2018/10/01

Monday, October 01, 2018 11:33 AM

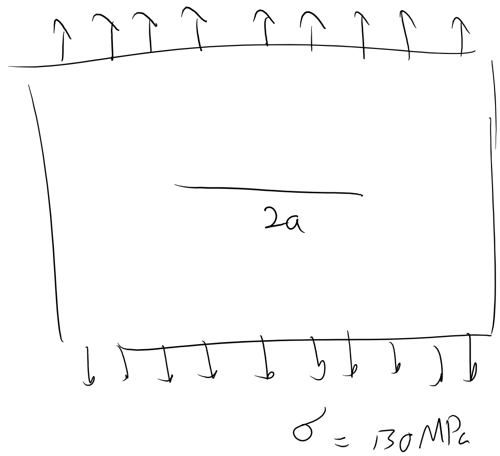
Example on how we can extend the applicability of LEFM just a bit further -> modifying the crack length by Irwin's correction:

For a mid-crack in an infinite domain:

$$K_{eff} = \frac{\sqrt{\pi a}\ \sigma}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}$$

Consider a large central cracked plate subjected to a uniform stress of 130 MPa. The fracture toughness K_c=50MPa√m, the yield strength σ_{ys}=420MPa.

(a) What is the maximum allowable crack length?
(b) What is the maximum crack length if plastic correction is taken into account. Plane stress and Irwin's correction.



$K_c = 50\ MPa\sqrt{m}$

$\sigma = 130\ MPa$

$\sigma_{ys} = 420\ MPa$

What is the maximum allowable crack size (LEFM, no correction)

$$K = \underset{\downarrow\ 50}{\sigma}\sqrt{\pi a} = \underset{\downarrow\ 130}{K_c} \rightarrow \quad 50\ MPa\sqrt{\pi a} = 130\ MPa\sqrt{m}$$

$$\rightarrow \quad a = \frac{1}{\pi}\left(\frac{50}{130}\right)^2 m \rightarrow a = 0.0471\ m$$

$$\rightarrow \boxed{2a_{LEFM} = .0942\ m = 94.2\ mm}$$

(b) max length with Irwin's correction

$$K_{eff} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1-.5\left(\frac{\sigma}{\sigma_{ys}}\right)^2}} \qquad \sqrt{1-.5\left(\frac{\sigma}{\sigma_{ys}}\right)^2} = \sqrt{1-.5\left(\frac{130\ MPa}{420\ MPa}\right)^2}$$

$$= .9758$$

for fracture to initiate we have $K_{eff} = K_c$

$$50\ MPa\sqrt{m} = \frac{130\sqrt{\pi a}\ MPa}{.9758} \rightarrow a = 0.0448 \rightarrow$$

1) 94.2 ≈ (2a)...

$$\boxed{2a_{cr} = .0897\,m = 89.7\ mm} < 94.2 = (2a)_{LEFM}$$

no correction

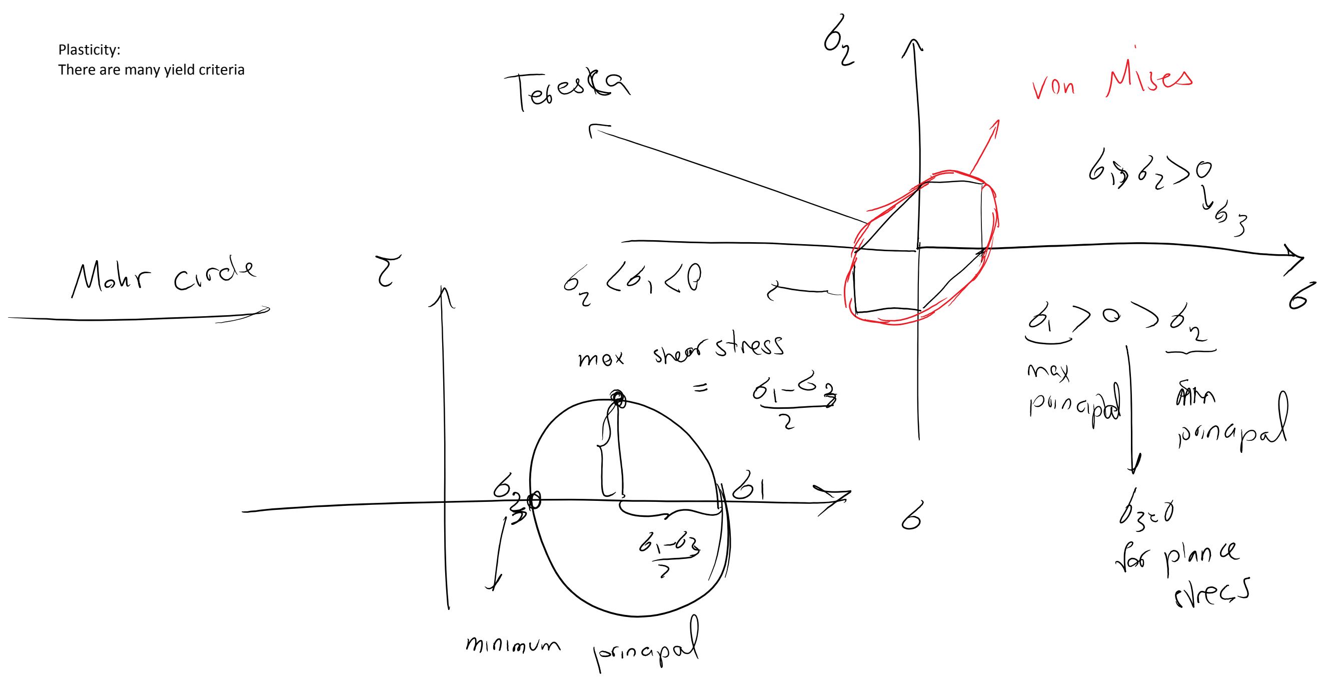
$$\frac{r_p}{r_s} \propto \left(\frac{\sigma}{\sigma_{ys}}\right)^2 = \left(\frac{130}{470}\right)^2 = .002$$

Typically $\frac{r_p}{r_s} > .1 \to .2$ .. LEFM is not very accurate

5.2.2 Plastic zone shape: 2D models

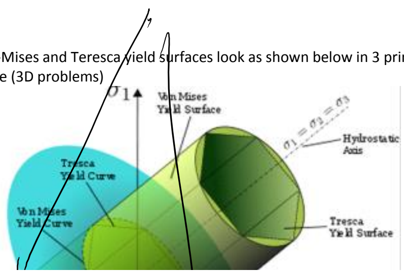
- 2D models
- plane stress versus plane strain plastic zones

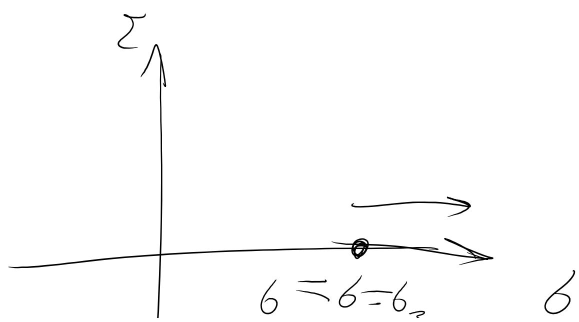
Plasticity:
There are many yield criteria

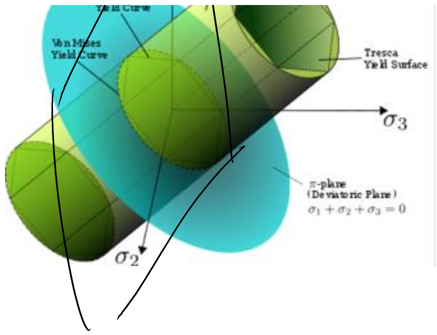


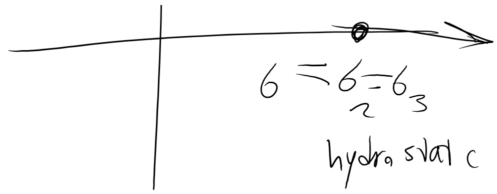
Yield with Teresca criterion occurs when maximum shear stress reaches τ_{ys}

Von-Mises and Teresca yield surfaces look as shown below in 3 principal stress space (3D problems)







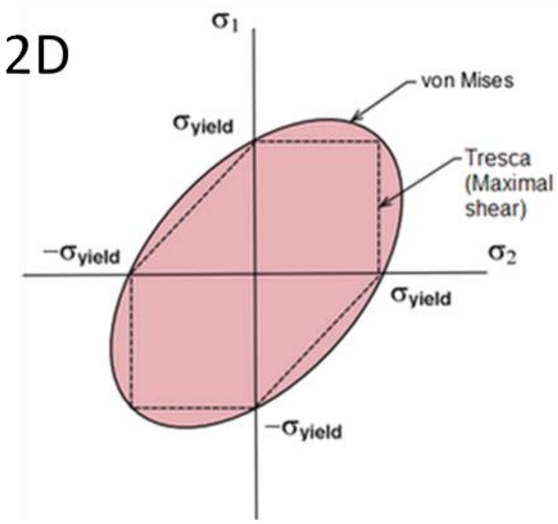


$\sigma = \sigma_2 = \sigma_3$ $\quad \sigma$

hydrostatic loading

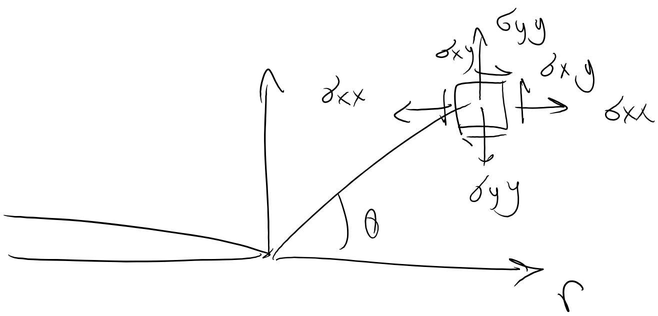
no yielding occurs

For plane stress $\quad \boxed{\sigma_3 = 0}$



plane - stress

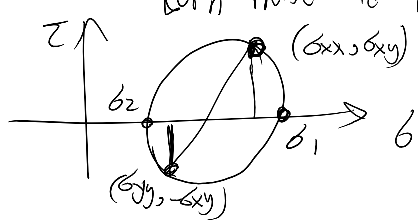
Now instead of 1D plasticity models that only looked ahead of the crack, we want to investigate what zone around the crack in all directions is yielding.



$$\left\{\begin{aligned}
\sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] \\
\sigma_{yy} &= \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] \\
\sigma_{xy} &= \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)
\end{aligned}\right.$$

turn these to principle stresses

$$\sigma_1, \sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$



$\sigma_1, \sigma_2 = ?$

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right)$$
$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)$$
$$\sigma_3 = \begin{cases} 0 & \text{plane stress} \\ \dfrac{2\nu K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

principal stresses for Mode I

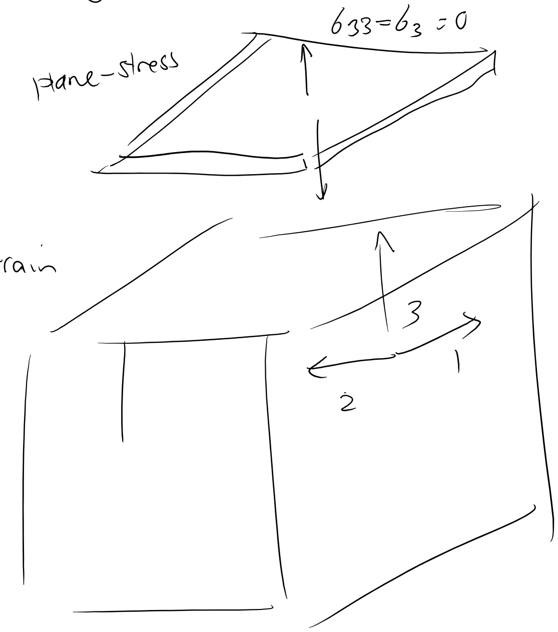
- All we need to do, is to plug them into a yield criterion.

Before that: how σ_3 is derived

plane-stress

$\sigma_{33} = \sigma_3 = 0$

plane-strain

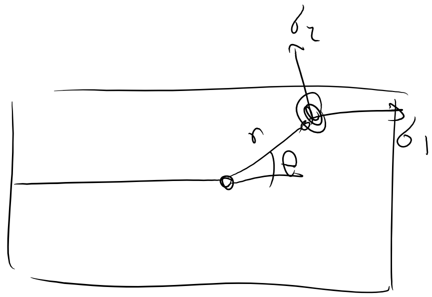


$\epsilon_{33} = 0$

$$\epsilon_{33} = \frac{1}{E}(\sigma_{33} - \nu\sigma_{11} - \nu\sigma_{22}) = 0$$

$$\boxed{\sigma_{33} = \nu(\sigma_{11} + \sigma_{22})}$$

$\sigma_3 = \nu(\sigma_1 + \sigma_2) \leftarrow$



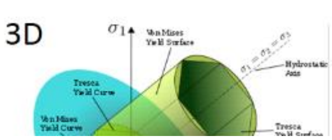
Now all we need to do is plug principal stresses into a yield criterion.

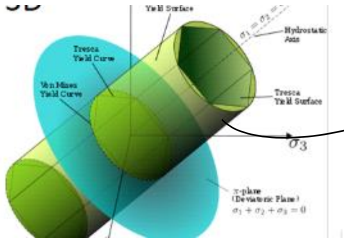
$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right)$$
$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)$$
$$\sigma_3 = \begin{cases} 0 & \text{plane stress} \\ \dfrac{2\nu K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

For example for Von-Mises we have

$$\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}}$$

$\sigma_e = \sigma_Y$





$\sigma_e = \sigma_Y$

We get the following equations for the locus of yield region

$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1 + \cos\theta + \frac{3}{2}\sin^2\theta\right]}$$ plane stress

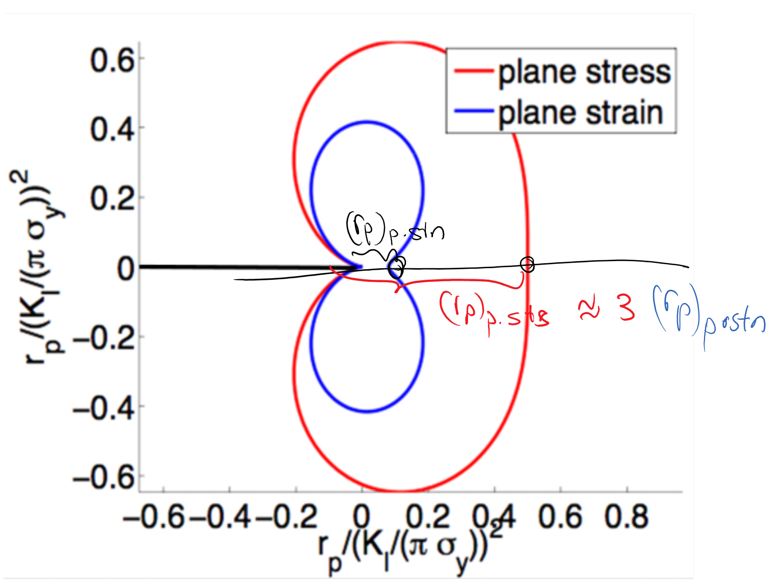
$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[(1-2\mu)^2(1+\cos\theta) + \frac{3}{2}\sin^2\theta\right]$$ plane strain

similar to 1D models

this angle-dependent part is added in 2D analysis

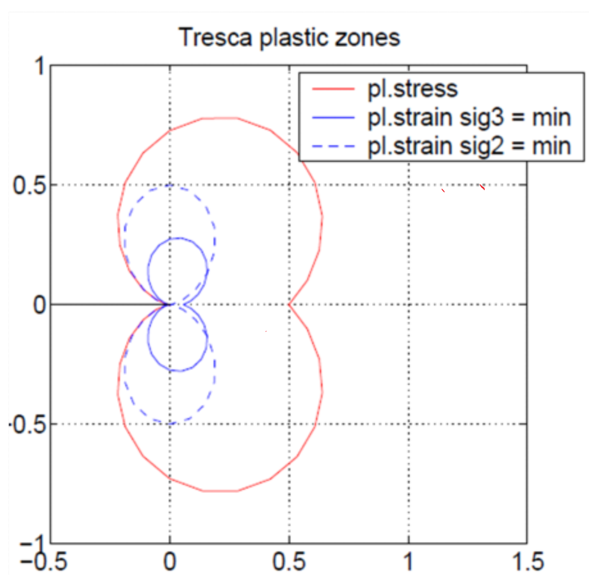
von-Mises criterion

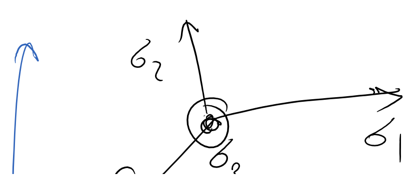
$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1 + \cos\theta + \frac{3}{2}\sin^2\theta\right]}$$



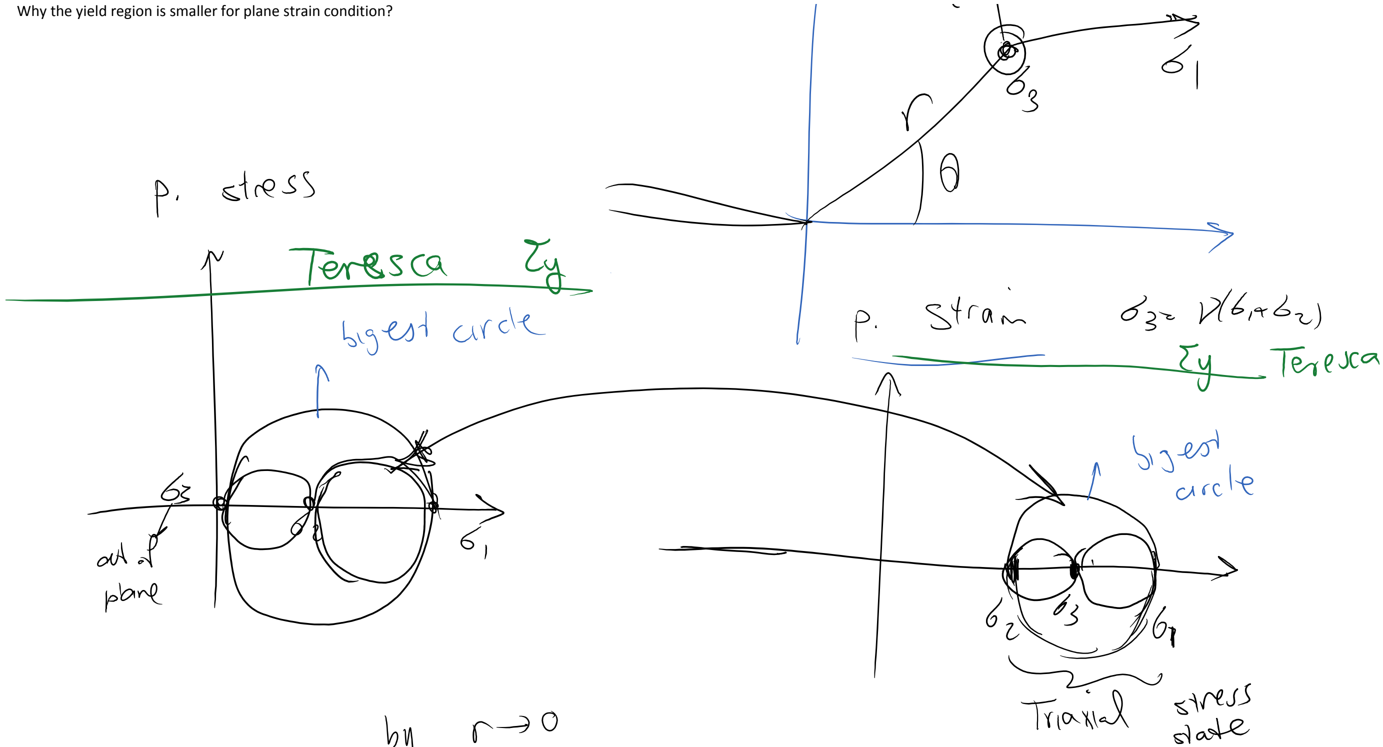
Why the yield region is smaller for plane strain condition?

Tresca criterion





Why the yield region is smaller for plane strain condition?

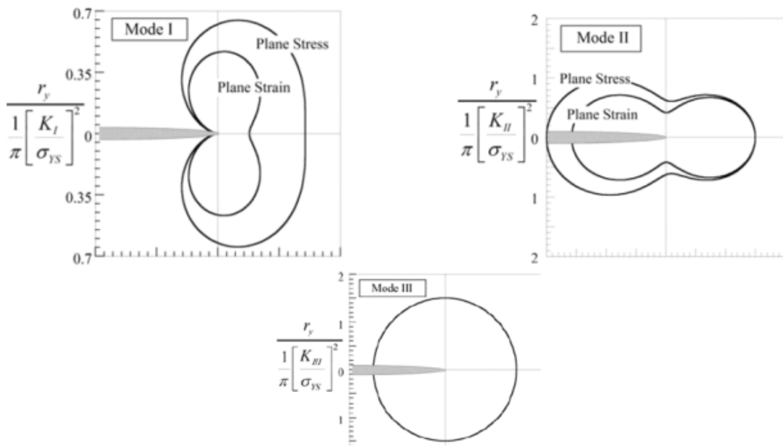


by $r \to 0$
circles grows
until they hit the yield surface,

For plane stress this happens for larger r $\to$ it has a larger yield area

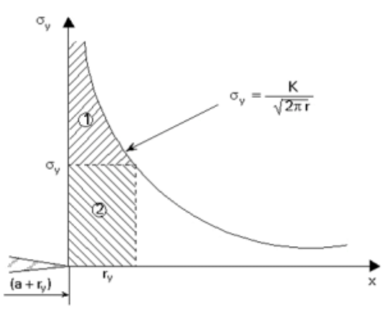
Triaxial stress state (e.g. plane strain) is less prone to yielding and more likely to fail by brittle fracture

Plastic zone shape: Mode I-III

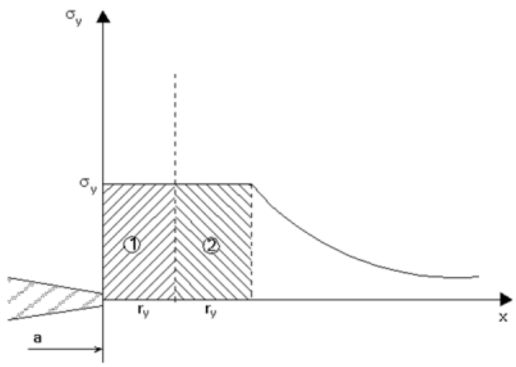


So we did above for 2D is pretty much the 2D generalization of 1st order approximation that does not take stress redistribution into account:

Recall from 1D models:



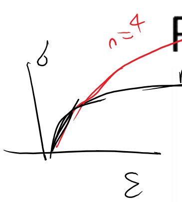
(a) Elastic crack



(b) Real crack with plastic zone

To take stress redistribution due to yielding, we really need to solve the problem taking yielding into account from the beginning.

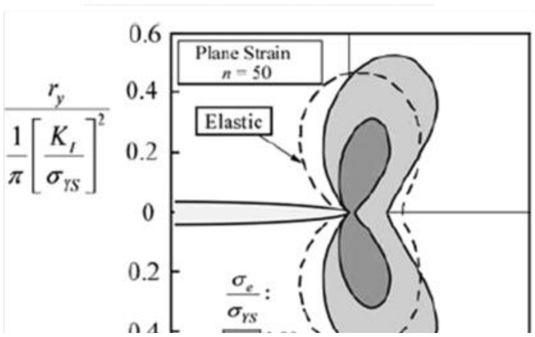
Dodds, 1991, FEM solutions
Ramberg-Osgood material model

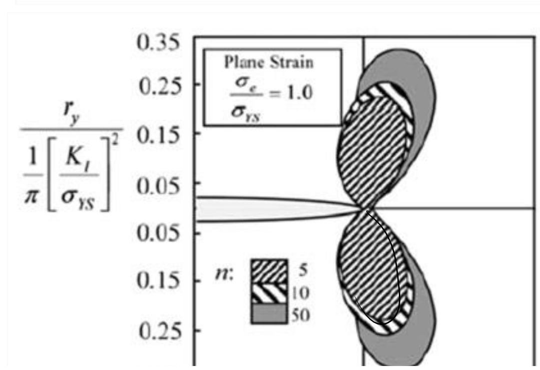


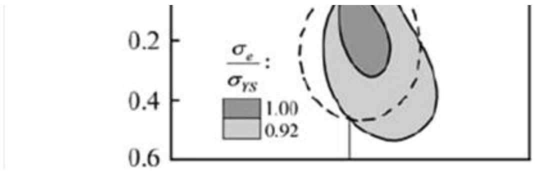
$n \to \infty$

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n$$

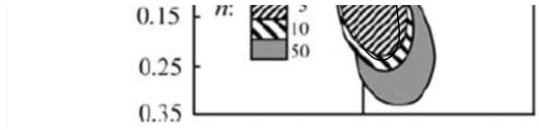
- Low n: High strain-hardening.
- $n \to \infty$: Similar to elastic perfectly plastic.







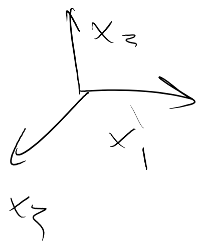
Effect of definition of yield (some level of ambiguity)

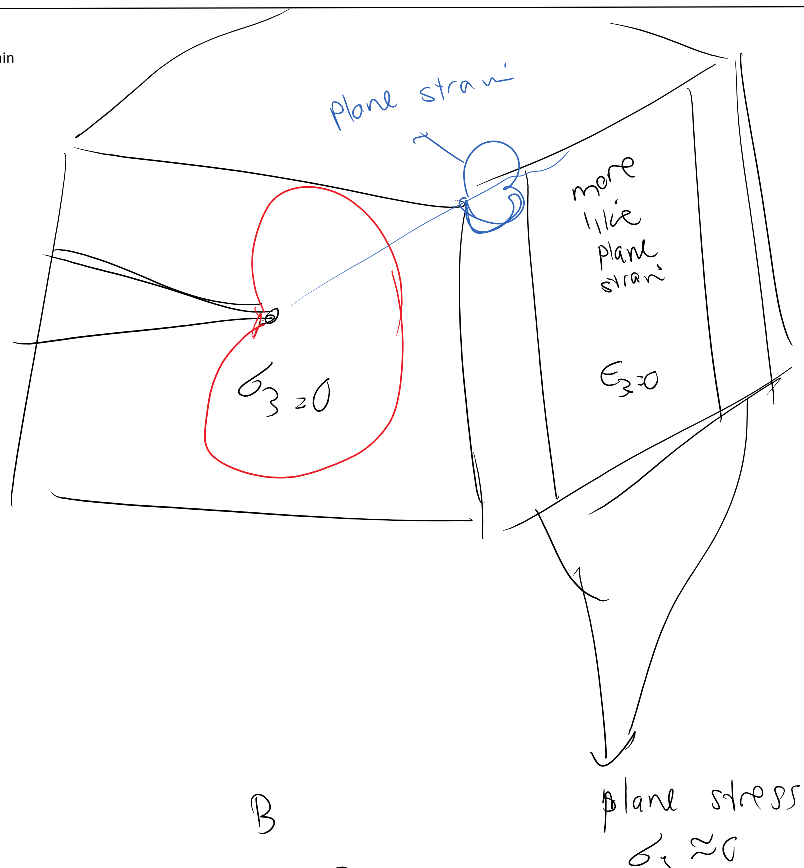


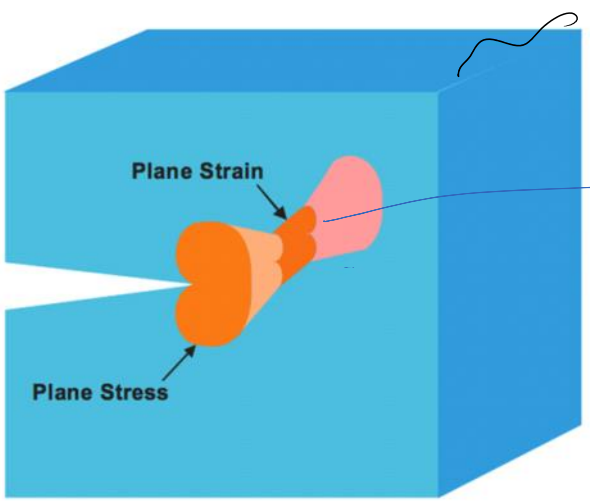
Effect of strain-hardening:
Higher hardening (lower n) => smaller zone

These solutions show that even the crude 2D models we covered before this (without considering stress redistribution) provide a decent estimate on the size of "plastic yielding"

Continuing the discussion on plane stress vs. plane strain





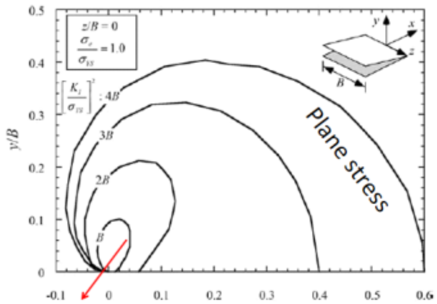


dog-bone yield region

B must be compared with a fracture-related length scale to decide if the plate is in plane-strain or plane-stress condition for fracture-response

Plane stress/plane strain: What thicknesses are plane stress?

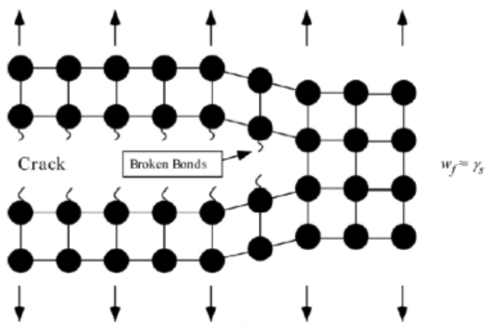
- As $\frac{\left(\frac{K}{\sigma_{ys}}\right)^2}{B}$ increases:
  1. The plastic zone expands (load is increasing)
  2. Plastic zone transitions from plane strain to plain stress
- Note that $r_p \propto \left(\frac{K}{\sigma_{ys}}\right)^2$

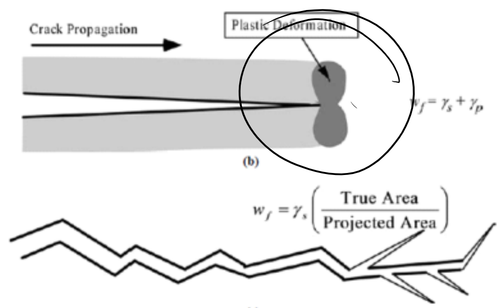


$$\frac{\left(\frac{K}{\sigma_{ys}}\right)^2}{B} \propto \frac{r_p}{B} : \begin{cases} \text{low (high } B) & \text{plane strain} \\ \text{high (low } B) & \text{plane stress} \end{cases}$$

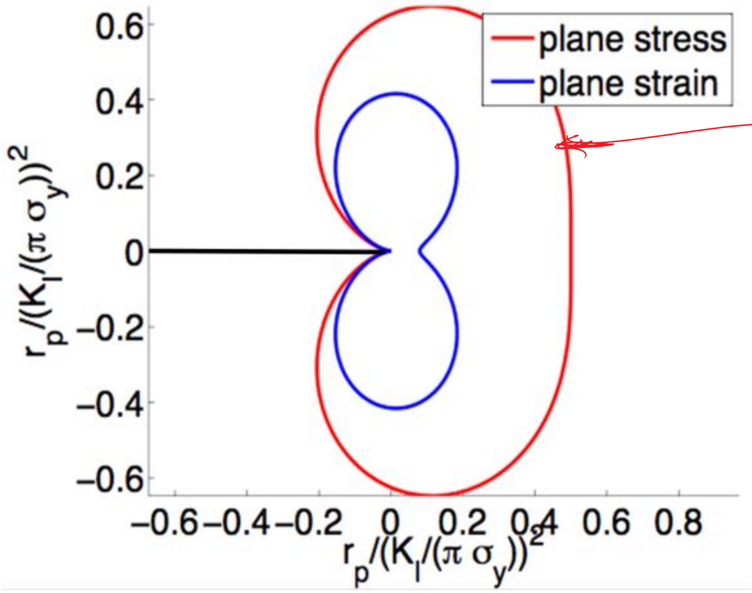
Change of plastic loci to plane stress mode as "relative B decreases". Nakamura & Park, ASME 1988

$$\boxed{\text{For plane strain condition we must have } B > \left(\frac{K}{\sigma_{ys}}\right)^2 \propto r_p}$$



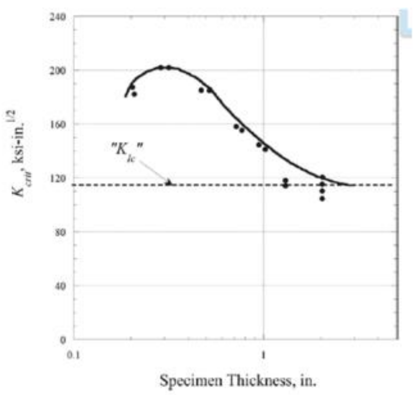


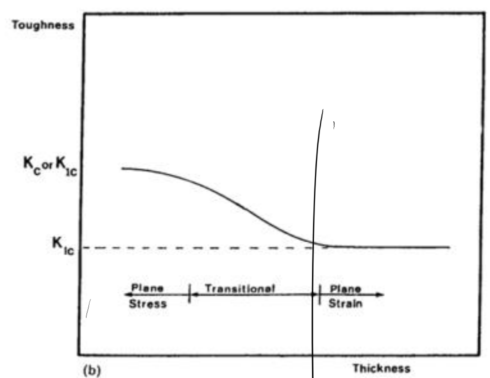
Plane stress or plane strain which one dissipate more energy per unit area of crack advance



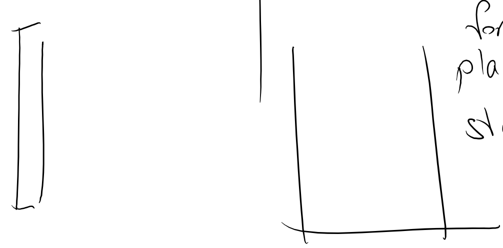
larger yield region

→ higher fracture toughness





K_{Ic} is constant for plane strain



r_p^2

Plane strain fracture toughness **lowest K (safe)**

$$K_c = K_{Ic}\left(1+\frac{1.4}{B^2}\left[\frac{K_{Ic}}{\sigma_{ys}}\right]^4\right)^{1/2}$$

Note that $\frac{1}{B^2}\left[\frac{K}{\sigma_{ys}}\right]^4 \propto \left(\frac{r_p}{B}\right)^2$

fracture toughness for plane strain

$\propto \left(\frac{r_p}{B}\right)^2$

plane strain $\frac{r_p}{B} \to 0$

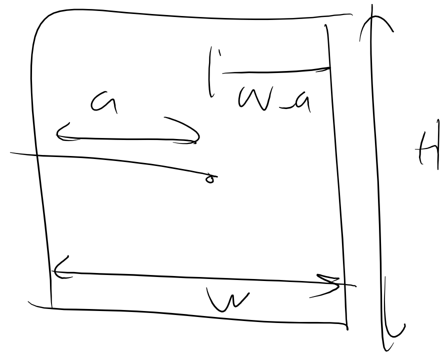
plane stress $\frac{r_p}{B}$ not negligible

Experimentalists want to stay in the plane-strain region

For good experiments we want all relevant length scales to be much larger than r_p?

$W-a,\ a,\ W,\ H \gg r_p$ SSY

$B \gg r_p$ plane-strain



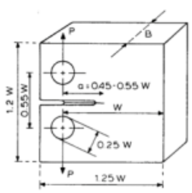
Fracture toughness tests

- Prediction of failure in real-world applications: need the value of fracture toughness
- Tests on cracked samples: PLANE STRAIN condition!!!

Compact Tension Test

$$K_I = \frac{P}{B\sqrt{W}} \frac{\left(2+\frac{a}{W}\right)\left[0.886 + 4.64\frac{a}{W} - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4\right]}{\left(1-\frac{a}{W}\right)^{3/2}}$$

$a, B, (W-a) \geq 2.5\left(\frac{K_{Ic}}{\sigma_{YS}}\right)^2$



ASTM (based on Irwin's model) for plane strain condition:

$$\boxed{a, B, (W-a) \geq 2.5\left(\frac{K_{Ic}}{\sigma_{Y}}\right)^2}$$

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