2018/10/10

Wednesday, October 10, 2018 11:29 AM

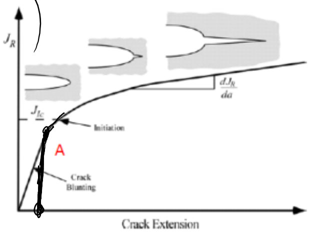
Last time:

We learnt how to calculate energy release rate J even for nonlinear elastic response

We need to compare J with resistance to see if a crack grows

Crack growth resistance curve

- A: R curve is nearly vertical:
  - small amount of apparent crack growth from blunting
- J_{I_c} measure of ductile fracture toughness
- Tearing modulus $T_R = \frac{E}{\sigma_0^2}\frac{\mathrm{d}J_R}{\mathrm{d}a}$ is a measure of crack stability



non dimensional parameter that determines how fast resistance (J_R) grows

Why resistance increases?

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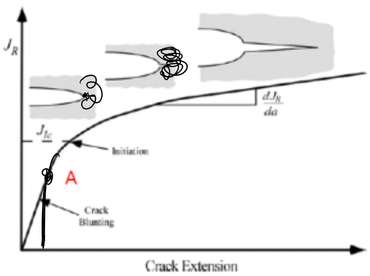
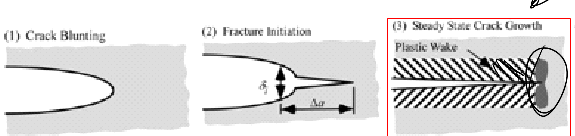
resistance grows because we're developing a region ahead of crack that is yielded



If the crack propagates longer we even observe a flag R value



Rare in experiments because it requires large geometries!

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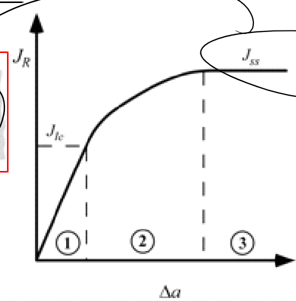
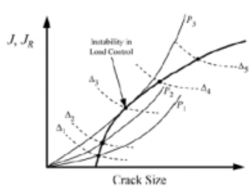


constant J_R region

Crack growth and stability

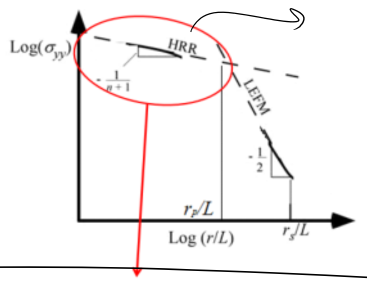


- The J_R and J are similar to R and G curves for LEFM:
  - Crack growth can happen when $J = J_R$
  - Crack growth is unstable when $\frac{\mathrm{d}J}{\mathrm{d}a} > \frac{\mathrm{d}J_R}{\mathrm{d}a}$

Back to the validity of HRR solution around the crack tip:

Back to the validity of HRR solution around the crack tip:

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n, \theta)$$

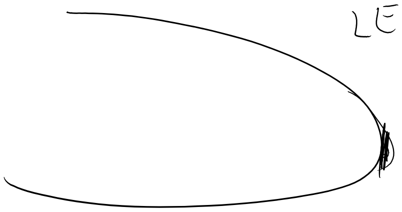


HRR solutions depend on J

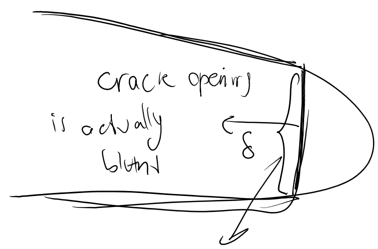
US

Crack Tip Opening Displacement (CTOD)

was introduced in parrall in UK as another measure of nonlinear response:



LEFM solution of crack opening

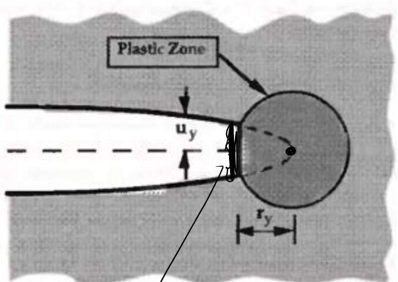


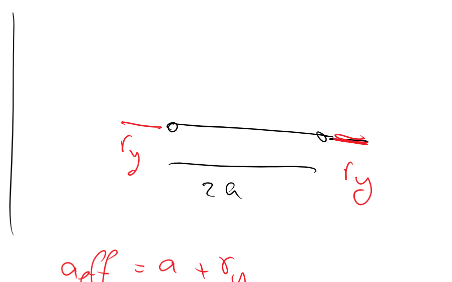
crack opening is actually blunt

CTOD

It's an indication of nonlinear yielding response

Estimate for CTOD (δ):





$a_{eff} = a + r_y$

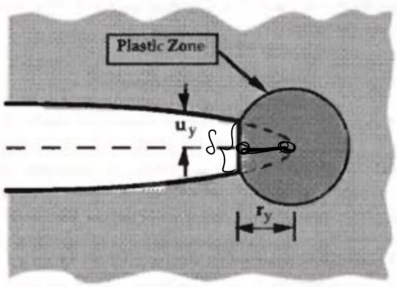
~~we~~ artificially made the crack longer to extend the applicability of LEFM

$2u_y$ (at $r = -r_y$) is how much CTOD would be because of plastic yielding

accurate but not exact, because we're not taking stress redistribution into account

$u_y = \frac{\kappa + 1}{\ } K \sqrt{r}$

into account



$$u_y = \frac{\kappa+1}{2\mu} K_I \sqrt{\frac{r}{2\pi}}$$

$$r = r_y$$

$$\delta = 2u_y(r = r_y) = 2\frac{\kappa+1}{2\mu} K_I \sqrt{\frac{r_y}{2\pi}}$$

$$r_y = \left(\frac{K_I}{\sigma_y}\right)^2 \Longrightarrow$$

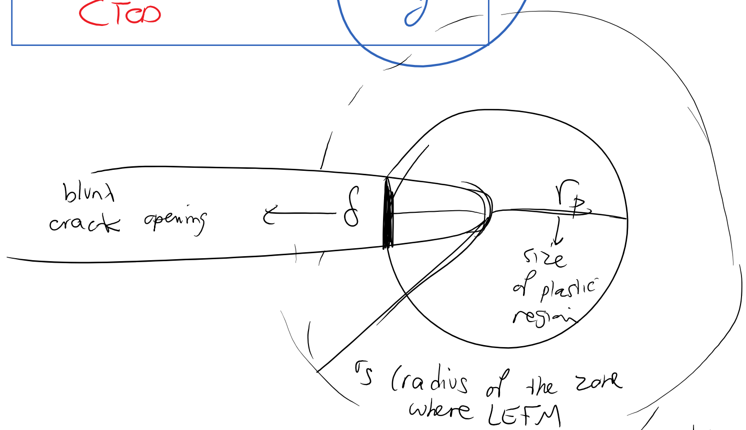
$$\delta = \frac{\kappa+1}{\sqrt{2\pi}} \frac{K_I^2}{\mu \sigma_y}$$

$\kappa = \frac{3-\nu}{1+\nu}$ p. strain

$$\mu = \frac{E}{2(1+\nu)} \Longrightarrow$$

$$\boxed{\delta \approx \frac{4}{\pi} \frac{K_I^2}{E\sigma_y}}$$

$\delta \to$ CTOD



$$\delta = \frac{K_I^2}{E\sigma_y}$$

$$\wedge$$

$$r_p = \frac{K_I^2}{\sigma_y^2}$$

$$r_s = \frac{K_I^2}{\sigma_o^2} \rightarrow$$ for field stress loading

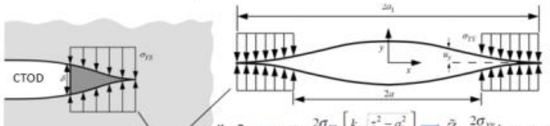
It's the competition between these length scales that determines LEFM, PFM, ... are applicable.

Before the discussion of SSY

More accurate formulas for δ:

Strip yield model that takes stress redistribution into account on the crack line provides a more accurate estimate for CTOD:

Crack Tip Opening Displacement: Strip yield model



Stresses that yielded K = 0

K = 0 $\rightarrow Z = \frac{2\sigma_{YS}}{\pi}\left[\frac{k}{z}\sqrt{\frac{z^2 - a_1^2}{1-k^2}}\right] \rightarrow \tilde{Z} = \frac{2\sigma_{YS}}{\pi}[z\omega_1 - a\omega_2]$

$\rightarrow u_y = \frac{2}{E}\mathrm{Im}\tilde{Z} = \frac{4\sigma_{YS}}{\pi E}\left[a\coth^{-1}\left(\frac{1}{a_1}\sqrt{\frac{a_1^2 - z^2}{1-k^2}}\right) - z\coth^{-1}\left(\frac{k}{z}\sqrt{\frac{a_1^2 - z^2}{1-k^2}}\right)\right]$

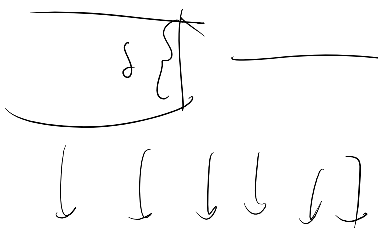
$z = a \rightarrow \quad \delta = 2u_y = \frac{8\sigma_{YS}a}{\pi E}\ln\left(\frac{1}{k}\right) = \frac{8\sigma_{YS}a}{\pi E}\left[\frac{1}{2}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{YS}}\right)^2 + \frac{1}{12}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{YS}}\right)^4 + \cdots\right] \rightarrow$

$$\boxed{\text{For } \sigma/\sigma_{YS} \rightarrow 0 \qquad \delta = \frac{K_I^2}{\sigma_{YS}E}}$$

our less accurate analysis gave $\quad \delta \approx \frac{4}{\pi}\frac{K_I^2}{\sigma_y E}$

Relating J (USA) and CTOD (UK) measure for nonlinear response around the crack tip?

$\delta = J^{\alpha} \ldots$



$\longrightarrow J = f(\delta) \rightarrow J$ energy release rate

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What is the relation if we use LEFM solution without considering stress redistribution.

$$\left.\begin{aligned} \delta &= \frac{K_I^2}{E\sigma_y} \\ J &= G = \frac{K_I^2}{E} \end{aligned}\right\} \Rightarrow \delta = \frac{J}{\sigma_y}$$

$[L]$ $\quad [\sigma][L]$ $\quad [\sigma]$

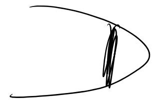
$$\boxed{J = \delta\,\sigma_y} \qquad \delta = \frac{J}{\sigma_y}$$

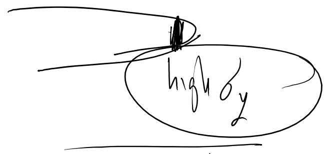
$J \nearrow$ (for field loading $\nearrow$) $\longrightarrow \delta \nearrow$

$\sigma_0 = \lambda \cdot \sigma_{0\,old} \rightarrow \delta = \lambda\,\delta_{old}$

σ_y gets higher $\delta \searrow$

low σ_y





displacement looks closer to that of LEFM

More accurate relations exist that if we have CTOD -> from them we can more accurately estimate J:

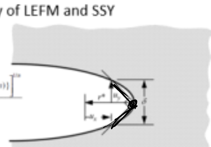
CTOD-J relation

- When SSY is satisfied G = J so we expect:

$$G = m\sigma_y\delta \quad \Rightarrow \quad \boxed{J = m\sigma_y\delta}$$

- In fact this equation is valid well beyond validity of LEFM and SSY
- E.g. for HRR solution Shih showed that:

$$u_i = \frac{\alpha\sigma_o}{E}\left(\frac{EJ}{\alpha\sigma_o^2 I_n r}\right)^{\frac{n}{n+1}} r\tilde{u}_i(\theta, n) \qquad d_n = \frac{2\tilde{u}_y(\pi,n)\left[\frac{\alpha\sigma_o}{E}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}\right]^{1/n}}{I_n}$$

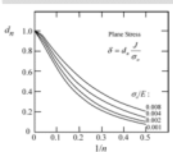


- δ is obtained by 90 degree method:
  Deformed position corresponding to $r^* = r$ and $\varphi = -\pi$ forms 45 degree w.r.t crack tip)

$$\frac{\delta}{2} = u_y(r^*, \pi) = r^* - u_x(r^*, \pi)$$

$$\boxed{r^* = \left(\frac{\alpha\sigma_o}{E}\right)^{1/n}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}^{\frac{n+1}{n}}\frac{J}{\sigma_o I_n}} \Rightarrow J = m\sigma_0\delta$$

for $\boxed{m = \frac{1}{d_n},\ d_n = \frac{2\tilde{u}_y(\pi,n)\left[\frac{\alpha\sigma_o}{E}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}\right]^{1/n}}{I_n}}$



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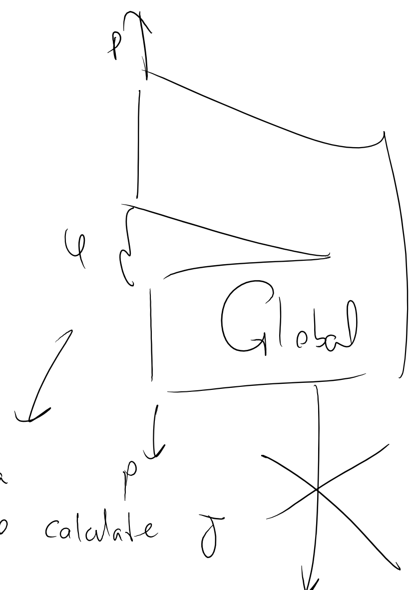


found a way to calculate J

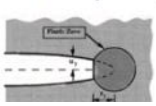
local

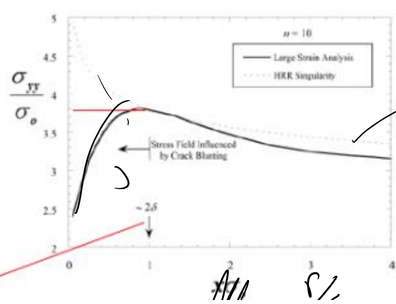
can still locally measure δ ⟹ get J

Limitations of HRR solution

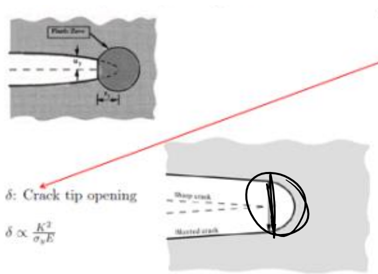
Limitations of HRR analysis

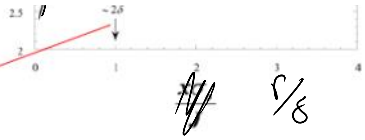
- Small strain: $\epsilon = \frac{1}{2}(\nabla\mathbf{u} + \nabla^T\mathbf{u})$ (accurate for $\epsilon \lessapprox 0.1$)
- Small deformation theory (*e.g.*, not using PK stresses, etc)
- Elastic HRR model instead of plastic model
- Crack tip blunting: $\Rightarrow \sigma_{xx} = 0$



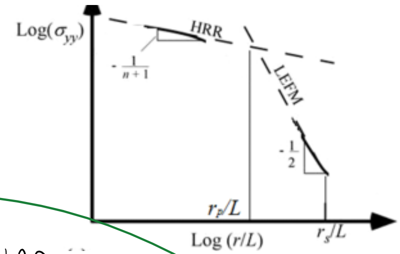


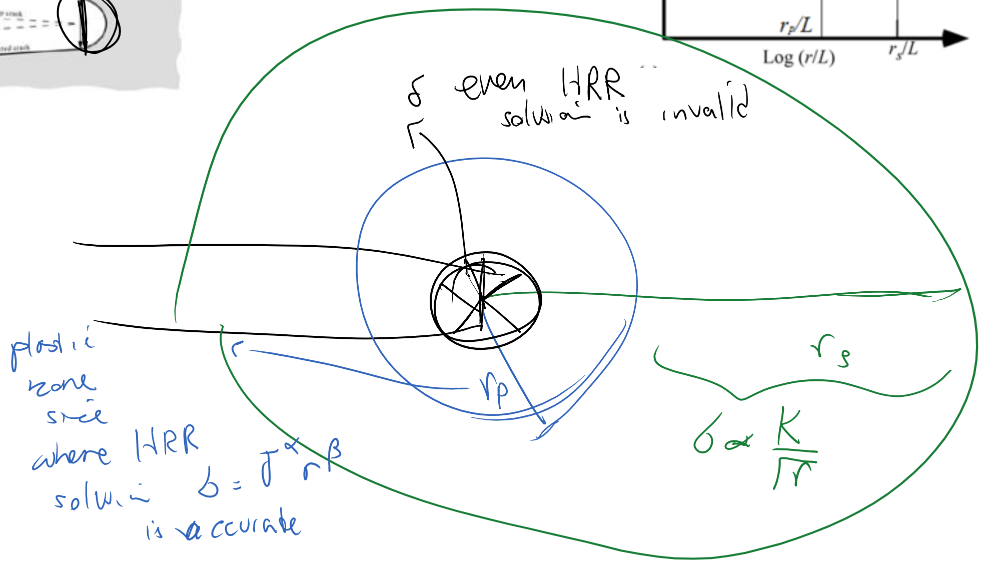
HRR solution

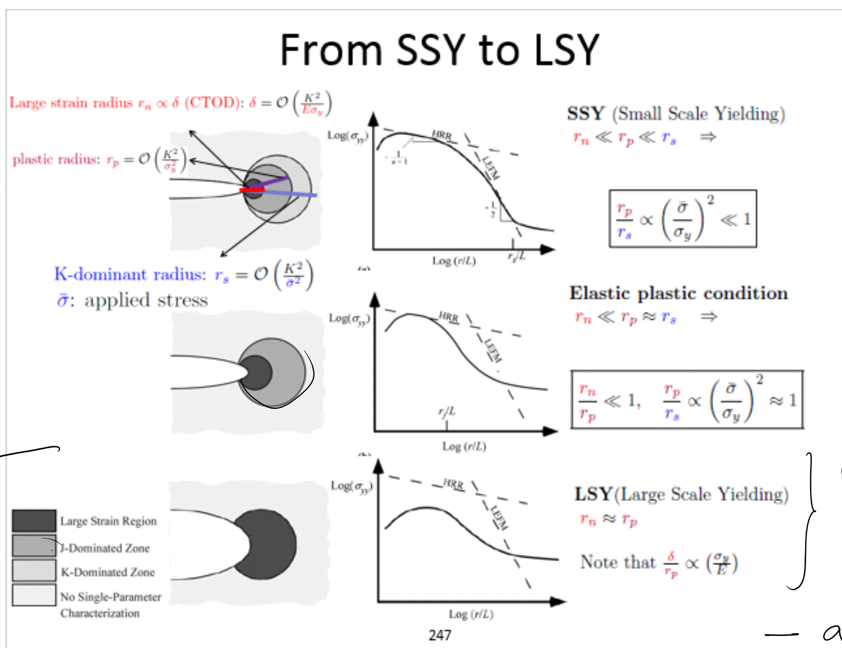




McMeeking and Parks, ASTM STP 668, ASTM 1979







LEFM ✓

J & HRR ✓

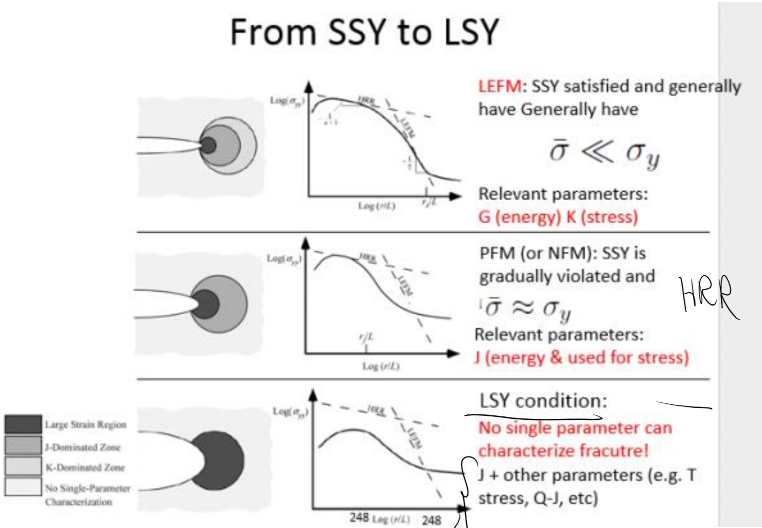
When does this happen?

- a lot of plastic unloading
- very large deformation

MORE ADVANCED THEORIES

J + ... some other things

↳ OR solve the problem numerically with plasticity, large deformation, ...



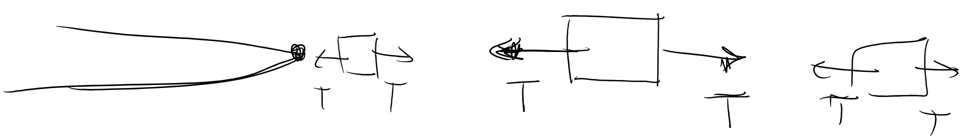
Couple of theories for that

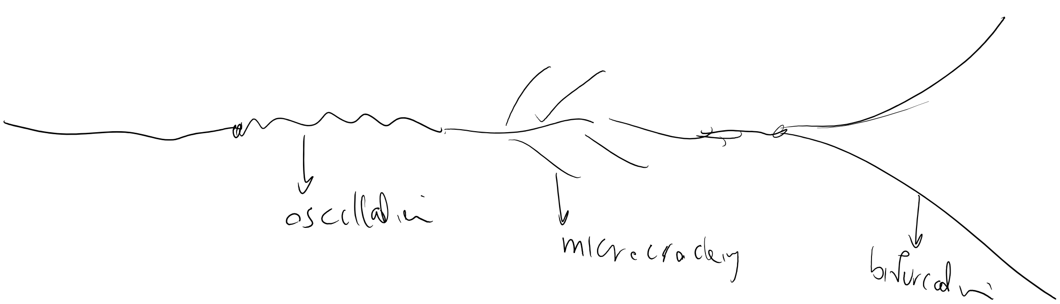
T - stress

$$\sigma_{ij}^{I} = \frac{K^{I}}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + T\delta_{i1}\delta_{j1}$$

0th term of expansion

1st term



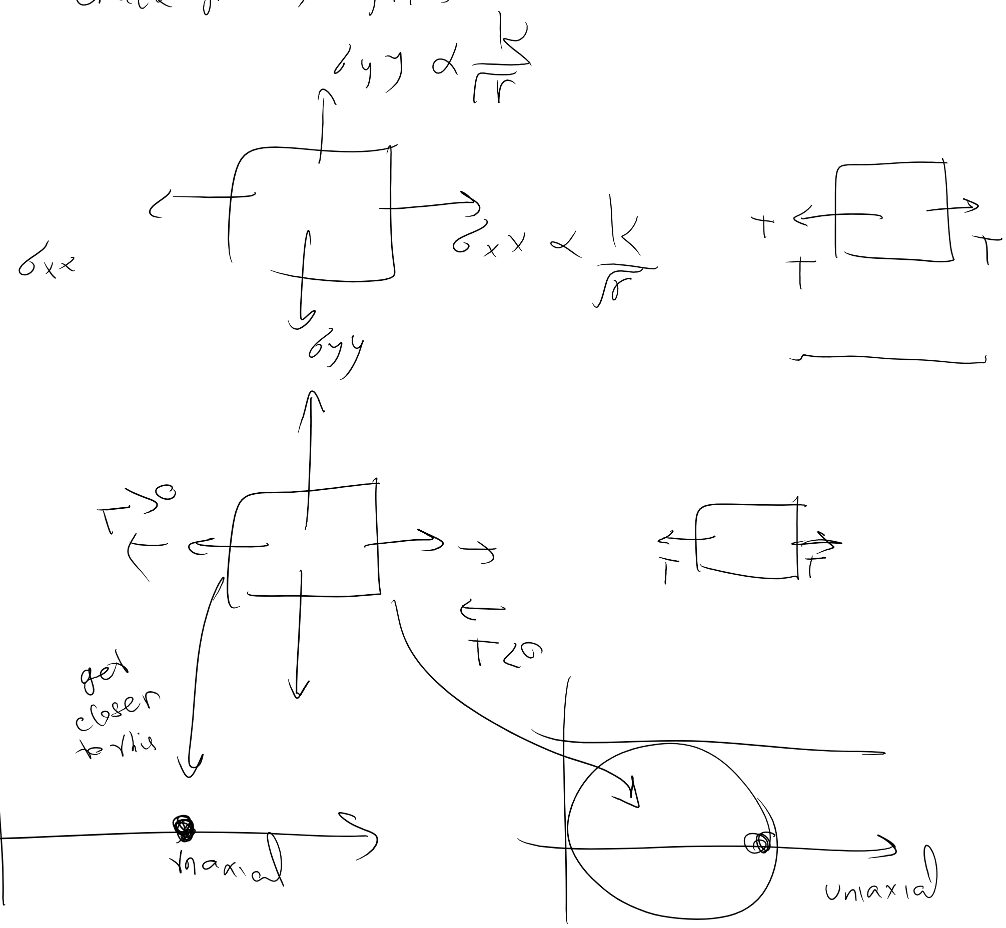


crack speed — $\dot{a}$

T stress ↗ crack oscillate less & its path becomes more stable!

side note

How about the effect of T stress on crack growth, toughness



toughness ↘

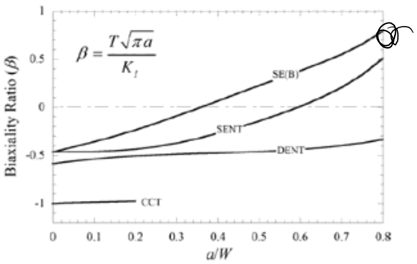
better for yielding

toughness ↗

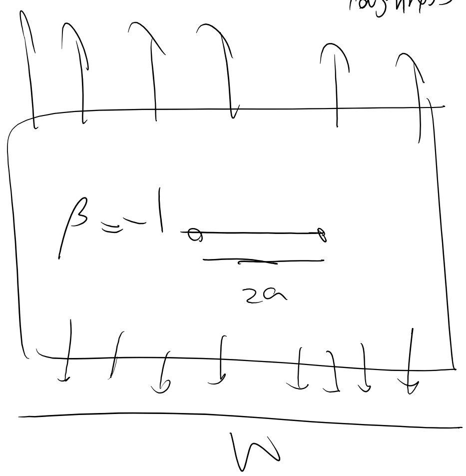
if want to use T stress

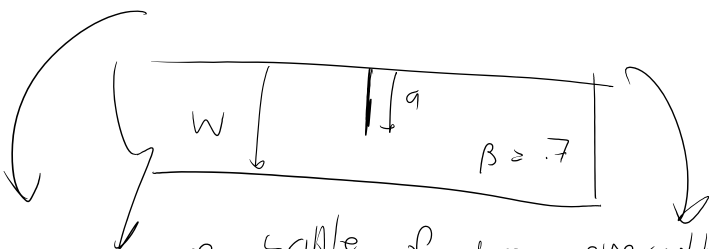
$T > 0$ triaxiality ↗ → reduce toughness good $T<0$

" $\searrow$ → increase toughness



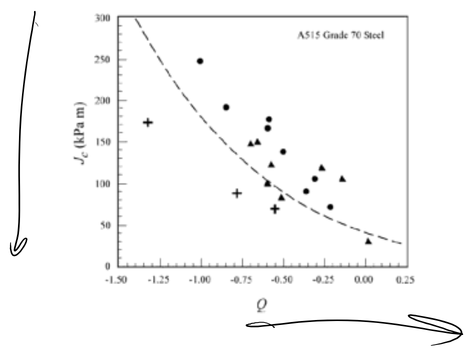
$\beta = \frac{T\sqrt{a}}{K}$





more brittle fracture everything being the same

β high ⟹ we need to reduce resistance



Q $\nearrow$ J_c $\searrow$