2018/10/15

Monday, October 15, 2018 11:35 AM

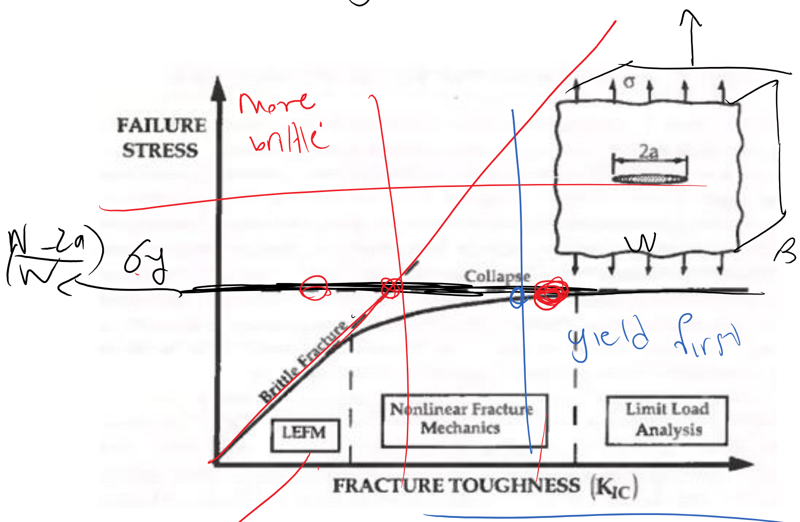
5.3. 7. Fracture mechanics versus material (plastic strength

Mechanical source of failure of a specimen / material:

1. Fracture
2. Fatigue
3. Plastic yielding
4. Buckling

...

Plastic yielding



$(W-2a)B\sigma_y = F = \sigma BW$

yielding $\boxed{\sigma_{Max} = \left(\frac{W-2a}{W}\right)\sigma_y}$

$K = f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a} \rightarrow$

$\boxed{\sigma_{fracture} = \frac{K_{Ic}}{f\left(\frac{a}{W}\right)\sqrt{\pi a}}}$

$\frac{K}{\sqrt{a}}/\sigma_y$ is low

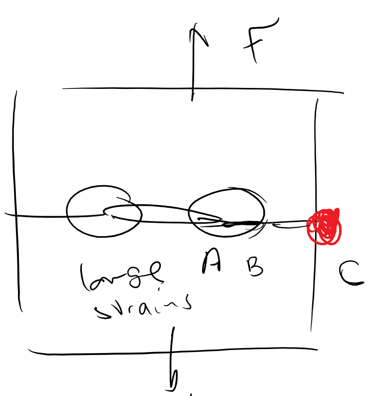
it fractures first

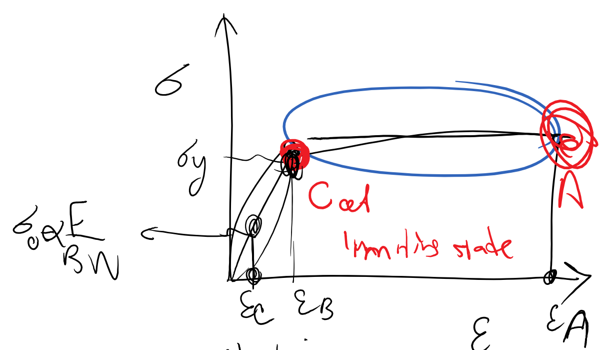
"many times failure mechanism for brittle materials"

more ductile

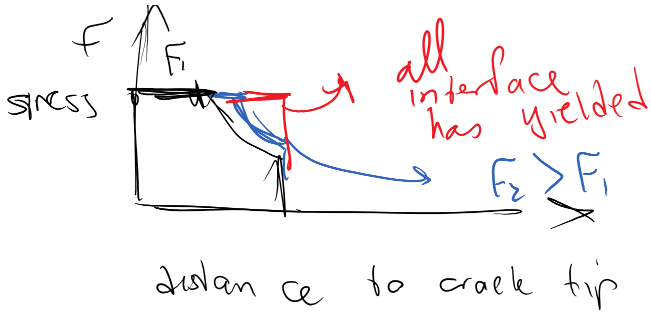
K_{Ic} high

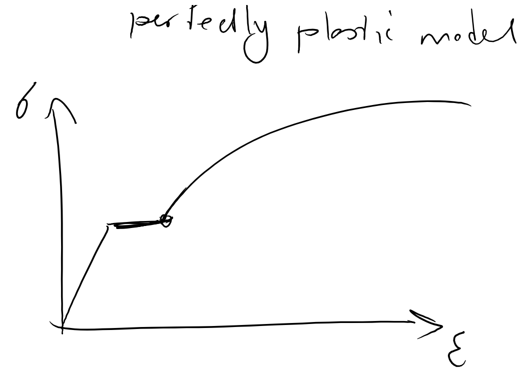
σ_y lower





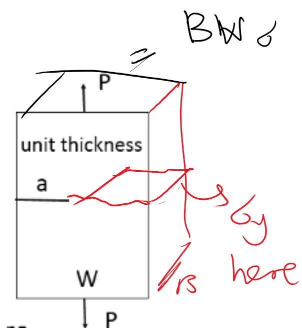
F F1 all interface





yielding max load:

— assume all interface is yielded



$$P_{yielding} = B(W-a)\,\sigma_y$$

$a \to W \qquad P_{yielding} \to 0$

for fracture $\quad \sigma = \dfrac{P}{\text{total area}} = \dfrac{P}{BW}$

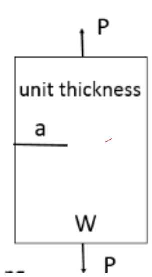
$$K = \sigma\sqrt{\pi a}\; f\left(\frac{a}{W}\right) \text{ @ failure}$$

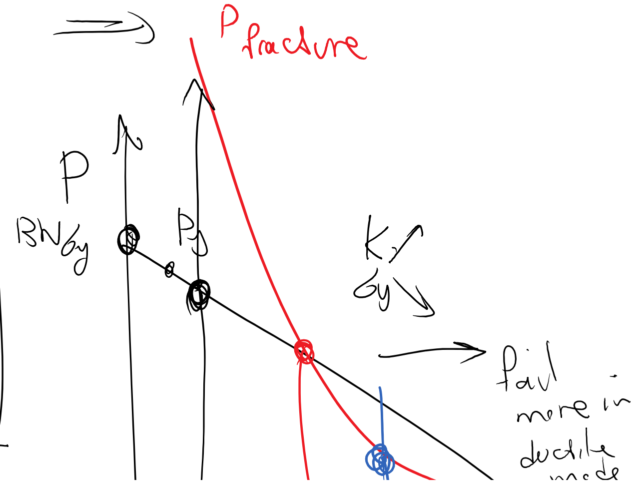
$$K_{IC} = \sigma_f \sqrt{\pi a}\; f\left(\frac{a}{W}\right) \Longrightarrow$$

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi a}\; f\left(\frac{a}{W}\right)}$$

$P_f = BW\sigma_f$

$$\boxed{\begin{aligned} P_{yield} &= B(W-a)\,\sigma_y \\ P_{frac.} &= \frac{K_{IC}\, BW}{\sqrt{\pi a}\; f\left(\frac{a}{W}\right)} \end{aligned}}$$



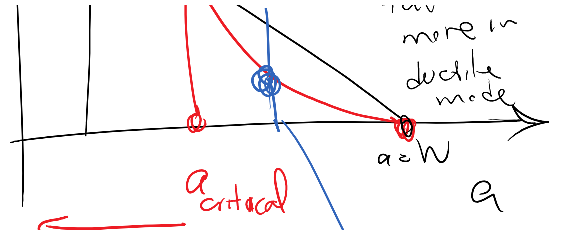


W

P

$f(\frac{a}{W}) \to 0$

as $a \to W$



$a < a_{critical}$

It yields first

ductile fracture

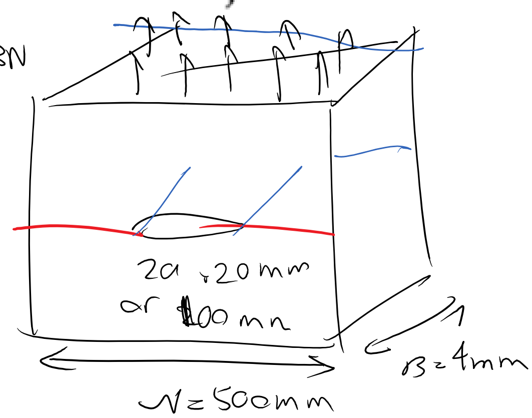
$a > a_{crit}$

It fractures first

brittle fracture

Example 4.11 Estimate the failure load under uniaxial tension for a centre-cracked panel of aluminium alloy of width W=500 mm, and thickness B=4 mm, for the following values of crack length $2a$ = 20 mm and $2a$ = 100 mm. Yield stress σ_y=350 MPa and fracture toughness K_{Ic}=70 MPa $\sqrt{m}$

$F_f = \sigma_f BW$



$K_{IC} = 70 MPa\sqrt{m}$

$\& \sigma_y = 350 MPa$

a = 20 or 100mm which failure mode dominates in each case?

2a = 20mm

$F_{yield} = \sigma_y \times (W - 2a) B = 350 \text{ MPa} \times (500 - 20) \times 10^{-3} \text{ m} \times 4e\text{-}3 \text{ m}$

$\longrightarrow \boxed{F_{yield} = 672 \text{ kN}}$

Fracture

$K = \sigma \sqrt{\pi a}\, f(\frac{a}{W})$, $f = \sqrt{\sec(\frac{\pi a}{W})}$

$\Longrightarrow$ at fracture state

$\sigma_f = \frac{K_{IC}}{\sqrt{\pi a}\sqrt{\sec(\frac{\pi a}{W})}} = \frac{70 \text{ MPa}\sqrt{m}}{\sqrt{\pi \times 10e\text{-}3 \text{ m}}\sqrt{\sec(\frac{\pi\, 10}{500})}} =$

394.6 MPa

$F_f = \sigma_f \times B \times W \Longrightarrow \boxed{F_f = 790 \text{ kN}}$

for $a = 10m$ it will yield first $(F_{max} = F_{yield} = 672kN)$

for $2a = 100mm$

$F_{yield} = 560kN$

$\boxed{F_p = 172.2kN}$ it will fracture first.

6. Computational fracture mechanics

6.1. Fracture mechanics in Finite Element Methods

6.2. Traction Separation Relations (TSRs)

6.1Fracture mechanics in Finite Element Methods (FEM)

6.1.1. Introduction to Finite Element method

6.1.2. Singular stress finite elements

6.1.3. Extraction of K (SIF), G

6.1.4. J integral

6.1.5. Finite Element mesh design for fracture mechanics

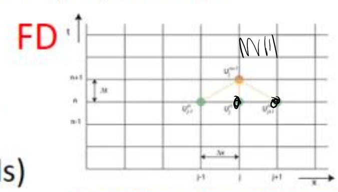
6.1.6. Computational crack growth

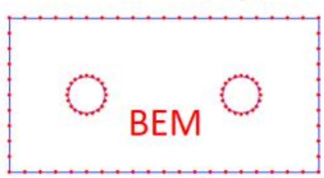
6.1.7. Extended Finite Element Method (XFEM)

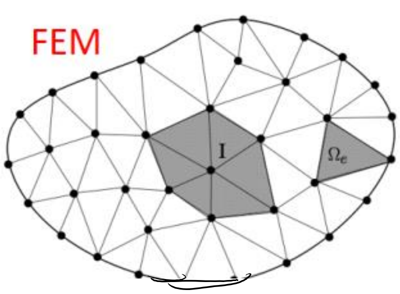
Numerical methods for solving PDEs

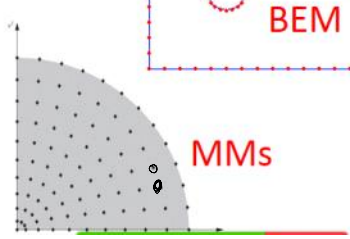
Numerical methods to solve PDEs

- Finite Difference (FD) & Finite Volume (FV) methods
- FEM (Finite Element Method)
- BEM (Boundary Element Method)
- MMs (Meshless/Meshfree methods)





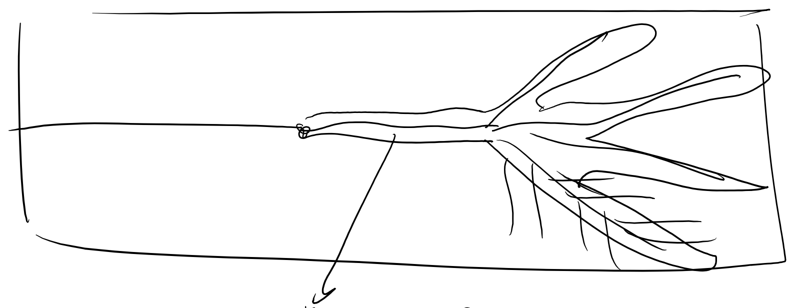




How failure is modeled in computational setting:

How failure is modeled in computational setting:

1. Bulk damage & phase field methods

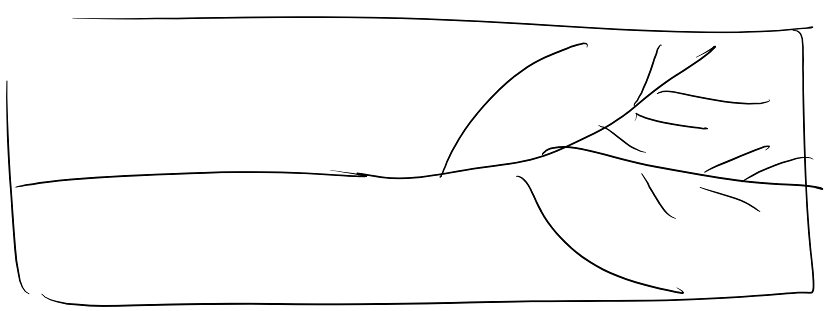


thick finite thickness regions approximate cracks

for bulk damage

$$\sigma = \underbrace{(1-D)}_{\text{Damage parameter}} \underbrace{C\varepsilon}_{\text{elastic strain}}$$

2. Sharp interface models



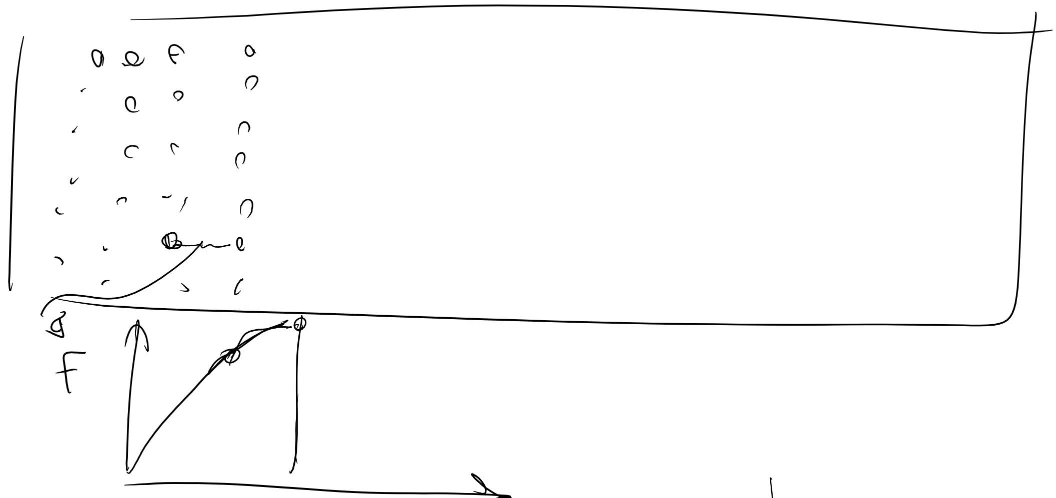
Some equations govern crack nucleation & propagations

- LEFM / PFM
- Traction separation Relations
- Interfacial damage model

1. Interfacial damage model

3. Discrete models:

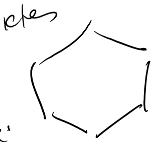
Material is treated as a collection of interacting particles, similar to Molecular Dynamics



bond is broken here Δu displacement jump

- Peridynamics
- Discrete element (rock mechanics)

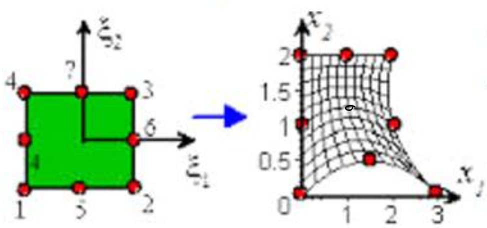
particles → specific geometry



can be elastic or rigid

We will only cover sharp interface models through LEFM and TSRs

Isoparametric Elements



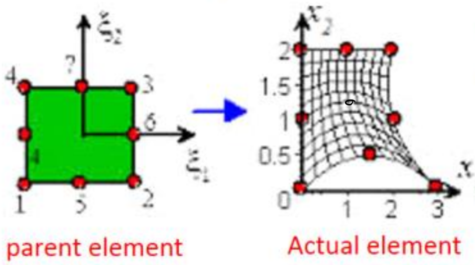
parent element

Actual element

- Geometry is mapped from a parent element to the actual element
- The same interpolation is used for geometry mapping and FEM solution (in the figure 2nd order shape functions are used for solution and geometry)
- Geometry map and solution are expressed in terms of ξ

Order of element

Isoparametric Elements



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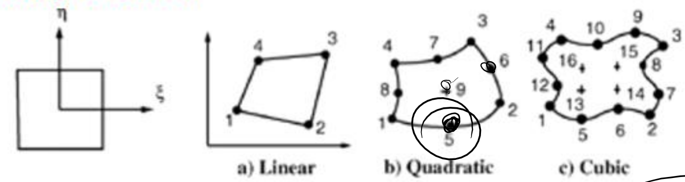
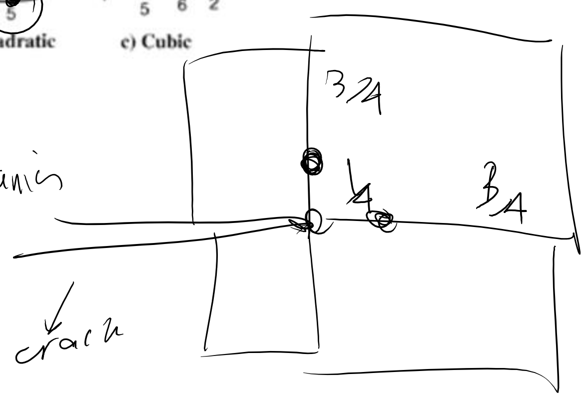
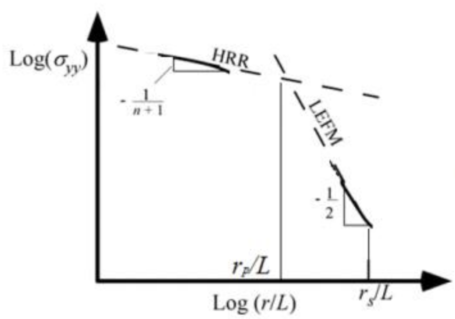


in Fracture Mechanics



they can reproduce singular response of LEFM

Singular crack tip solutions



$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n, \theta)$$

$$\epsilon_{ij} = \frac{\alpha \sigma_0}{E} \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\epsilon}_{ij}(n, \theta)$$

- NLFM (PFM): For HRR solution stress $\frac{1}{r^{\frac{1}{n+1}}}$ and strain $\frac{1}{r^{\frac{n}{n+1}}}$ are still singular $\Rightarrow$
  - for elastic-perfectly plastic ($n \to \infty$) stress is bounded and strain is $\frac{1}{r}$ singular

Linear Elastic FM (LEFM) ε & $\sigma \propto \frac{1}{\sqrt{r}}$
(n=1 above)

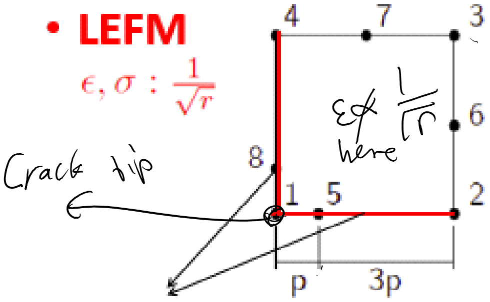
A. LEFM: reproducing 1/sqrt(r) singularity for strain:

Isoparametric singular elements

- **LEFM**

$\epsilon, \sigma : \frac{1}{\sqrt{r}}$

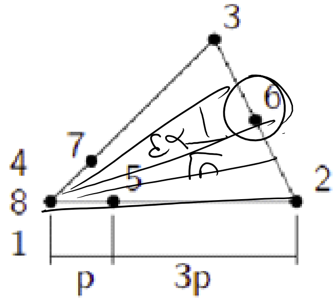
Quarter point Quad element



singular form $\frac{1}{\sqrt{r}}$ only along these lines

NOT recommended

Quarter point **collapsed** Quad element



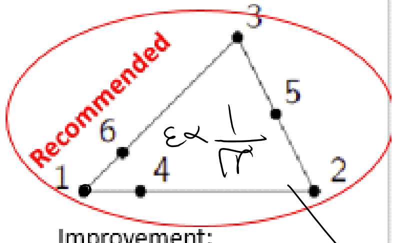
Improvement:

- $\frac{1}{\sqrt{r}}$ from inside all element

Problem

- Solution inaccuracy and sensitivity when opposite edge 3-6-2 is curved

Quarter point Tri element



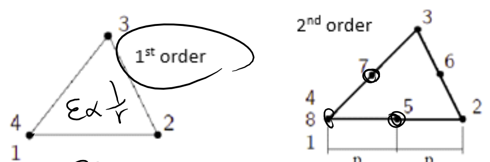
Improvement:

- Better accuracy and less mesh sensitivity

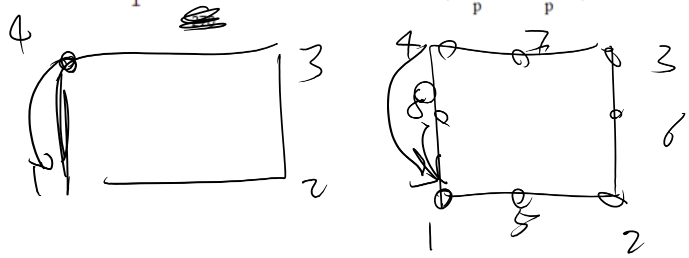
inside the element

B. 1/r singularity for elastic perfectly plastic

Collapsed Quad elements



better choice



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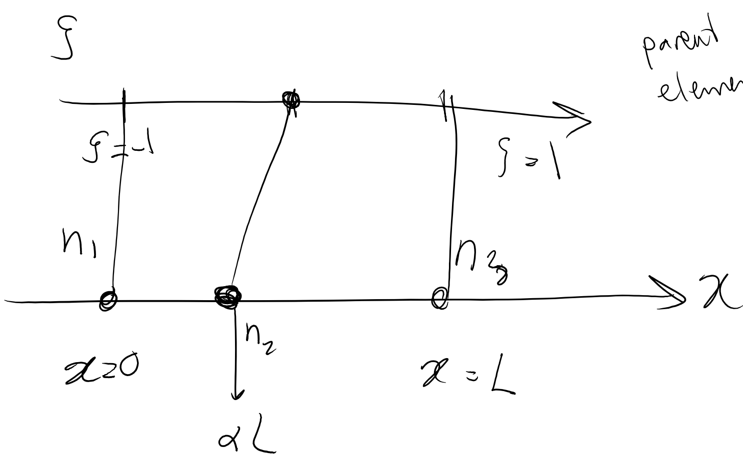
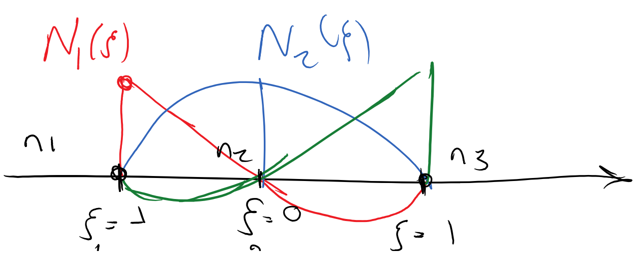
parent element

Shape functions





$$N_1(\xi) = \frac{(\xi - \xi_2)(\xi - \xi_3)}{(\xi_1 - \xi_2)(\xi_1 - \xi_3)}$$

$$N_1(\xi_2) = N_1(\xi_3) = 0$$

$$N_1(\xi_1) = 1$$

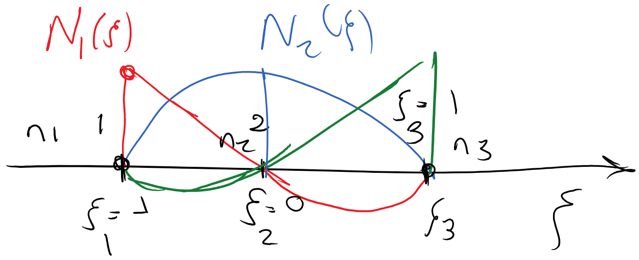
$\xi_1 = -1 \qquad \xi_2 = 0 \qquad \xi_3 = 1$

$N_1(\xi_1) = 1$

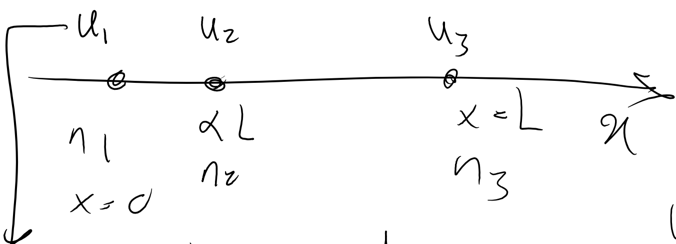
$$N_1 = \frac{(\xi - 0)(\xi - 1)}{(-1-0)(-1-1)} = \frac{\xi(\xi-1)}{2}$$

$$N_2(\xi) = \frac{(\xi-\xi_1)(\xi-\xi_3)}{(\xi_2-\xi_1)(\xi_2-\xi_3)} = 1-\xi^2$$

$$N_3(\xi) = \frac{(\xi-\xi_1)(\xi-\xi_2)}{(\xi_3-\xi_1)(\xi_3-\xi_2)} = \frac{\xi(\xi+1)}{2}$$



$$\boxed{\begin{aligned} N_1(\xi) &= \frac{\xi(\xi-1)}{2} \\ N_2(\xi) &= 1-\xi^2 \\ N_3(\xi) &= \frac{\xi(\xi+1)}{2} \end{aligned}}$$



nodal displacements

$$u(\xi) = u_1 N_1(\xi) + u_2 N_2(\xi) + u_3 N_3(\xi)$$

$$u(\xi_1) = u_1 \underbrace{N_1(\xi_1)}_{1} + u_2 \cancel{N_2(\xi_1)}_{0} + u_3 \cancel{N_3(\xi_1)}_{0}$$

$= u_1$

$$\boxed{u(\xi_i) = u_i}$$

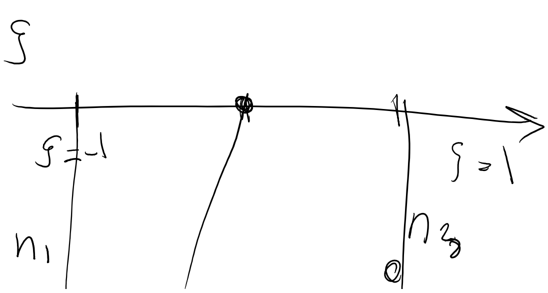
$$\boxed{u(\xi) = \left\{ u_1\left(\frac{\xi(\xi-1)}{2}\right) + u_2\left(1-\xi^2\right) + u_3\left(\frac{\xi(\xi+1)}{2}\right)\right\}}$$

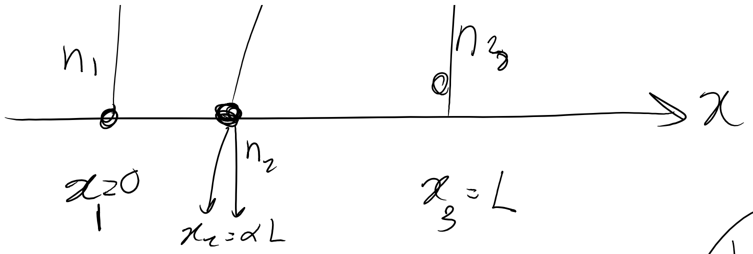
$$\varepsilon = \frac{du(\xi)}{dx}$$

we want $\varepsilon \to \infty$

at $x \to 0$

or $\xi \to -1$





$$\varepsilon = \frac{du(\xi)}{dx} = \frac{du(\xi)}{d\xi} \cdot \frac{d\xi}{dx} \quad ?$$

$$\xi(x)$$

$$x(\xi) \rightarrow \frac{d\xi}{dx} = \frac{1}{\frac{dx}{d\xi}}$$

$$x = x_1 N_1(\xi) + x_2 N_2(\xi) + x_3 N_3(\xi)$$

$$x(\xi_1) = x_1$$

$$x(\xi_2) = x_2 \qquad x(\xi_3) = x_3$$