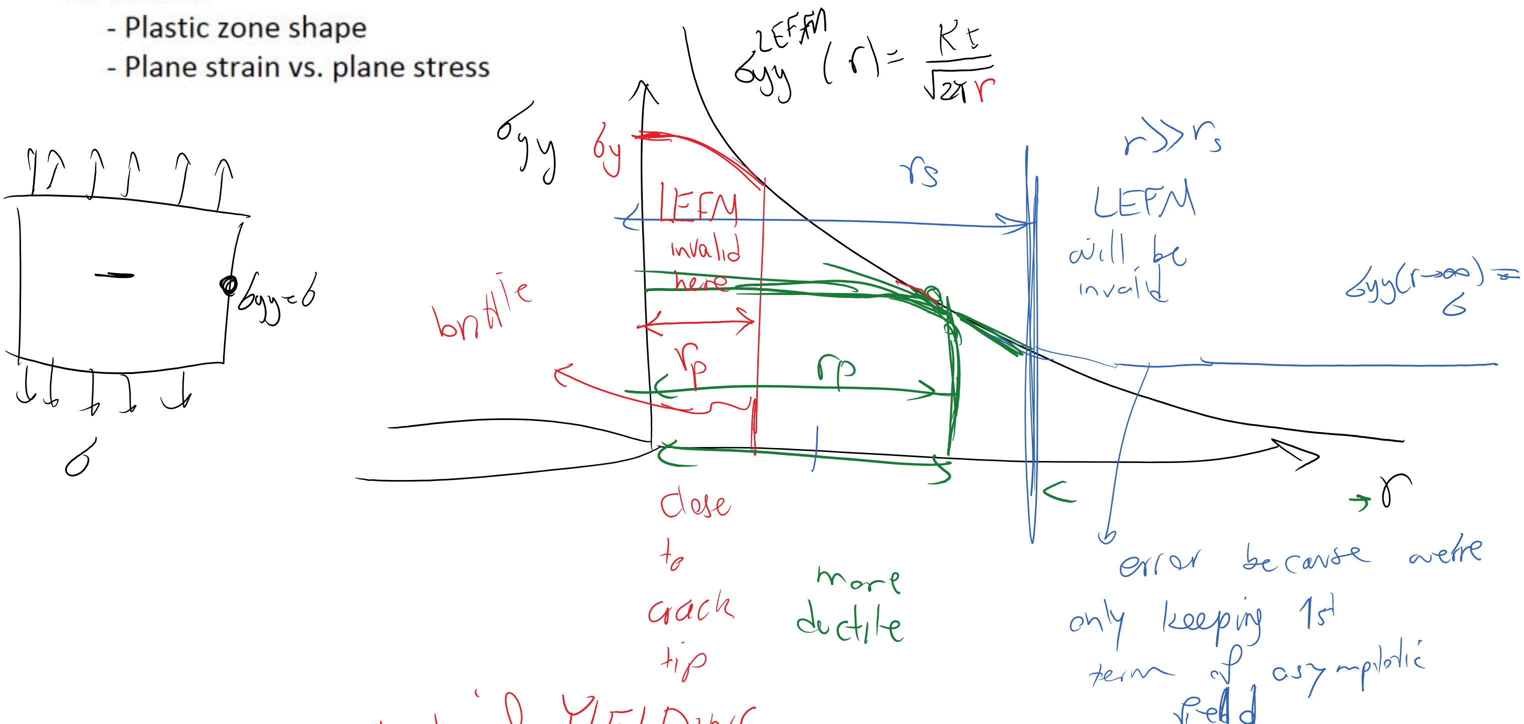
5.3. J Integral

5.4. Crack tip opening displacement (CTOD)

Material yields OR undergoes nonlinear response around the crack tip. We want to get estimates for that:

5.2. Plastic zone models

- 1D Models: Irwin, Dugdale, and Barenbolt models
- 2D models:
  - Plastic zone shape
  - Plane strain vs. plane stress



material YIELDING

or other nonlinear mechanisms

limit the stress around the crack tip

Small Scale Yielding (SSY)

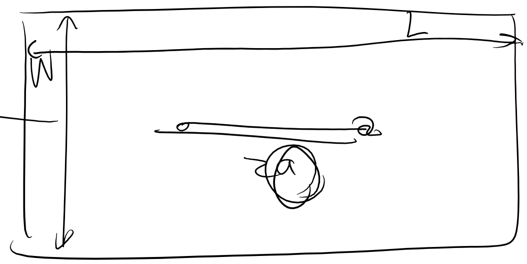
caveat states:

if r_p is much smaller than ALL relevant length scales of the problem SSY caveat holds &

LEFM is accurately represent the overall response

& stress, ... fields outside r_p.

relevant length scales



may not be relevant in dynamics

An intrinsic length scale is r_s

SINGULAR DOMINANT ZONE RADIUS

$\frac{r_p}{r_s} \ll 1$ as a necessary condition for SSY

$r_s = ?$