2018/10/29

Monday, October 29, 2018 11:23 AM

- Direct incorporation of singular terms: e.g. enriched elements by Benzley (1974), shape functions are enriched by K_I, K_{II} singular terms

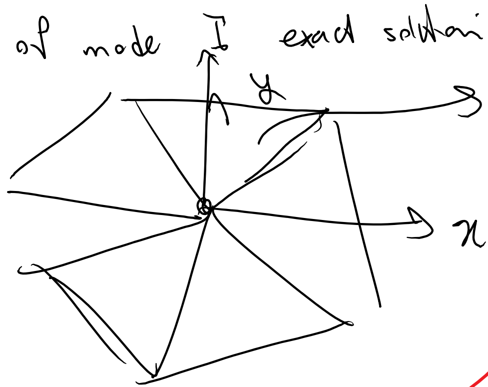
$$u_i = \sum_{k=1}^{4} f_k \bar{u}_{ik} + K_1 \left(Q_{1i} \sum_{k=1}^{4} f_k \bar{Q}_{1ik} \right) + K_{11} \left(Q_{2i} \sum_{k=1}^{4} f_k \bar{Q}_{2ik} \right)$$

$$Q_{ij} = \frac{u_{ij}}{k_i}$$

$$u' = \left(1 + \frac{2x}{L} - 3\sqrt{\frac{x}{L}}\right) u'_1 + \left(4\sqrt{\frac{x}{L}} - 4\frac{x}{L}\right) u'_2 + \left(\frac{2x}{L} - \sqrt{\frac{x}{L}}\right) u'_3$$

$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$
$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

Example of mode I exact solution



normal solution
$u = a + bx + cy$

$+ \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}} \dots$

Problem with this approach is that the crack tip is fixed

Not appropriate for crack propagation problems.

Belytschko et al 1999 set of enriched nodes

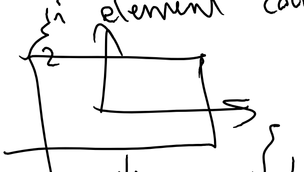
Generalized FEM (GFEM)

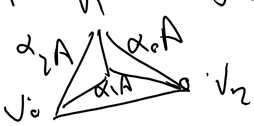
$$\mathbf{u}^h(\mathbf{x}) = \underbrace{\sum_{I \in \mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I}_{\text{standard part}} + \sum_{J \in \mathcal{S}^c} N_J(\mathbf{x})\Phi(\mathbf{x})\mathbf{a}_J$$

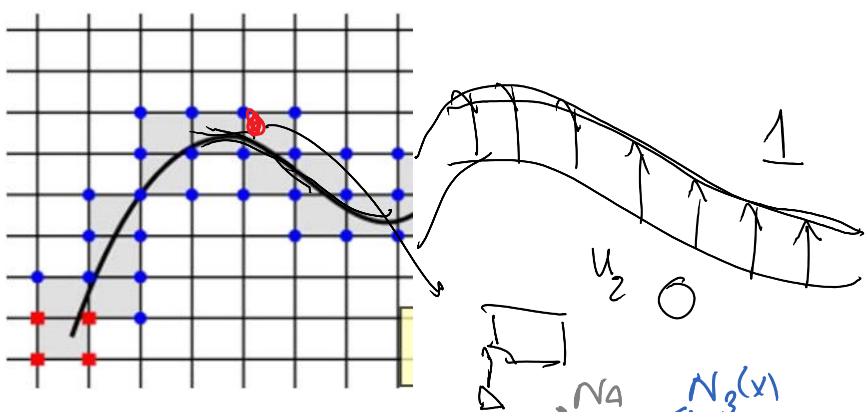
standard part enrichment part

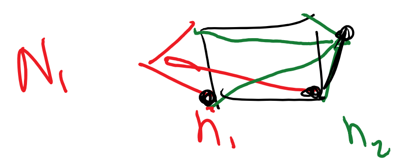
normal FEM solution

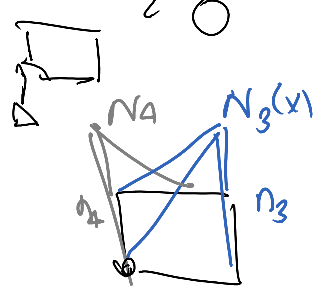
often polynomials in element coordinate





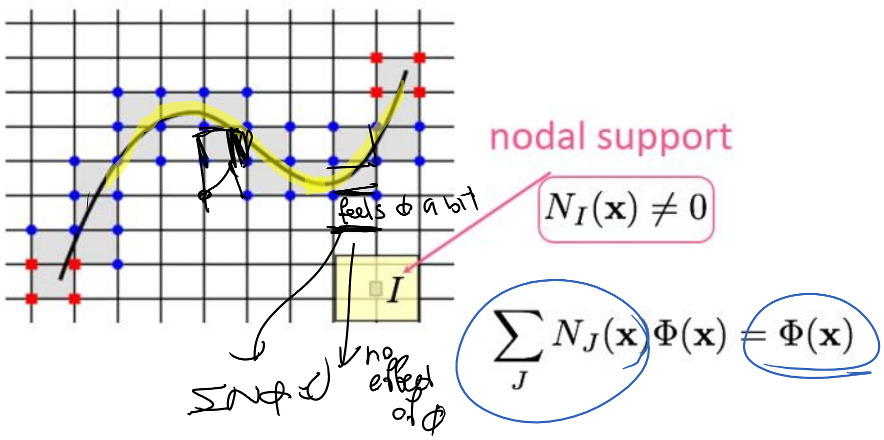


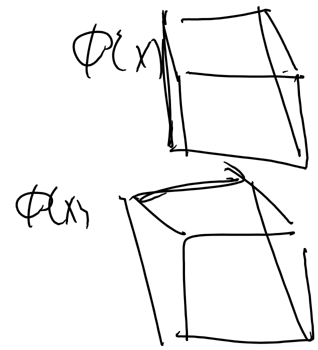


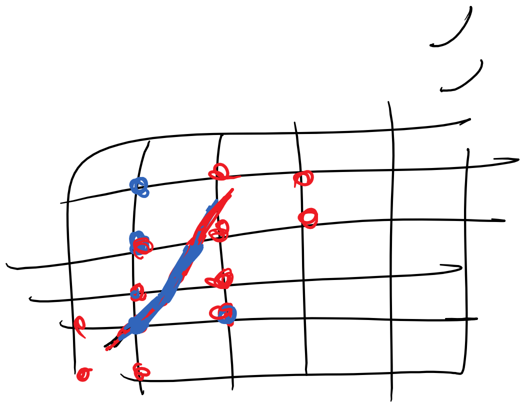


$\phi(x) \quad H(\vec{x} - \bar{x}_0)$

$$\phi(x)\underbrace{\left(N_1(x) + N_2(x) + N_3(x) + N_4(x)\right)}_{1}$$







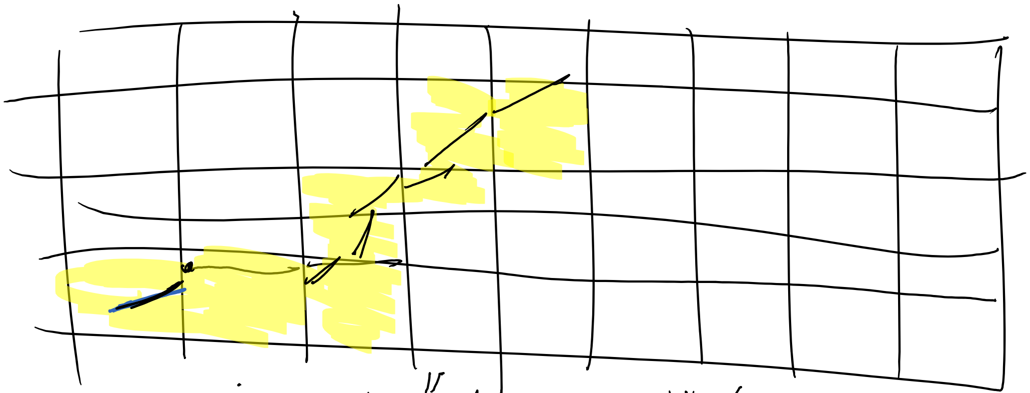
Partition of Unity (PUM) enrichment function

$$\sum_J N_J(\mathbf{x}) = 1 \longrightarrow \sum_J N_J(\mathbf{x})\Phi(\mathbf{x}) = \Phi(\mathbf{x})$$

Another method that allows crack propagation inside elements:

Embedded discontinuities

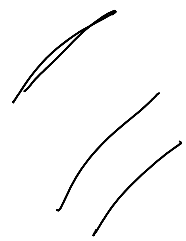
"element centered



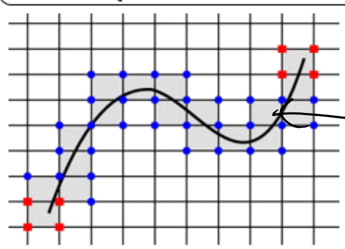
because it's not "node centered" (Partition of unity methods like XFEM), crack path is not continuous

Types of enrichment:

1. Crack surface opening



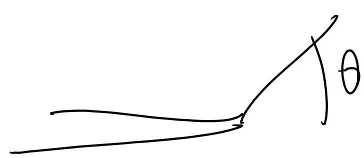
$$H(\mathbf{x}) = \begin{cases} +1 & \text{if } (\mathbf{x} - \mathbf{x}^*) \cdot \mathbf{n} \geq 0 \\ -1 & \text{otherwise} \end{cases}$$



all the blue nodes

2. Crack tip inside an element (where this very different from 1974 work, because now the crack tip can be inside the element)

$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$
$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$



$\sqrt{r}\cos\frac{\theta}{2}$, $\sqrt{r}\sin\theta\sin\frac{\theta}{2}$

$\sqrt{r}\,\overbrace{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}^{\sin\theta}\sin\frac{\theta}{2}$

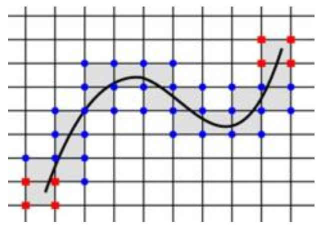
$$\sqrt{r}\,\sin\frac{\theta}{2},\ \sqrt{r}\,\cos\frac{\theta}{2}\,\sin\theta$$

So these four enrichment functions can reproduce the exact crack tip displacement field at **any location**:

$$[B_\alpha] = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

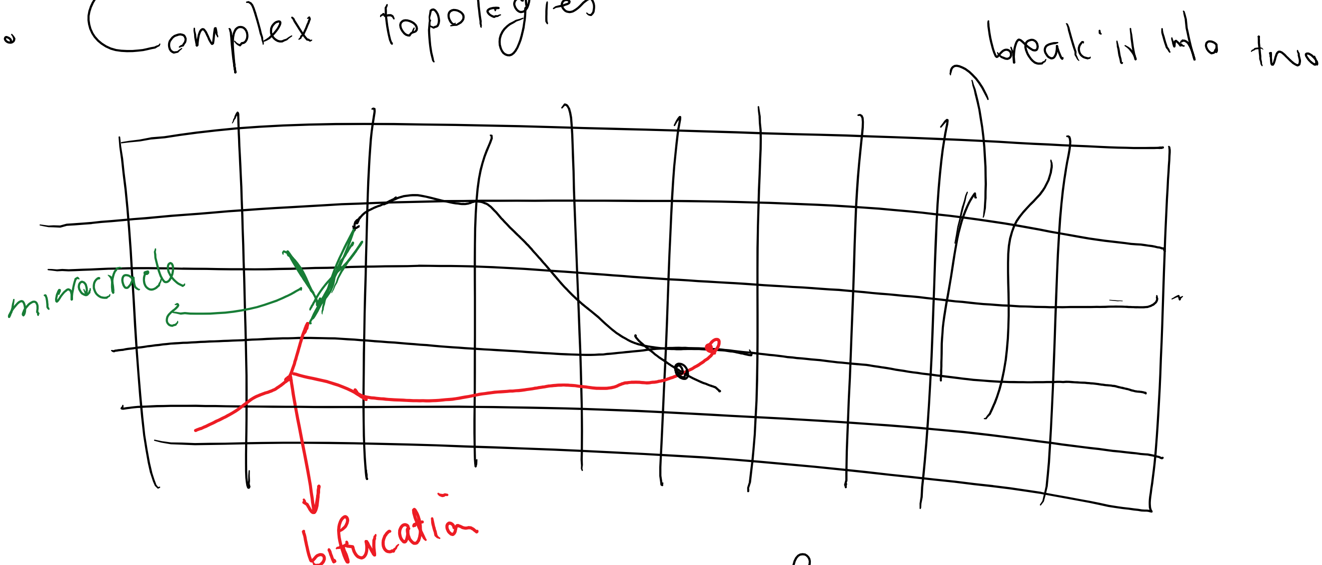
S^c	blue nodes
S^t	red nodes

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I\in S} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J\in S^c} N_J(\mathbf{x})H(\mathbf{x})\mathbf{a}_J + \sum_{K\in S^t} N_K(\mathbf{x})\left(\sum_{\alpha=1}^{4} B_\alpha \mathbf{b}_K^\alpha\right)$$



Potential difficulties with XFEM methods

1. Complex topologies



2. Enrichment is based of LEFM

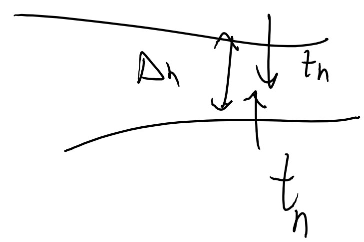
$\frac{1}{\sqrt{r}}$ singularity

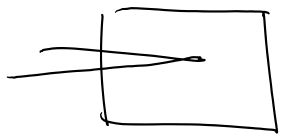
not very accurate when SSY is violated

- PFM strain $\frac{1}{r^x}$ stress $\frac{1}{r^y}$

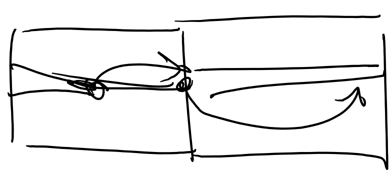
Cohesive models

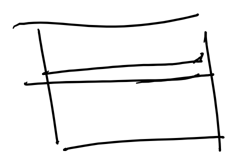
$t_n = f(\Delta_n)$





Don't know how to enrich element solution for more complex fracture models

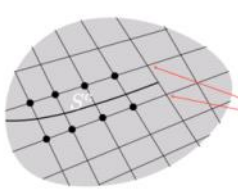




XFEM for cohesive cracks

Wells, Sluys, 2001

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in \mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J \in \mathcal{S}^c} N_J(\mathbf{x}) H(\mathbf{x}) \mathbf{a}_J$$



No crack tip solution is known, no tip enrichment!!!

not enriched to ensure zero crack tip opening!!!

$$H(\mathbf{x}) = \begin{cases} +1 & \text{if } (\mathbf{x} - \mathbf{x}^*) \cdot \mathbf{n} \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

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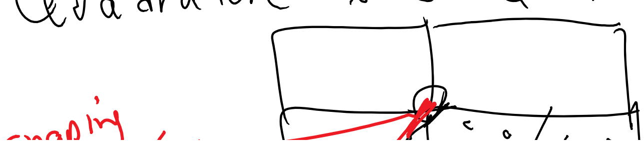
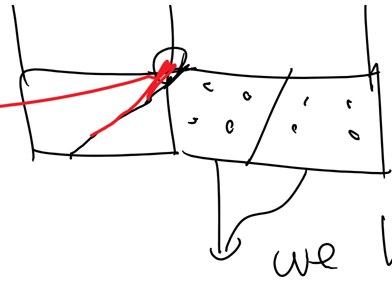
(3) Quadrature issues & other numerical issues

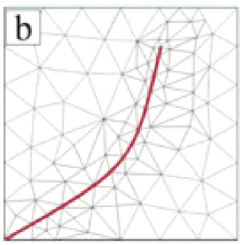




snapping on node

we have to be careful with numerical integration of enriched elements

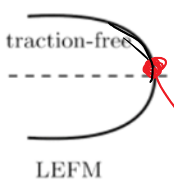
In contrast, mesh adaptive methods can address all these issues but they are very difficult to implement, particularly in 3D



Crack tracking

6.2. Traction Separation Relations (TSRs)

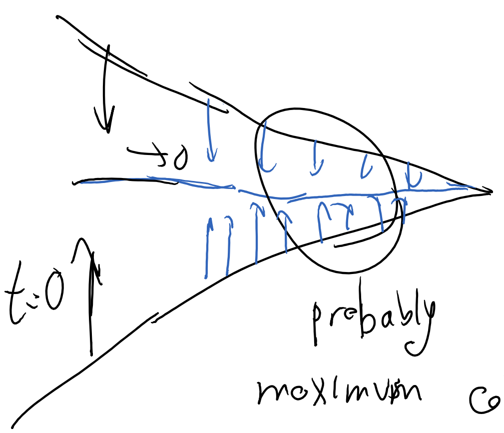
Cohesive models



$v_y \propto \sqrt{r}$

process zone size ~0

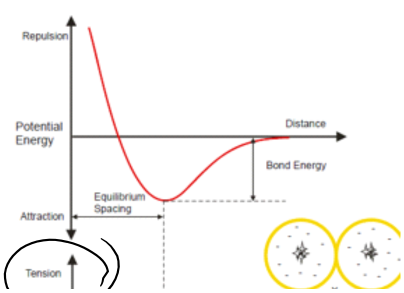
$$s \propto \frac{1}{\sqrt{r}}$$

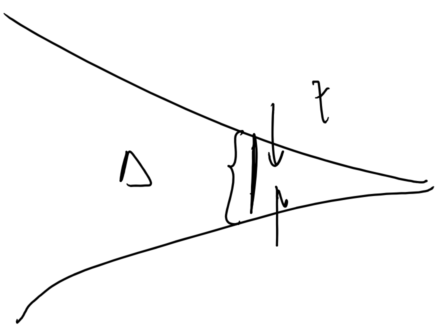


realistic crack opening

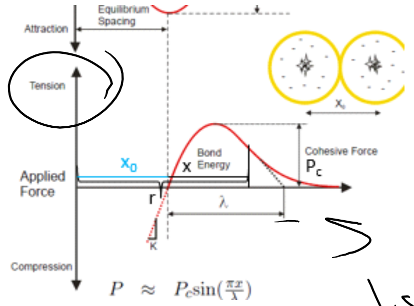
probably maximum cohesive tractions

Motivation:
From atomistic potential





$t = f(\vec{\Delta})$

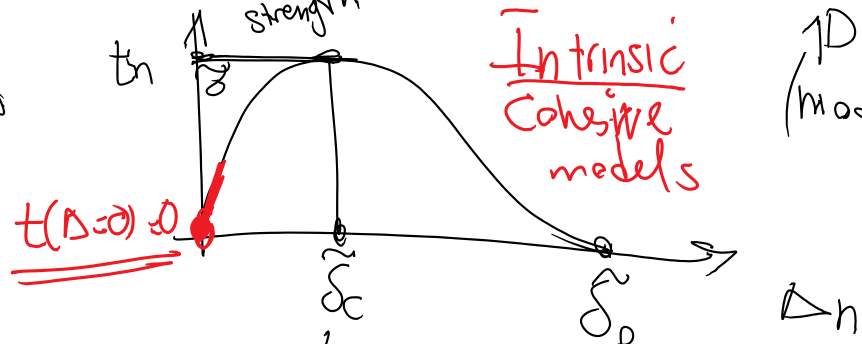


displacement jump

Two forms of strength of TSRs

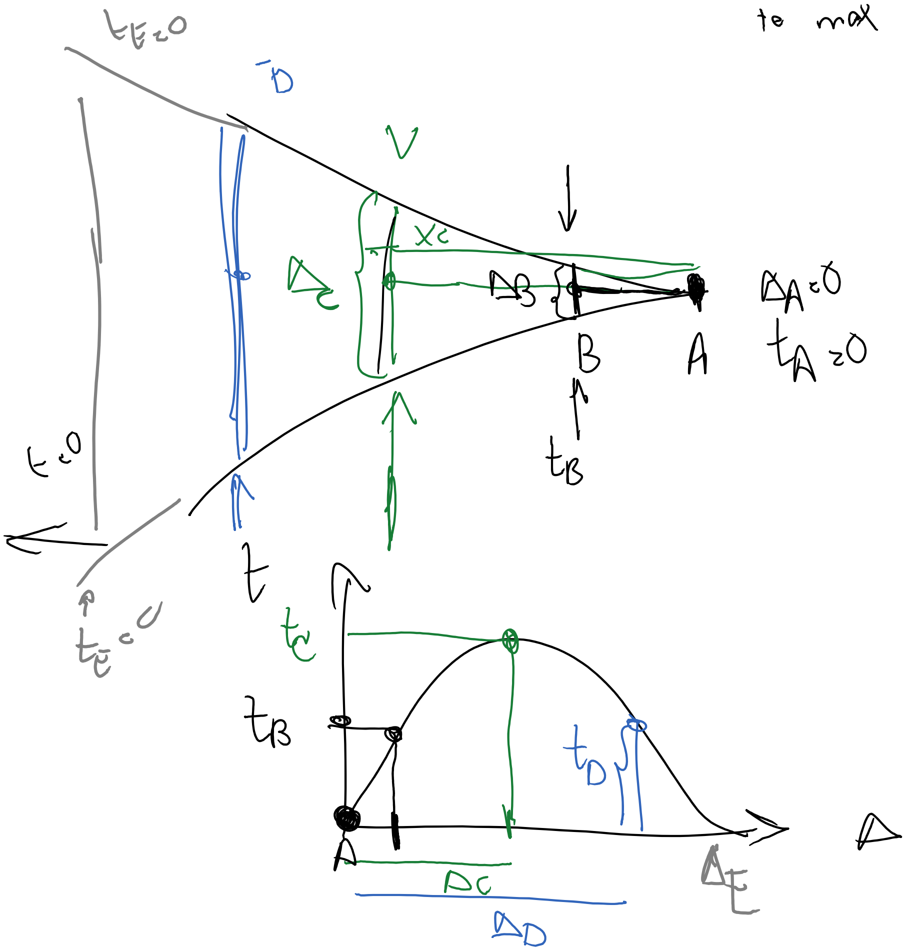
Intrinsic Cohesive models

1D (mode I)



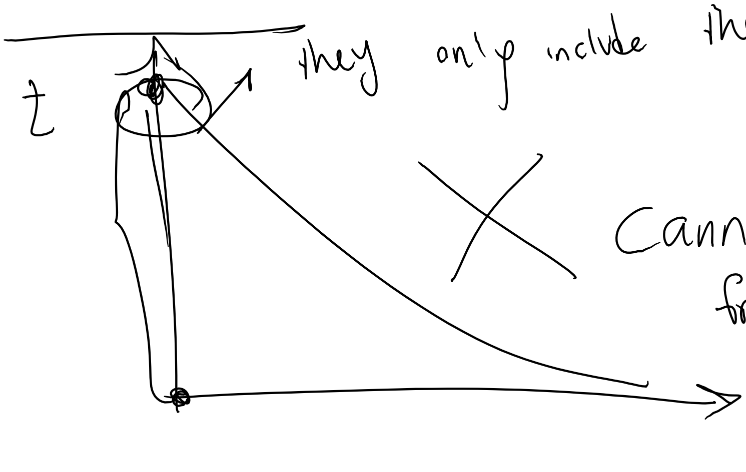
separation corresponding to max traction

separation for zero traction



2nd type of TSRs:

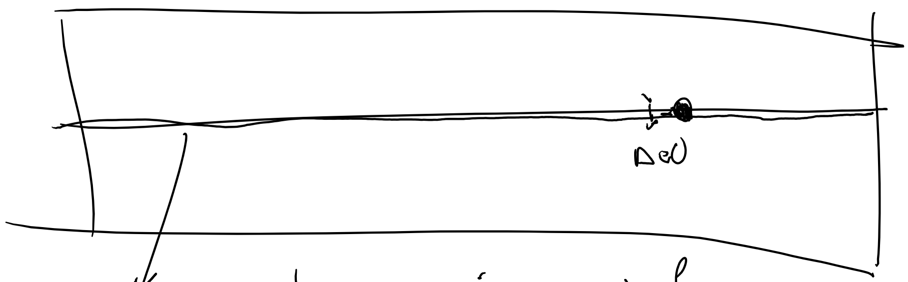
Extrinsic cohesive model



they only include the softening part of t, Δ

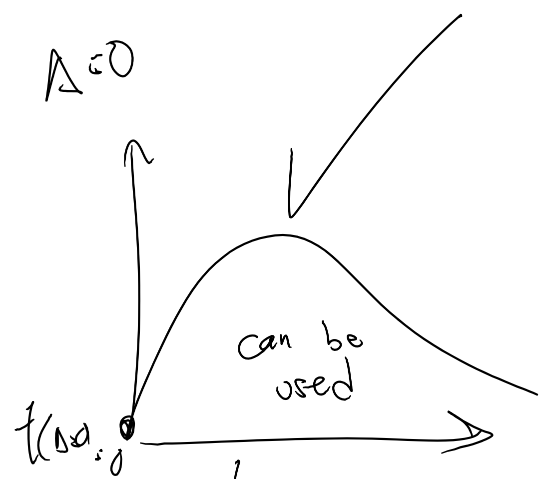
Cannot be used from the begining of the analysis

on pre-inserted crack surfaces



preinserted cohesive interfaces

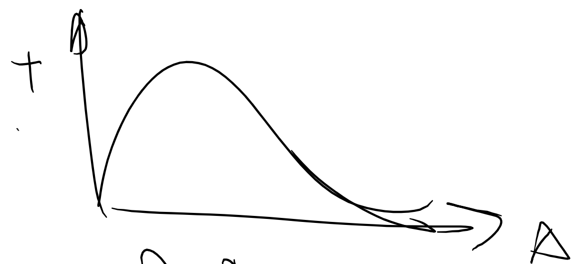
Δ=0

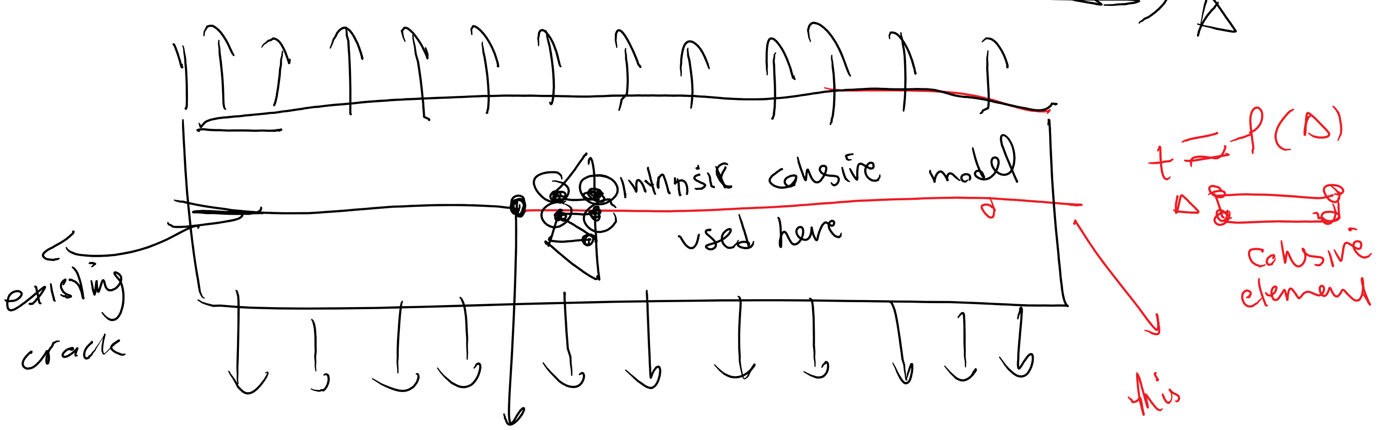


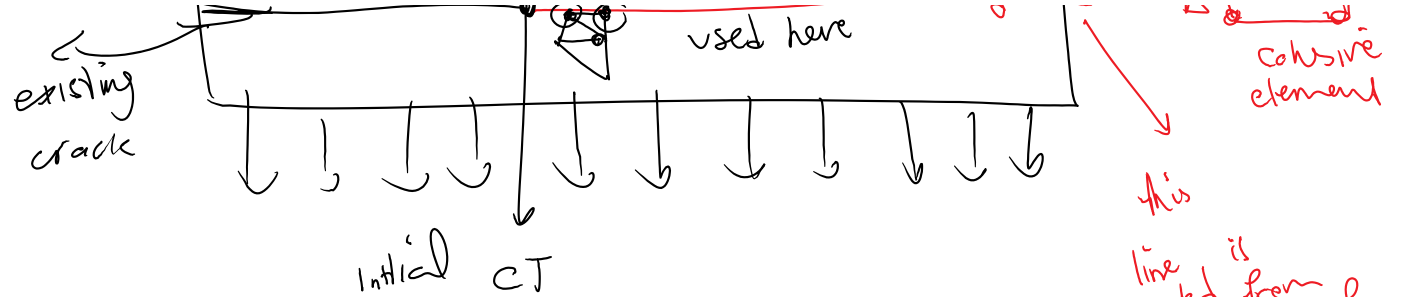
this one can be used

How each model is used?

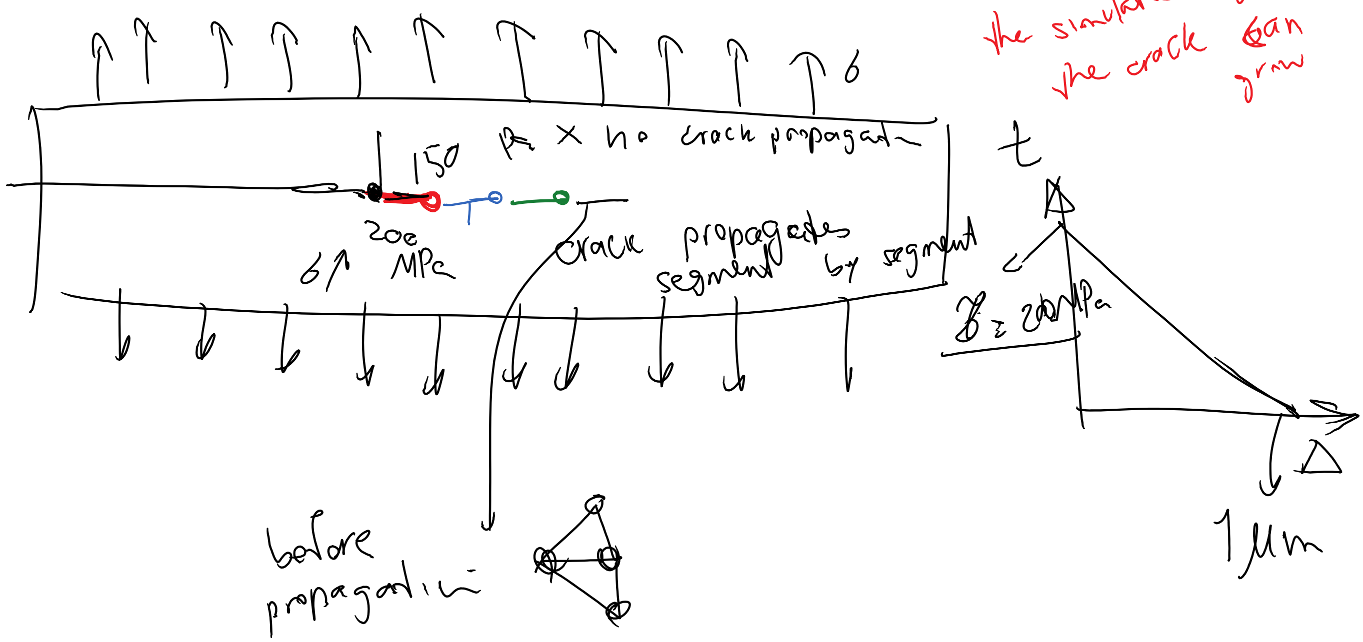
— Intrinsic

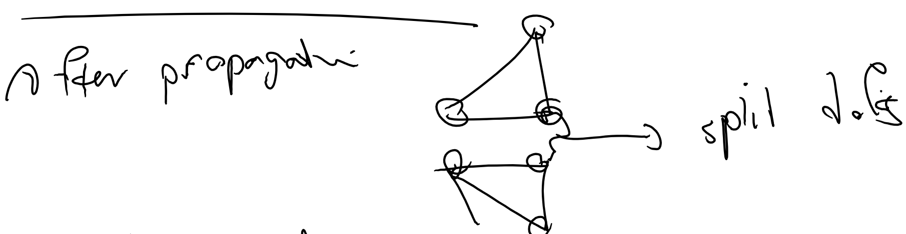






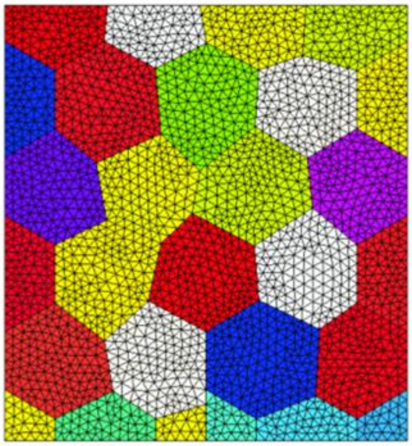
how extrinsic models are used?

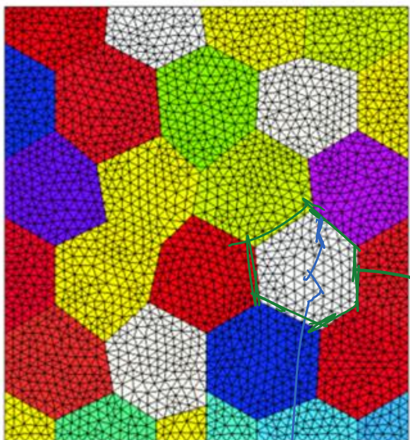




Extrinsic cohesive models require continuous modification to FE nodes & element connectivity as the crack propagates.

How do we simulate problems like this?





Inter granular cohesive surfaces

Intragranular cohesive surfaces