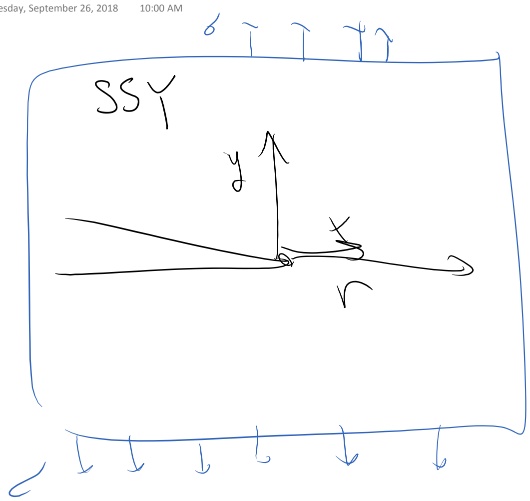
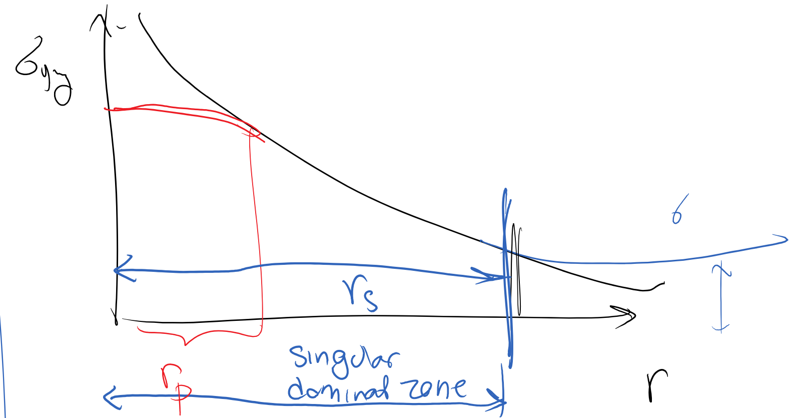
2018/09/26

Wednesday, September 26, 2018 10:00 AM





SINGULAR DOMINANT ZONE: the region where the 1st term of the asymptotic expansion is dominant

$r_p \ll$ all relevant length scale : domain size (static), crack length

r_s: radius of singular dominant zone

SSY $r_p \ll r_s$

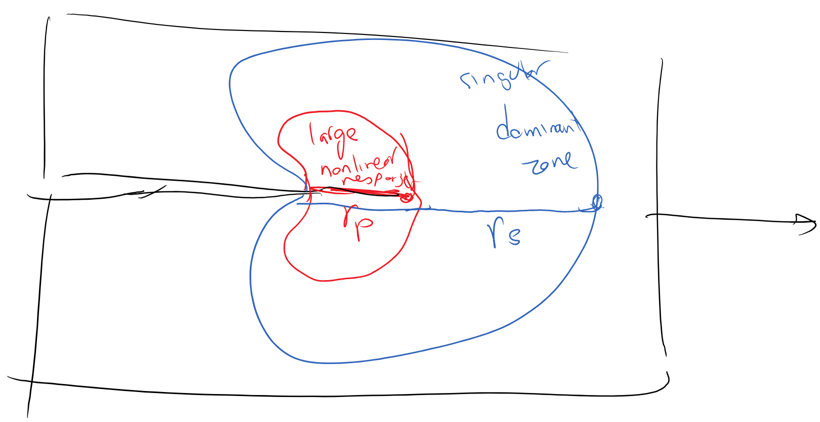
$r_p \ll a$, $r_p \ll W$

So LEFM is accurate

$r_p = ?$

$r_s = ?$

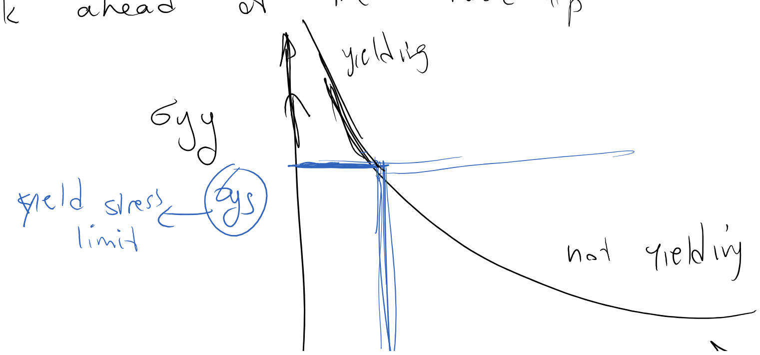
- 1D Models: Irwin, Dugdale, and Barenbolt models



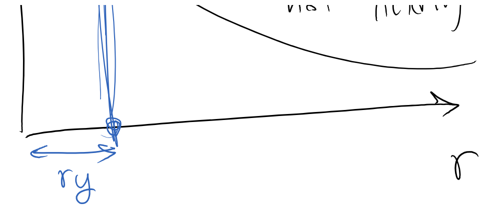
1D models only look ahead of the crack tip

- 1st order approximation

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$



at r_y



at r_y

$\sigma_{yy}(r_y) = \sigma_{ys}$

$\rightarrow \quad \frac{K_I}{\sqrt{2\pi r_y}} = \sigma_{ys} \rightarrow \boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$

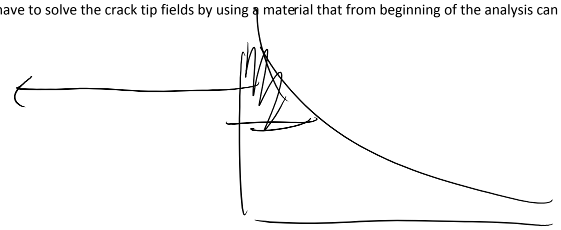
$K_I \rightarrow \sigma\sqrt{L}$

$\sigma_{ys} \rightarrow \sigma$

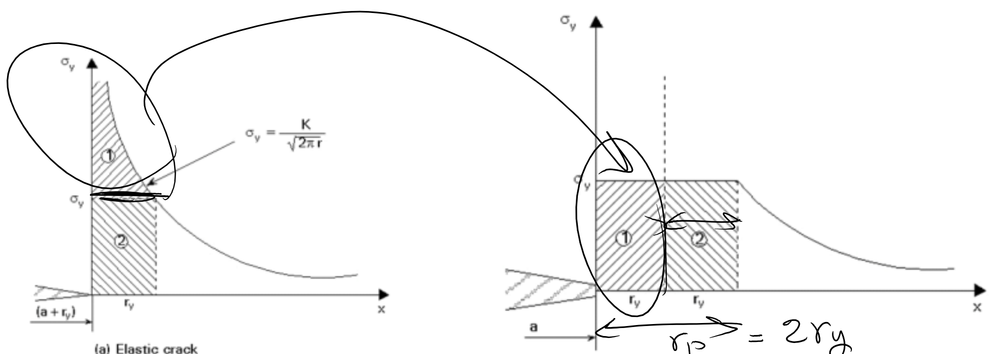
For brittle materials σ_{ys} (or ultimate strength) is very high → their r_y is smaller → LEFM is more accurate for brittle materials

The problem with this model is that stress is not redistributed (we really have to solve the crack tip fields by using a material that from beginning of the analysis can yield)

In particular, lose some force above the yield surface



2.Irwin's plastic correction



(a) Elastic crack

(b) Real crack with plastic zone

$$\boxed{r_p = 2r_y = \frac{1}{\pi}\left(\frac{K}{\sigma_{ys}}\right)^2}$$

While keeping the force the force, it really does not re distribute stresses

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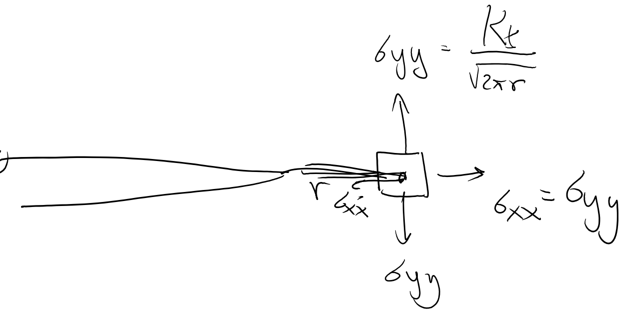
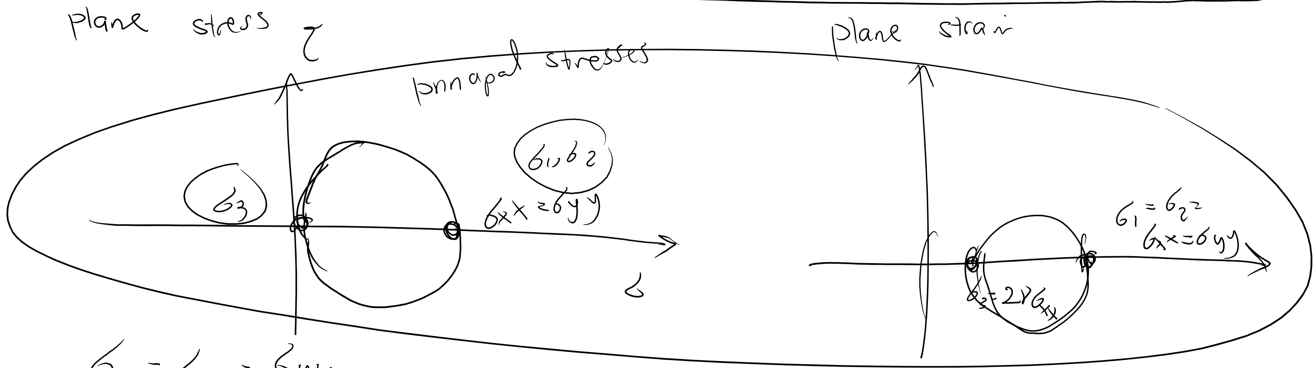
$\sigma_{zz} = 0$ plane stress

$\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$ " strain

$\epsilon_{zz} = 0$ $\epsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu\sigma_{xx} - \nu\sigma_{yy}) = 0$





$\sigma_1 = \sigma_2 = \sigma_{yy}$
$\sigma_3 = 0$

$\sigma_1 = \sigma_2 = \sigma_{yy}$
$\sigma_3 = 2\nu\sigma_y$

We need to use stress tensor (or principal stresses) in a YIELD CRITERION to see if a material is yielding

Von Mises criterion.

$$\underset{\text{equivalent}}{\sigma_e} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2} = \sigma_Y$$

σ_e : plane stress $\sigma_1 = \sigma_2$ $\sigma_3 = 0$ $\boxed{\sigma_e = \sigma_1}$

σ_e " strain $= \frac{1}{\sqrt{2}}\sqrt{(\sigma-\sigma)^2+((1-2\nu)\sigma_1)^2+((1-2\nu)\sigma_1)^2} =$

$\sigma_1 = \sigma_2$ $\sigma_3 = 2\nu\sigma_1$

$(1-2\nu)\sigma_1$

$\sigma_1 = \sigma_2 \quad \sigma_3 = 2\nu\sigma_1$ $\qquad (1-2\nu)\sigma_1$

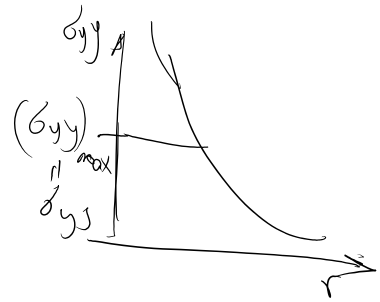
For yielding

plain stress $\quad \sigma_e = \sigma_1 = \sigma_Y$

" strain $\quad \sigma_e = (1-2\nu)\,\sigma_1 = \sigma_Y$

$$\underbrace{(\sigma_1)_{max}}_{\sigma_{ys}} = \begin{cases} \sigma_Y & \text{plane stress} \\ \dfrac{\sigma_Y}{1-2\nu} & \text{" strain} \end{cases}$$

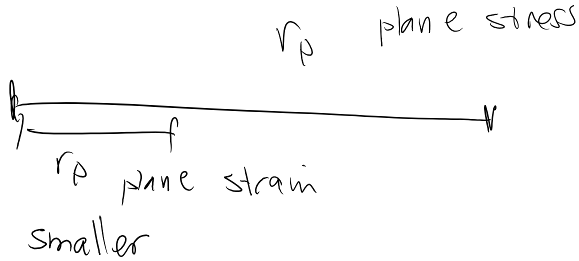
max σ_{yy} that yielding does not occur



$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

$$= \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{plane stress} \\ \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_Y/1-2\nu}\right)^2 = \dfrac{(1-2\nu)^2}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 \end{cases}$$

$$\boxed{r_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{plane stress} \\ \dfrac{(1-2\nu)^2}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 \approx \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \nu \simeq .2 \quad \text{plane strain} \end{cases}}$$



These equations imply that even for the same material plane strain condition is more brittle (more stress triaxiality)

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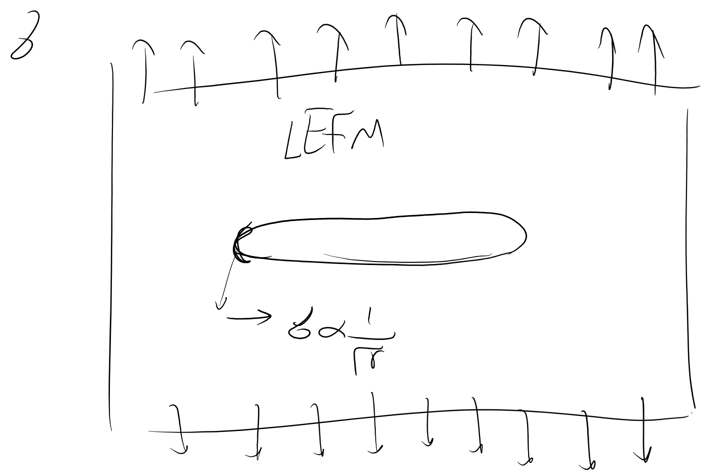
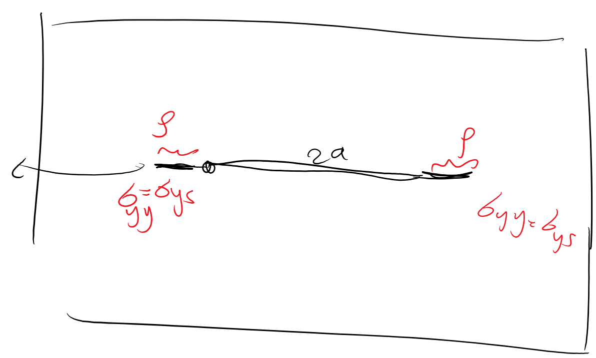
3. Strip Yield Model

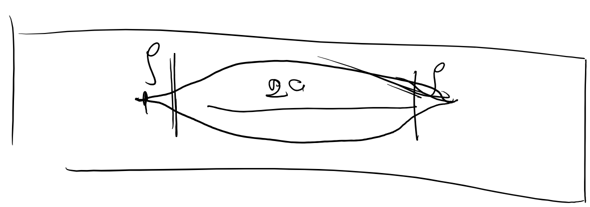
proposed by Dugdale and Barrenblatt

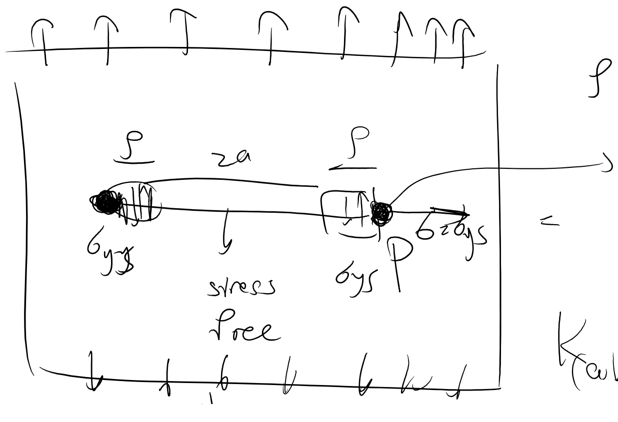


H's yielding



deformed shape





$\rho = ?$

σ_{yy}

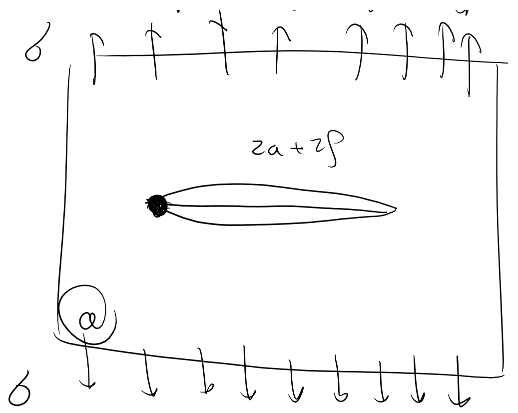
1st term in asymptotic expansion

$$\frac{K_{ab}}{\sqrt{2\pi r}} = 0$$

must be zero because stress @ ρ is finite

$$K_{(a b)} = K_a + K_b$$

=



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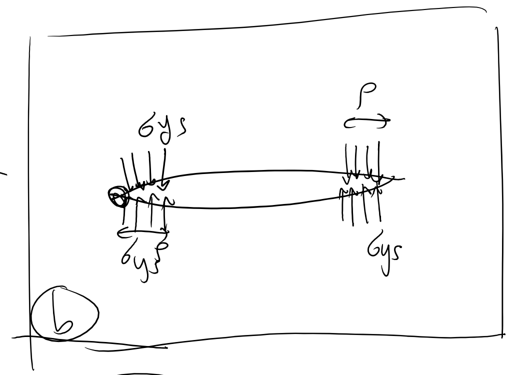
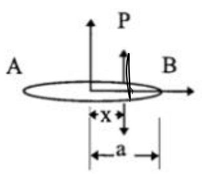


K_I^a + $K_I^b = ?$ > $K_I^{ab} = 0$

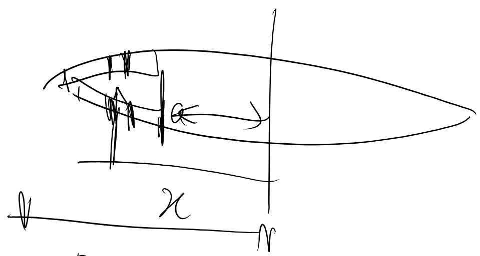
$K_I^a = \sigma\sqrt{\pi(a+\rho)}$



$$K_A = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$K_B = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$

Anderson, p64



$P = -\sigma_{ys}\, dx$

$a < x < a+\rho$

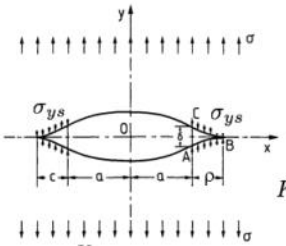
$$K_I^{\sigma_{ys}} = -\frac{\sigma_{ys}}{\sqrt{\pi c}}\int_a^c \left(\sqrt{\frac{c-x}{c+x}} + \sqrt{\frac{c+x}{c-x}}\right) dx$$

$$K_I^{\sigma_{ys}} = -2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right)$$

loading case b

$$-2\sigma_{ys}\sqrt{\frac{\cancel{a+\rho}}{\pi}}\cos^{-1}\left(\frac{a}{\rho+a}\right) = \underbrace{\sigma\sqrt{\pi(\cancel{a+\rho})}}_{K_I^a}$$

manipulate this to get



Superposition principle

$$K_I = K_I^{\sigma} + K_I^{\sigma_{ys}}$$

$$K_I^{\sigma} = \sigma\sqrt{\pi(a+\rho)}$$

$$K_I^{\sigma_{ys}} = -2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right)$$

(derivation follows) $\sigma \ll \sigma_{ys}$

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}}f_{ij}(\theta) + \text{H.O.T}$$

$$\boxed{K_I = 0} \Rightarrow \frac{a}{a+\rho} = \cos\left(\frac{\pi\sigma}{2\sigma_{ys}}\right)$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \cdots$$

Irwin's result 0.318

$$\underbrace{\boxed{\rho = \frac{\pi^2\sigma^2 a}{8\sigma_{ys}^2} = \frac{\pi}{8}\left(\frac{K_I}{\sigma_{ys}}\right)^2}}_{\text{Dugdale's model}}$$

close to 0.392

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$$\boxed{r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

$\rho \ll a$

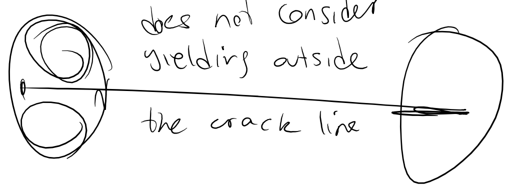
Dugdale's model

estimate for

process zone (where yielding occurs)

limitation

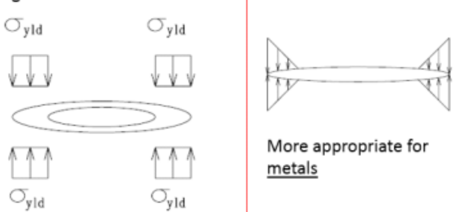
does not consider yielding outside the crack line



3. Strip Yield Model: Dugdale vs Barenblatt model

Dugdale: Uniform stress

Barenblatt: Linear stress



More appropriate for polymers

More appropriate for metals

Effective crack length model

To make the range of applicability of LEFM large

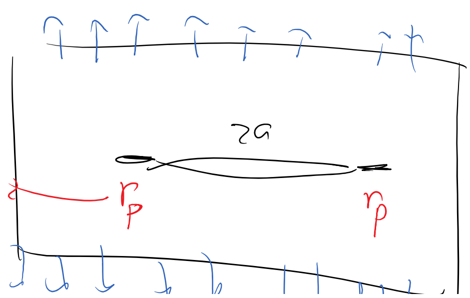
OR making it more accurate,

We add the yield zone to the effective crack length

$a_{eff} = a + r_p$

$a \nearrow$ $(a_{eff} > a)$

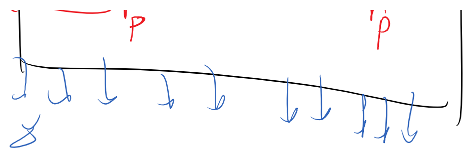
$\left\{ \begin{matrix} r_y \\ r_p \\ \rho \end{matrix} \right.$



$K \nearrow$ ... well

$a\nearrow$ ($a_{eff} > a$) $\nearrow P$

$K \nearrow$ as well



$$K_{eff} = \sqrt{\pi a_{eff}}$$

$$a_{eff} = a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2$$

1st order mod. $\leftarrow r_y$

?

$$K_{eff} = \sqrt{\pi\left(a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2\right)}$$

K_{eff} on two sides

square the equal $\dots$

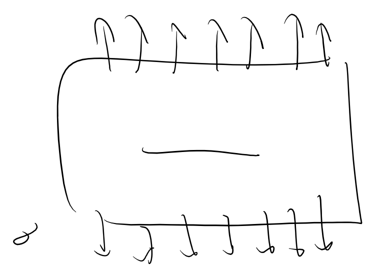
$$K_{eff}^2 = \pi a + \frac{1}{2}\frac{K_{eff}^2\,\sigma^2}{\sigma_{ys}^2}$$

$$\boxed{K_{eff} = \frac{\sqrt{\pi a}\,\sigma}{\sqrt{1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}}$$

$\rightarrow$ K for crack length a

K_{LEFM}

$$\boxed{\frac{K_{eff}}{K_{LEFM}} = \frac{1}{\sqrt{1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}}$$



When $\left(\frac{\sigma}{\sigma_{ys}}\right) = \frac{\text{LOAD}}{\text{YIELD STRESS}}$ is very small the two K's are very close & we don't need the modification

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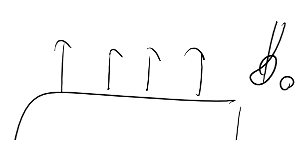
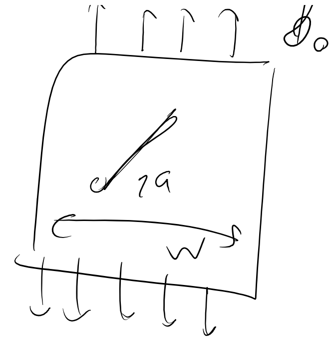
LEFM $\xrightarrow{\text{accuracy}}$ corrected a $\quad$ Elastoplastic $\rightarrow$ numerical

$$(K_{eff} = f(a_{eff}/W)\sqrt{\pi a_{eff}}$$



$$\begin{cases} K_{eff} = f(a_{eff}/W)\sqrt{\pi a_{eff}} \\ a_{eff} = a + \underbrace{\frac{1}{\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2}_{r_y} \end{cases}$$



nonlinear system of equations to be solved iteratively in practice

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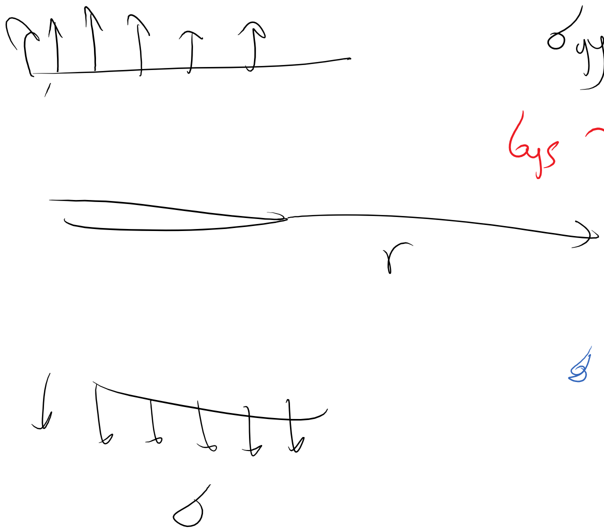
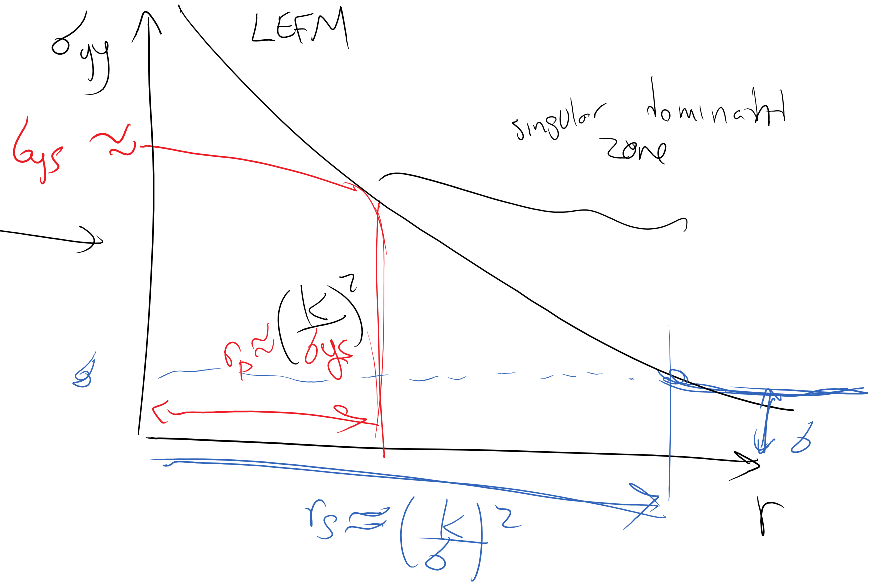




@ $r = r_s$ $\quad \sigma_{yy} = \sigma \rightarrow \dfrac{K}{\sqrt{2\pi r_s}} = \sigma \rightarrow r_s = \dfrac{1}{2\pi}\left(\dfrac{K}{\sigma}\right)^2$

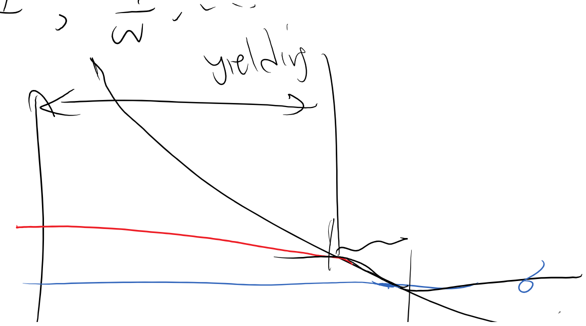
$$\boxed{\frac{r_p}{r_s} \propto \left(\frac{\sigma}{\sigma_{ys}}\right)^2}$$

SSY $\quad r_p \ll r_s$ (& all other relevant length scales)

$$\boxed{\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \ll 1}$$

SSY necessary

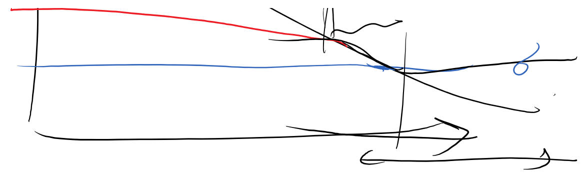
$\frac{r_p}{a}, \frac{r_p}{W}, \dots$



$r_p \approx r_s$

$r_p \approx r_s$

LEFM a bad model here



higher order terms need