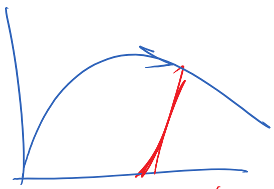
2018/10/22

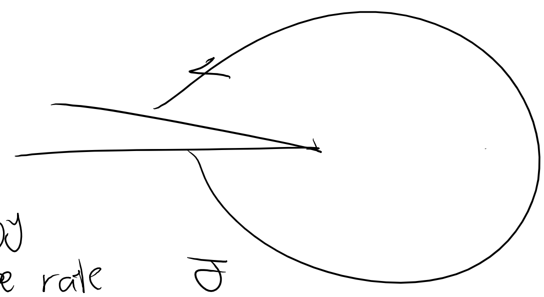
Monday, October 22, 2018 11:33 AM

– Path independence of J integral & relation to energy release rate were based on:

- Nonlinear elastic (no thermal strains, no plasticity)



- no inertia effects
- no body force
- no stresses on crack surfaces



J = energy release rate J

General form of J integral

$$J = \lim_{\Gamma_o \to 0} \int_{\Gamma_o} \left[(w+T)\delta_{1i} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} \right] n_i d\Gamma \qquad w = \int_0^{\varepsilon_{ij}^m} \sigma_{ij} d\varepsilon_{ij}^m$$

Inelastic stress

Kinetic energy density

$$T = \frac{1}{2}\rho \frac{\partial u_i}{\partial t} \frac{\partial u_i}{\partial t}$$

Can include (visco-) plasticity, and thermal stresses

$$\varepsilon_{ij}^{total} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} + \alpha\Theta\delta_{ij} = \varepsilon_{ij}^{m} + \varepsilon_{kk}^{t}$$

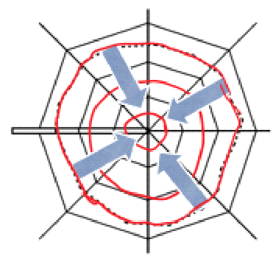
Elastic

Plastic

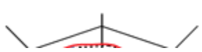
Thermal (Θ temperature)

before

new types of inelastic strain



$\Gamma_o \to 0$



total energy density = $\omega + T$

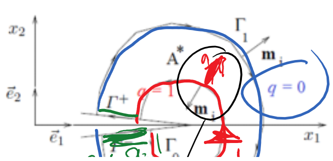
strain energy density $T = \frac{1}{2} \rho \vec{V} \cdot \vec{V}$

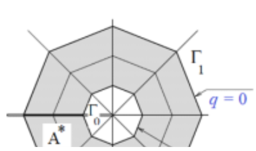
The only disadvantage is getting too close to the crack tip where numerical solutions are often not very accurate -> J will not be calculating very accurately.

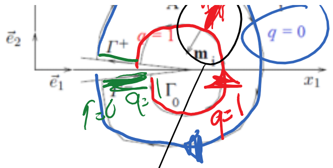
J integral: 2.Equivalent Domain Integral (EDI)

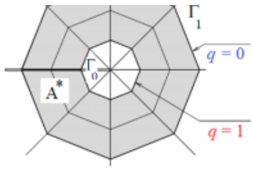
we turn line integral ⟹ surface integral

$$J = \int_{\Gamma_0} \left\{ (\omega + T)\delta_{1i} - \sigma_{ij} \frac{\partial u_i}{\partial x_1} \right\} n_j \, ds$$









$$= -\left(\int_{\Gamma_0} \left\{ (w+T)\delta_{1i} - \sigma_{ij}\frac{\partial u_i}{\partial x_1} \right\} m_j \, ds \right)$$

q = 1 on Γ_0

opposite directions for

m: normal to the surface

n: outward normal for Γ_0

$$\Rightarrow -J = \int_{\Gamma_0} \left\{ (w+T)\delta_{1i} - \sigma_{ij}\frac{\partial u_i}{\partial x_1} \right\} q\, m_j \, ds$$

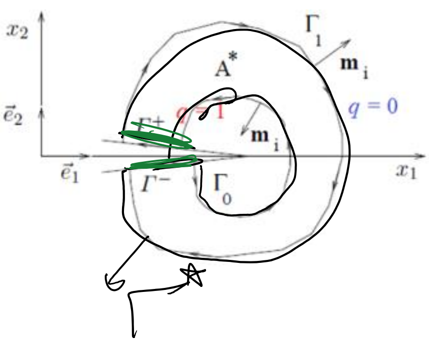
$$+ \int_{\Gamma_1} \left\{ (w+T)\delta_{1i} - \sigma_{ij}\frac{\partial u_i}{\partial x_1} \right\} q\, m_j \, ds$$

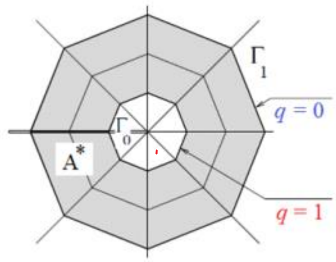
zero — q = 0 on Γ_1

genius idea!

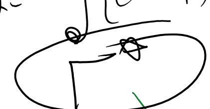
$$+ \int_{\Gamma^- \cup \Gamma^+} \left\{ (w+T)\delta_{1j} - \sigma_{ij}\frac{\partial u_i}{\partial x_1} \right\} q\, m_j \, ds$$

$$- \int_{\Gamma^- \cup \Gamma^+} (w+T)\delta_{1j} - \sigma_{ij}\frac{\partial u_i}{\partial x_1} \} q\, m_j \, ds$$

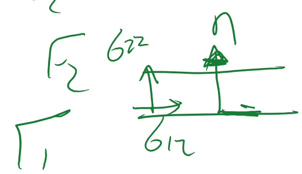




$$-J = \oint \left\{ (w+T)\delta_{1j} - \sigma_{ij}\frac{\partial u_i}{\partial x_1} \right\} q\, m_j^0 \, ds$$



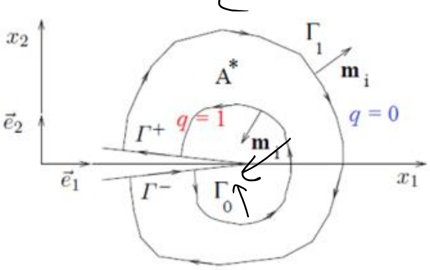
$$- \int_{\Gamma_1 \cup \Gamma_2} \left((w+T) n_1 - \sigma_{ij} m_i \frac{\partial u_i}{\partial x_1} q\, m_j \right) ds$$

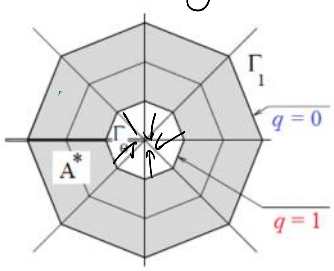


$n_1 = 0$

$2 = 1$

$$-J = \int \left[\left\{ (w+T)\delta_{1j} - \sigma_{ij}\frac{\partial u_i}{\partial x_1} \right\} q \right]_{,j} dV$$

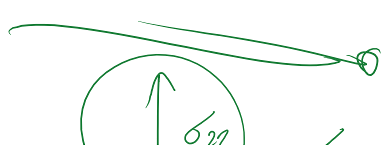




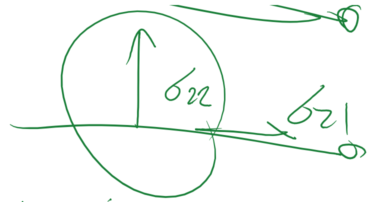
Divergence theorem

$$J = \int_{A^*} \frac{\partial}{\partial x_j} \left\{ \left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right] q \right\} dA - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j}\frac{\partial u_j}{\partial x_1} q \, d\Gamma$$

CT



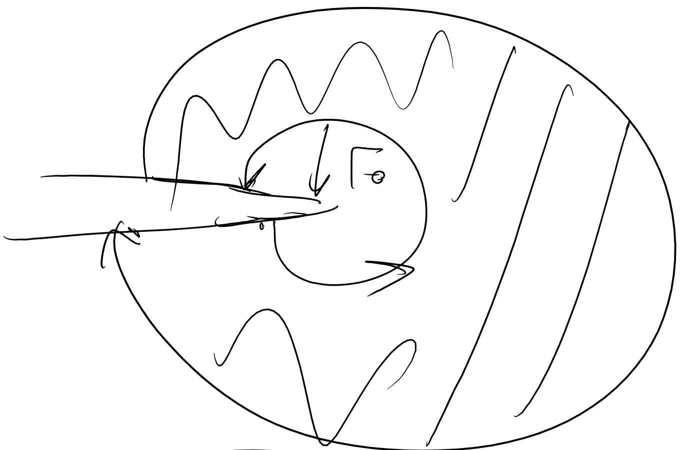
$$J = \int_{A^*} \frac{\partial}{\partial x_j}\left\{\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right]q\right\}dA - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j}\frac{\partial u_j}{\partial x_1} q\, d\Gamma$$

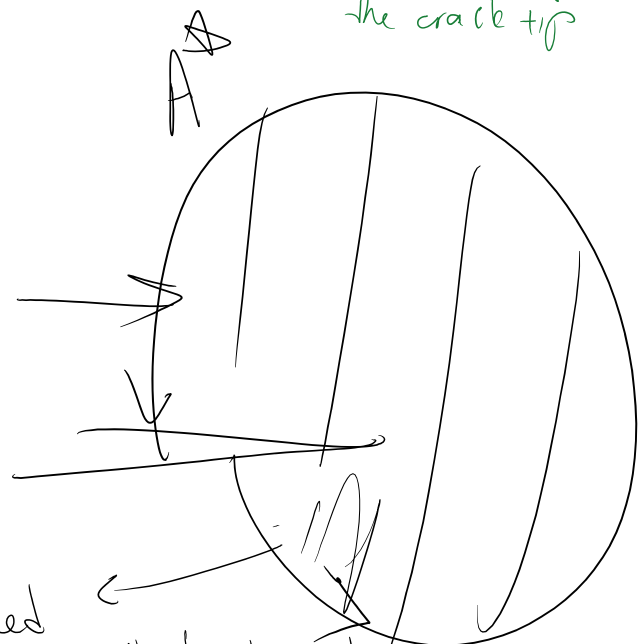


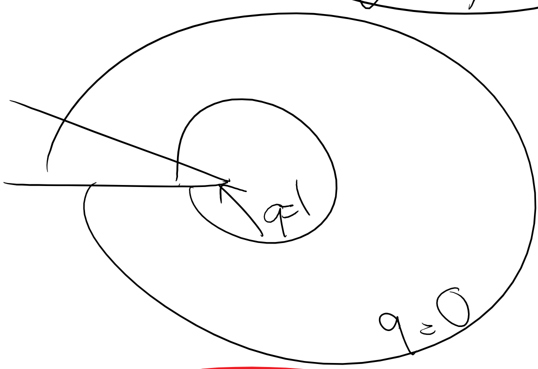
tractions applied behind the crack tip

We needed to tend to the crack tip for the J integral include all new effects (plasticity, thermal strain, body force, and kinetic energy)

$$J = \lim_{\Gamma_o \to 0} \int_{\Gamma_o} \left[(w+T)\delta_{1i} - \sigma_{ij}\frac{\partial u_j}{\partial x_1} \right] n_i \, d\Gamma$$







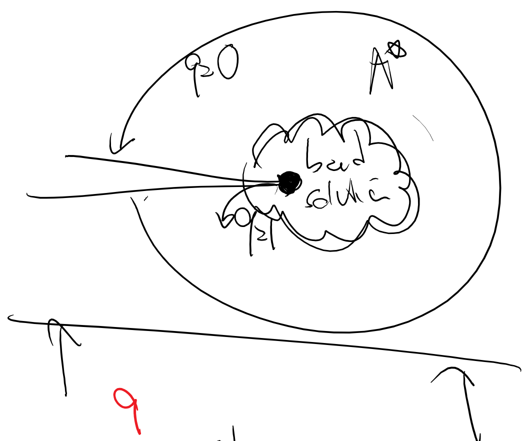
we need to integrate that integrand inside A* and is a very general J integral

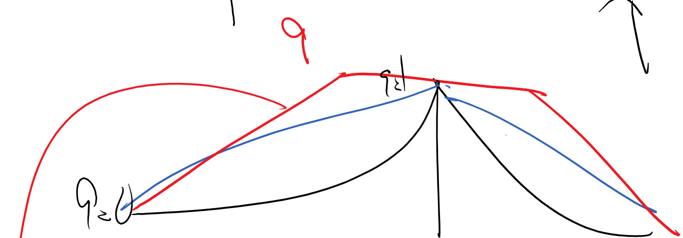
$$J = \int_{A^*} \frac{\partial}{\partial x_j}\left\{\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right]q\right\}dA - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j}\frac{\partial u_j}{\partial x_1} q\, d\Gamma$$

$$\frac{\partial}{\partial x_j}\left(\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right)$$

$$+ \left(\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right)\frac{\partial q}{\partial x_j}$$

zero for elastic response

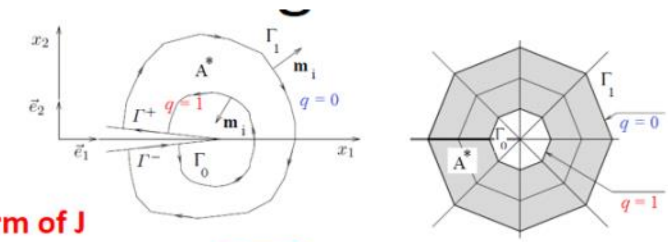




the best choice of q because

effectively gets rid of integral close to the crack tip

General formula



General form of J

$$J = \int_{A^*} \left\{ \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right] \frac{\partial q}{\partial x_i} + \left[\sigma_{ij} \frac{\partial \varepsilon_{ij}^p}{\partial x_1} - \frac{\partial w^p}{\partial x_1} + \alpha \sigma_{ii} \frac{\partial \Theta}{\partial x_1} - F_i \frac{\partial u_j}{\partial x_1} \right] q \right\} dA - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j} \frac{\partial u_j}{\partial x_1} q \, d\Gamma$$

Plasticity effects

Thermal effects

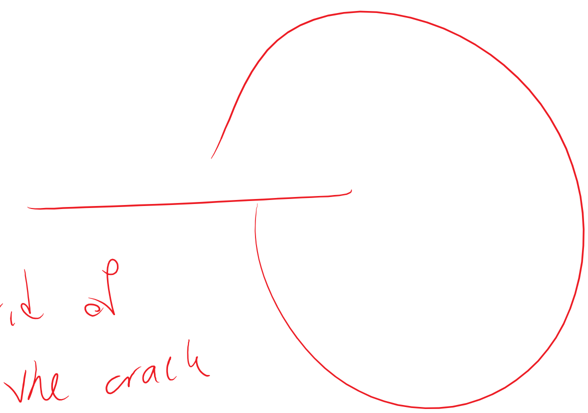
Body force

Nonzero crack surface traction

If all zero 0 0 0 & no stress on crack surface

we have an easier formula

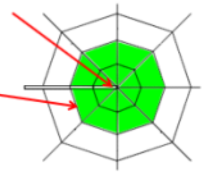
$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right] \frac{\partial q}{\partial x_i} dA$$

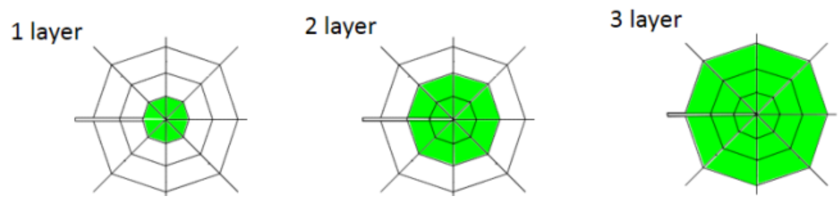


we can get rid of this close to the crack by choosing constant q

J integral: 2. EDI FEM Aspects

- Since $J_0 \rightarrow 0$ the inner J_0 collapses to the crack tip (CT)
- J_1 will be formed by element edges
- By using spider web (rozet) meshes any reasonable number of layers can be used to compute J:

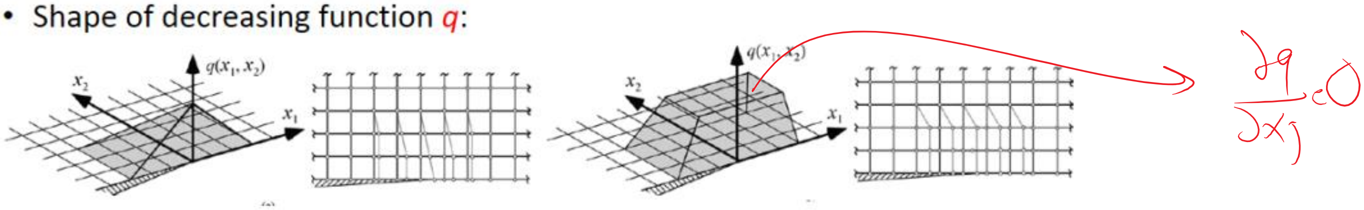




These regions are where we do the general integral (considering all the effects) or simple one (when such effects don't exist)

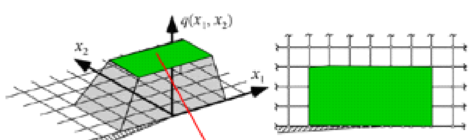
J integral: 2. EDI FEM Aspects

- Shape of decreasing function *q*:



Pyramid *q* function　　　　Plateau *q* function

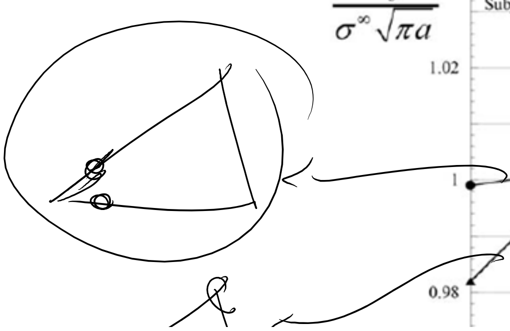
- Plateau q function useful when inner elements are not very accurate: e.g. when singular/quarter point elements are not used

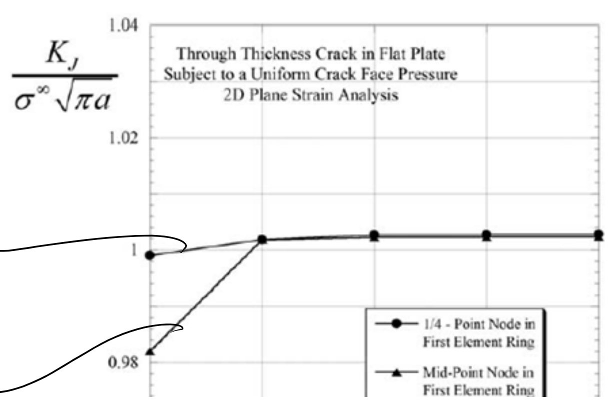


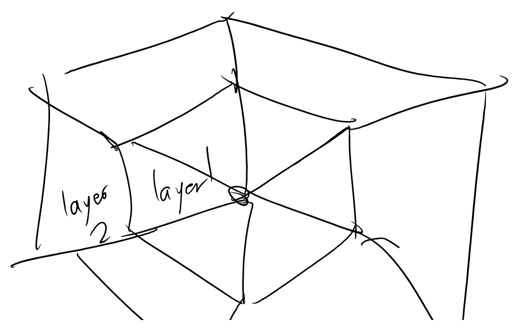
$$\frac{\partial q}{\partial x_i} = 0$$ These elements do not contribute to J

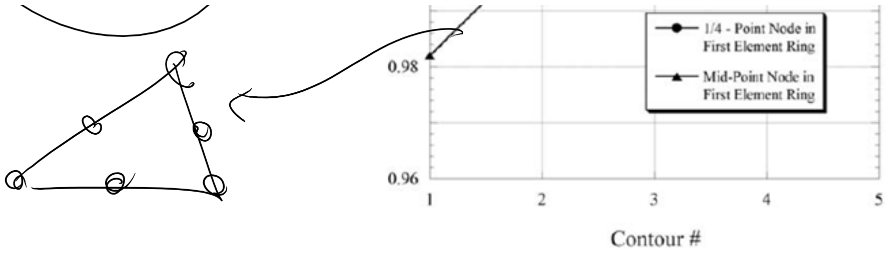
$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right] \frac{\partial q}{\partial x_i} dA$$

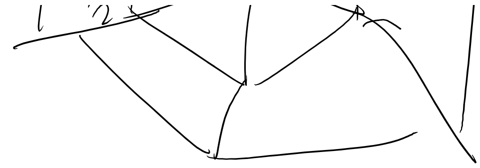
Comparison of different approaches:







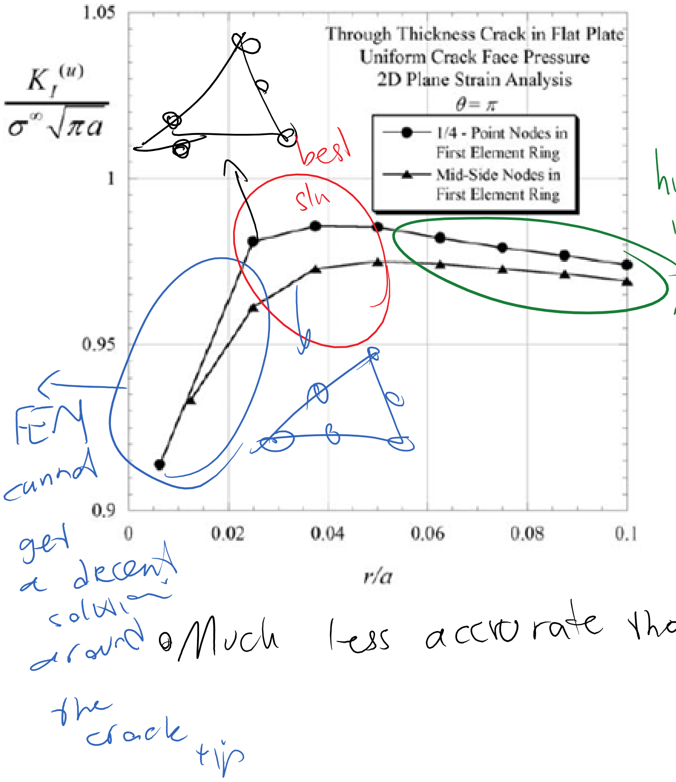




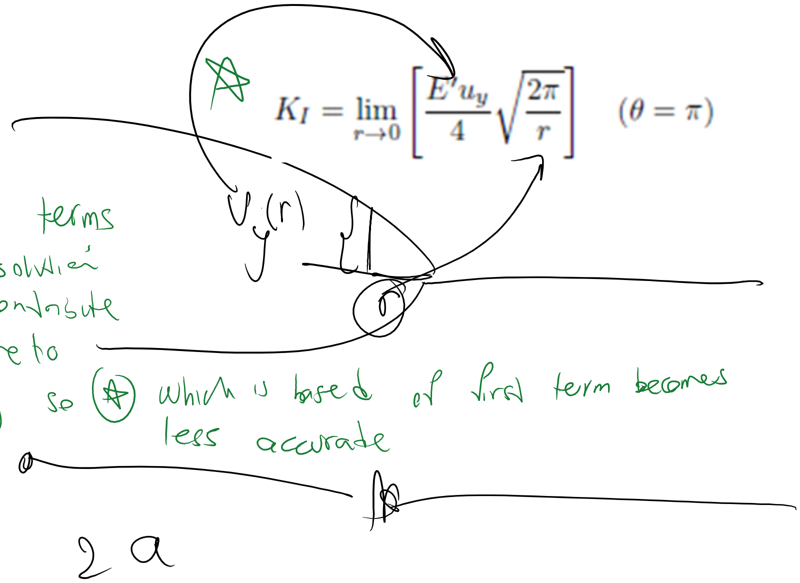
very good results

Very accurate approach

Next example, displacement based K calculation



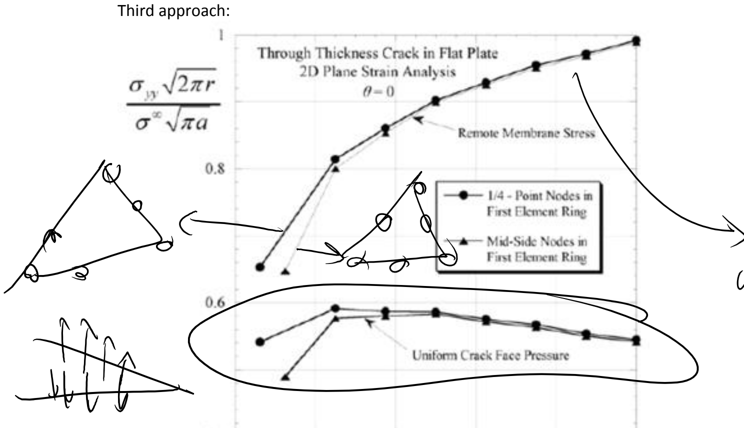
$$K_I = \lim_{r \to 0} \left[\frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$



higher terms in solution contribute more to u_y so (*) which is based on first term becomes less accurate

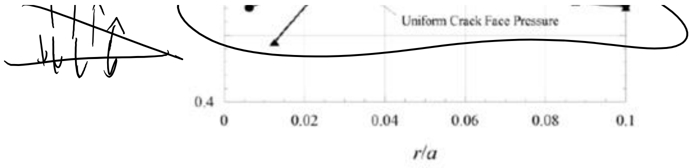
- Much less accurate than EDI
- Not easy to choose r that gives most accurate K

Third approach:



worse than displacement based

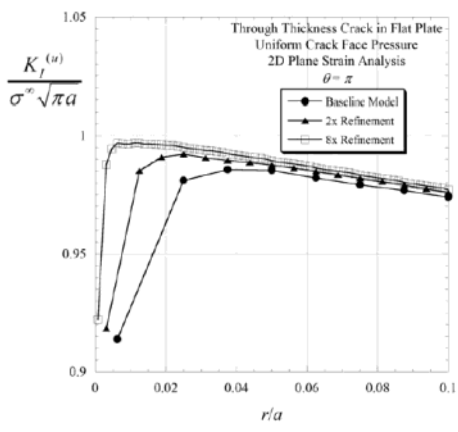
- singularity in σ makes

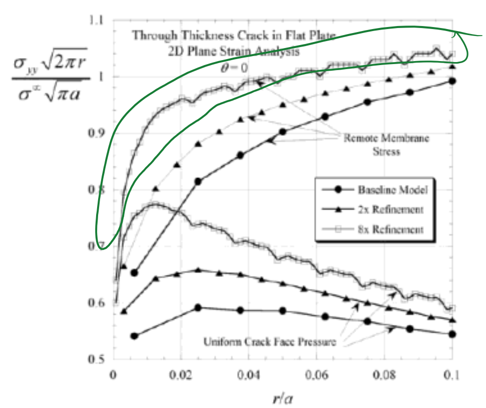


K from stress σ

$$K_I = \lim_{r \to 0} \left(\sqrt{2\pi r}\ \sigma_{22}|_{\theta=0} \right)$$

- singularity in σ makes σ_{22} it difficult to be captured by FEM
- σ is a derivative of u (using standard less accurate FEM sln)
- highly sensitive to crack surface tractions





even 8x finer mesh around CT still gives poor solution

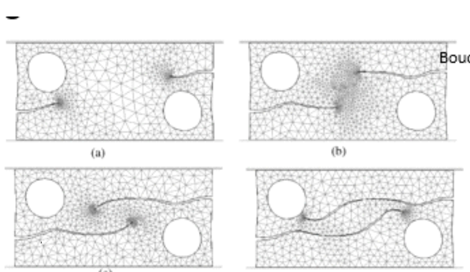
K from displacement *u*

$$K_I = \lim_{r \to 0} \left[\frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$

K from stress σ

$$K_I = \lim_{r \to 0} \left(\sqrt{2\pi r}\ \sigma_{22}|_{\theta=0} \right)$$

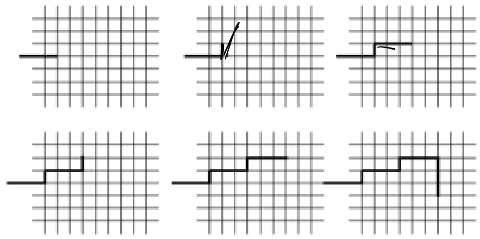
6.1.6. Computational crack growth



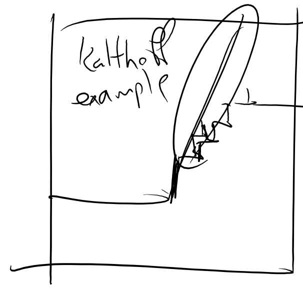
Bouchard et al. CMAME 2003

Mesh adaptive approaches that align element boundaries with crack path

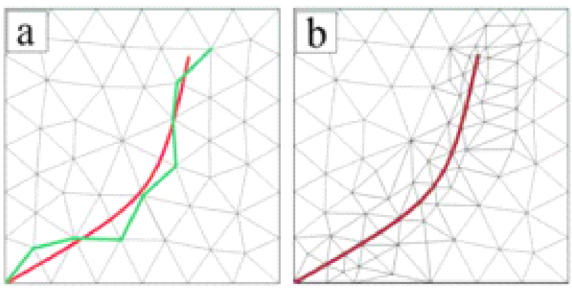
- Nodal release method (typically done on fixed meshes)
  - Crack advances one element edge at a time by releasing FEM nodes
  - Crack path is restricted by discrete geometry



- Jagged path
- making it difficult for the crack to grow



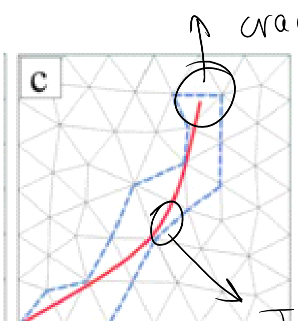
How much do we change total crack length?



Fixed mesh

Crack tracking

crack tip singularity inside the element



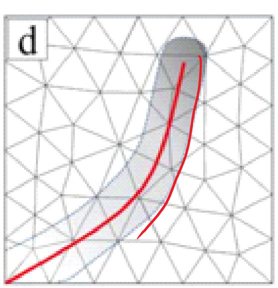
eXtended FEM (XFEM)

generalized FEMs (GFEM)

Jump in displacement

XFEM enriched elements

Last approach:



we don't model cracks explicitly but only model them in an averaged manner

- bulk damage model
- Phase field models

Crack/void capturing by bulk damage models

→ Phase field models

$\sigma = C\,\varepsilon$

↓

$\sigma = (1 - D)\,C\,\varepsilon$

D → damage parameter

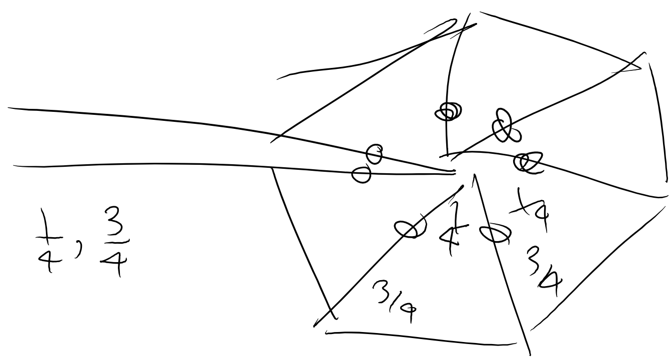
D = 0 undamaged material

D = 1 fully damaged

Brief overview on XFEM methods:

---- How to take care of stress singularity around a crack tip:

Approach 1 $\frac{1}{4}, \frac{3}{4}$



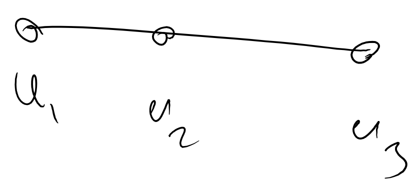
- Direct incorporation of singular terms: e.g. enriched elements by Benzley (1974), shape functions are enriched by K_I, K_{II} singular terms
  - XFEM method falls into this group (discussed later)

More accurate

$$u_i = \sum_{k=1}^{4} f_k \bar{u}_{ik} + K_I \left(Q_{1i} \sum_{k=1}^{4} f_k \bar{Q}_{1ik} \right) + K_{II} \left(Q_{2i} \sum_{k=1}^{4} f_k \bar{Q}_{2ik} \right)$$

$$Q_{ij} = \frac{u_{ij}}{k_i}$$

$$u' = \left(1 + \frac{2x}{L} - 3\sqrt{\frac{x}{L}}\right) u'_1 + \left(4\sqrt{\frac{x}{L}} - 4\frac{x}{L}\right) u'_2 + \left(\frac{2x}{L} - \sqrt{\frac{x}{L}}\right) u'_3$$



what if we make these our shape functions