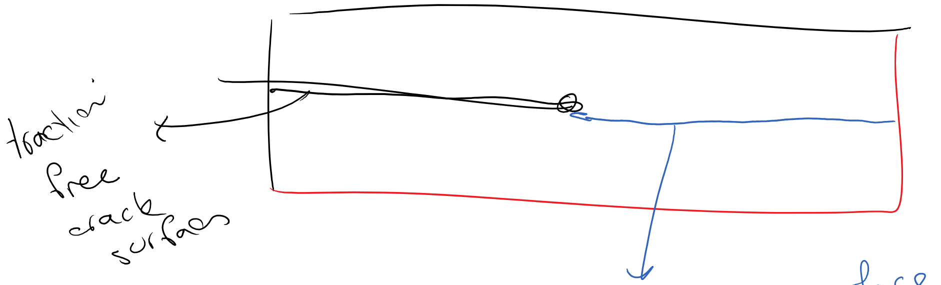
2018/10/31

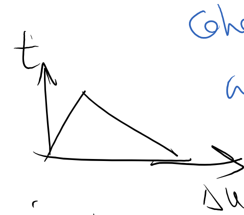
Wednesday, October 31, 2018 11:41 AM

Uses of cohesive model:

1. On pre-specified crack paths

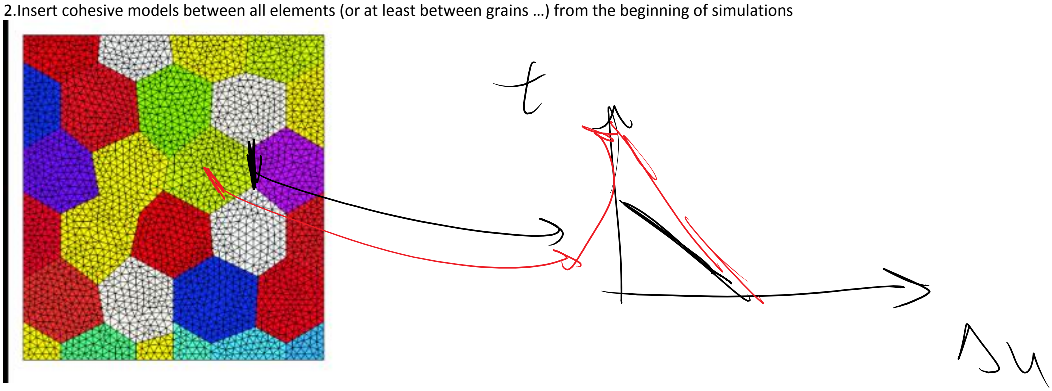


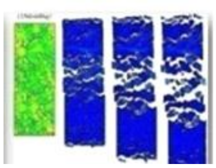
- Use intrinsic cohesive models



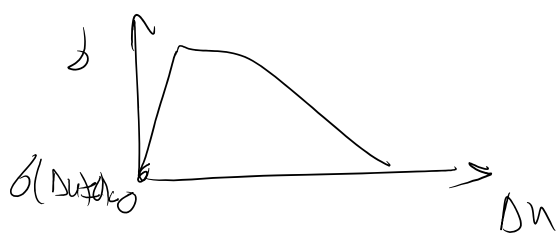
- OR insert extrinsic ones whenever the crack should grow

2. Insert cohesive models between all elements (or at least between grains …) from the beginning of simulations

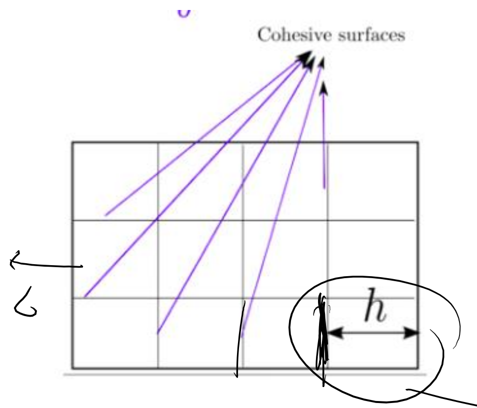


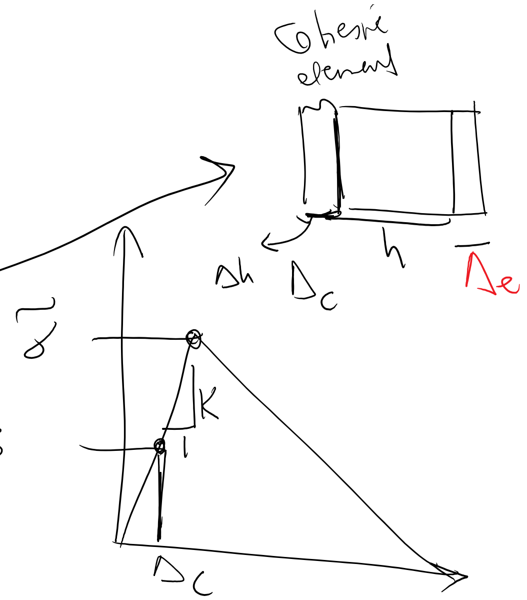


- these cohesive models should be intrinsic



- Problem: artificial compliance





$$\Delta_e = \varepsilon h$$
$$\Delta_c = \frac{\sigma}{K}$$

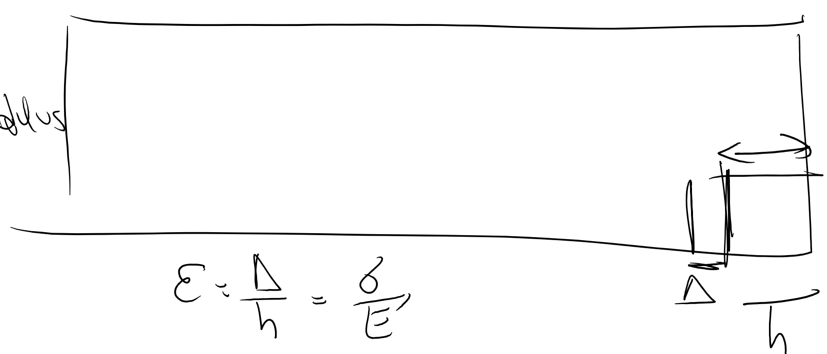
$$\frac{\sigma}{\Delta_c} = K \qquad \Delta_c = \frac{\sigma}{K}$$

$$\Delta = \Delta_e + \Delta_c =$$

$$\varepsilon h + \frac{\sigma}{K} = \frac{\overbrace{(\sigma/E)}^{\varepsilon}}{h} + \frac{\sigma}{K}$$

$$\Delta = \sigma\left(\frac{h}{E} + \frac{1}{K}\right) \quad (1)$$

E' effective elastic modulus



$$\varepsilon = \frac{\Delta}{h} = \frac{\sigma}{E'}$$

$$\rightarrow \Delta = \frac{\sigma}{E'} h \quad (2)$$

eqn 1

$$\sigma\left(\frac{h}{E} + \frac{1}{K}\right) = \frac{\sigma}{E'} h$$

$$\frac{1}{E'} = \frac{1}{E} + \frac{1}{Kh}$$

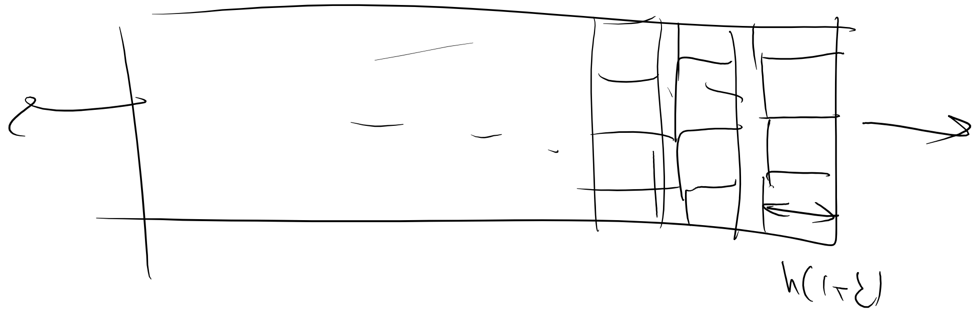
$$\underset{\sim}{C'} = C + \frac{1}{Kh}$$

compliance effective; $C \to \frac{1}{E}$

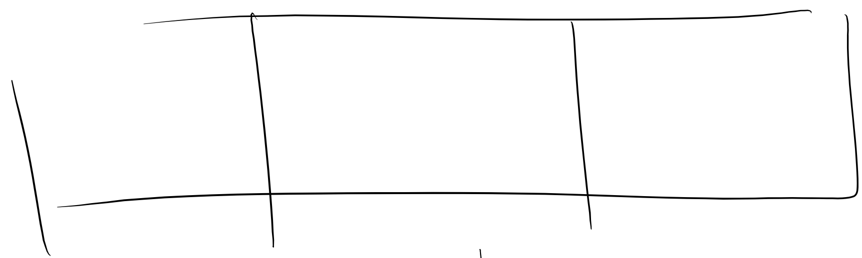
→ we make the material softer by all these small openings

how to address art. comp.

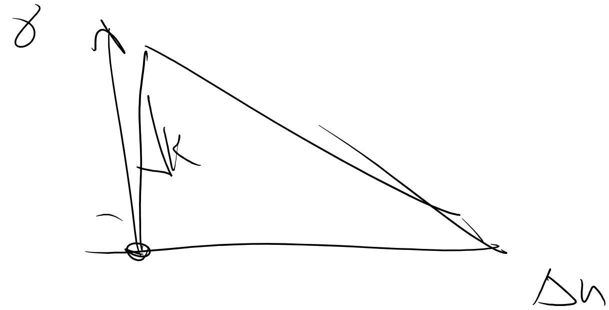
$Kh \to \infty$



$h \to \infty$



may not be very good because it introduces artificial compliance

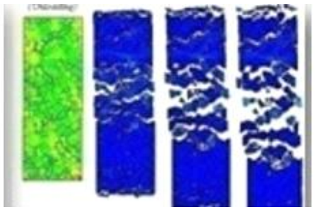


$K \to \infty$ $\qquad \frac{1}{Kh}$

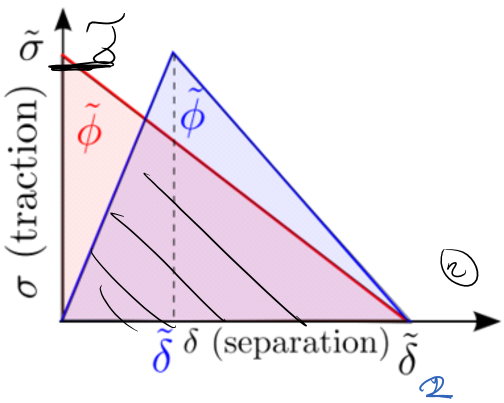
↓

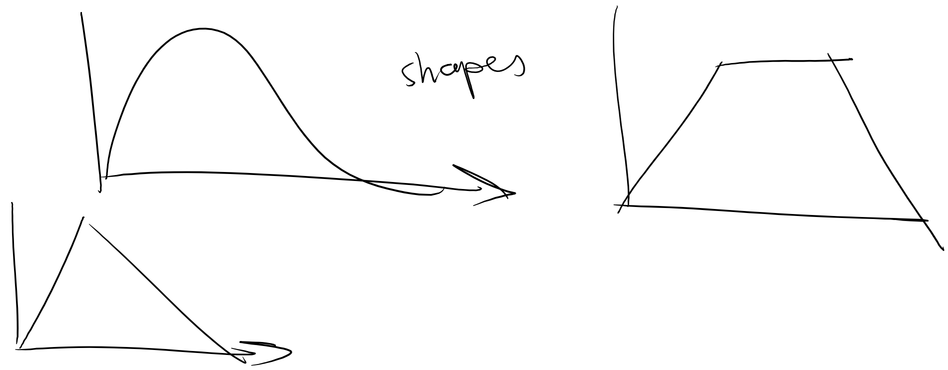
results in very stiff problems

If we want to use intrinsic cohesive surfaces we need to balance this (not for close cohesive surfaces)



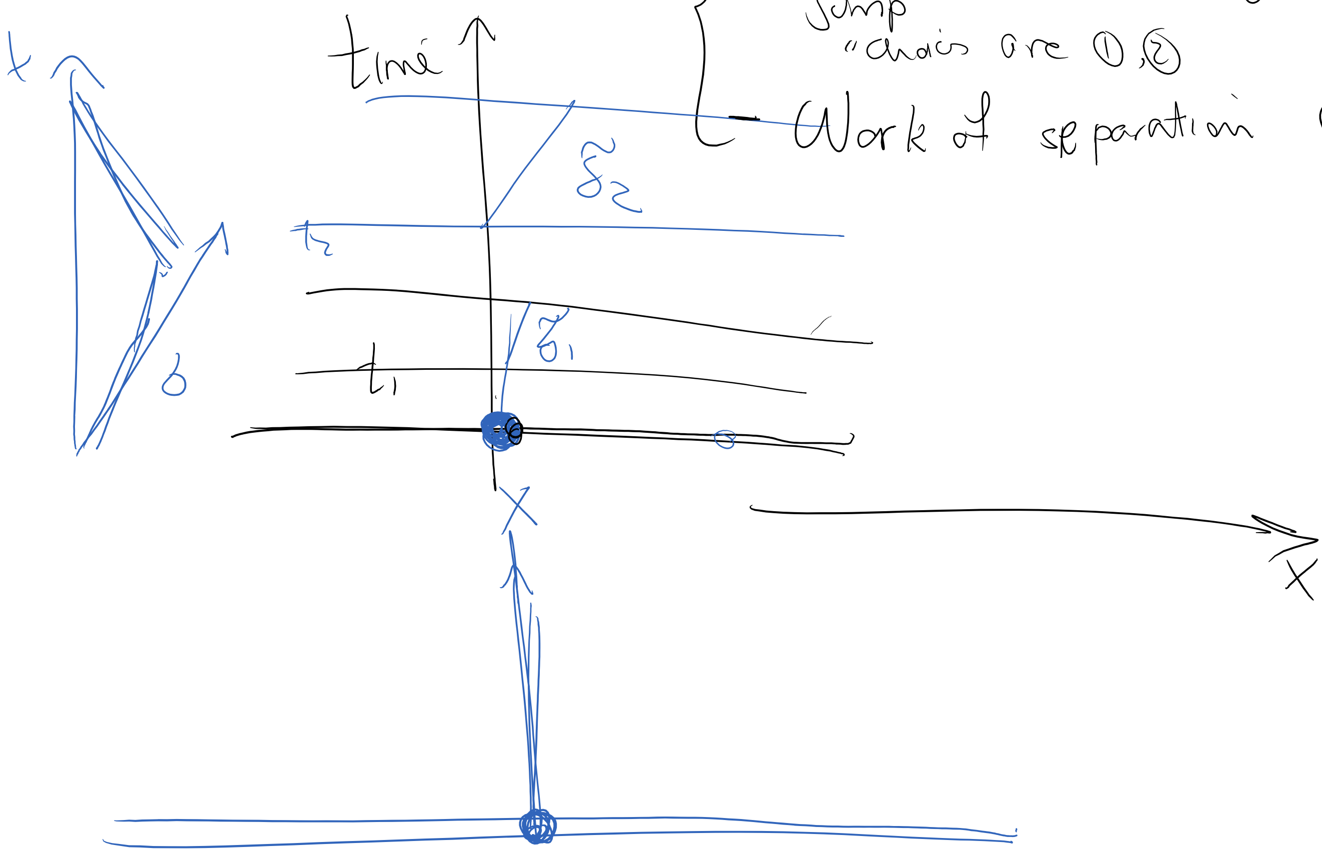
There are some models (some interfacial damage models) that can be inserted in the domain with any density, yet not introduce any artificial compliance





Important TSR scales

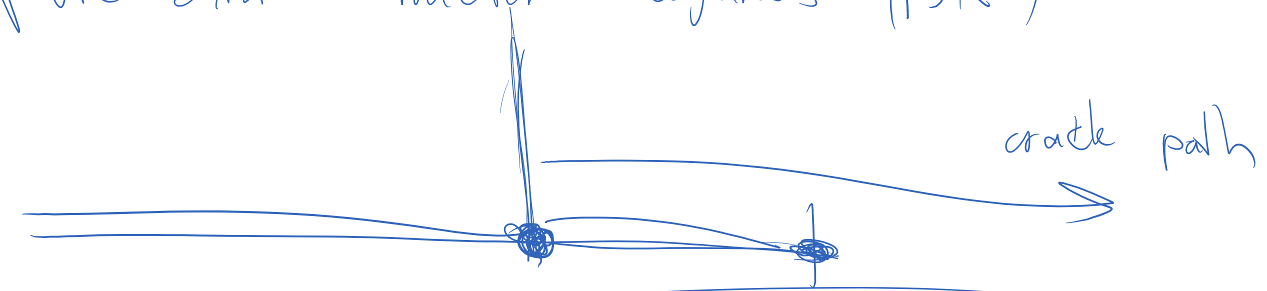
- strength $\tilde{\sigma}$
- characteristic displacement jump $\tilde{\delta}$
  "choices are ①,②"
- Work of separation $\tilde{\phi}$



Fixed point on crack path

$\tilde{\Phi}$ energy released once this point is fully debonded

compare with fracture toughness ($\Gamma = R$)



energy released per unit area of crack advance

For steady state crack propagation, energy dissipation per unit length advance of the crack = energy dissipated for one point from full bonding to full debonding

under this condition

$$\Gamma = \tilde{\Phi}$$

Γ → LEFM toughness

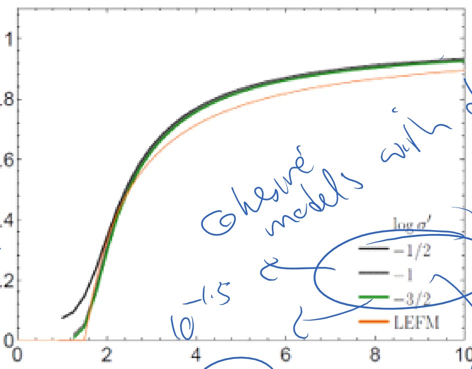
make sense

Example:

element

$(\tilde{\Phi} = \Gamma)$

$(\tilde{\phi} = 1)$



crack speed $\dot{a}'$

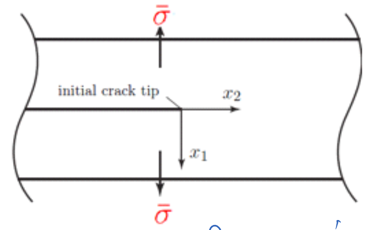
time t'

Cohesive models with different strength

$10^{-1.5}$

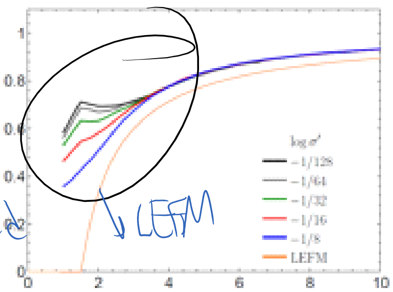
$\frac{\bar{\sigma}}{\tilde{\sigma}} \ll 1$

Low $\sigma' = \frac{\bar{\sigma}}{\tilde{\sigma}}$
SSY
LEFM ≈ CFM



$\sigma' = \frac{\bar{\sigma}}{\tilde{\sigma}}$ = ratio of loading to fracture strength

Cohesive (TSR)



crack speed

LEFM

time

High $\sigma' = \frac{\bar{\sigma}}{\tilde{\sigma}}$
LSY
LEFM ≠ CFM

$\bar{\sigma}$ → loading

$\tilde{\sigma}$ → strength

very high

violating SSY

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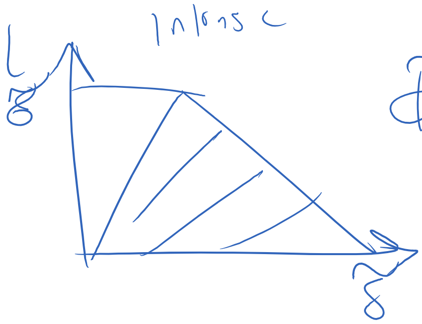
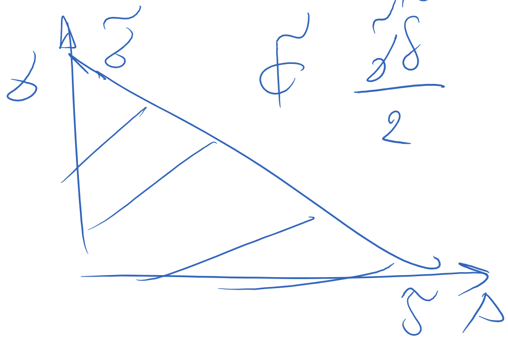
back to the discussion of scales

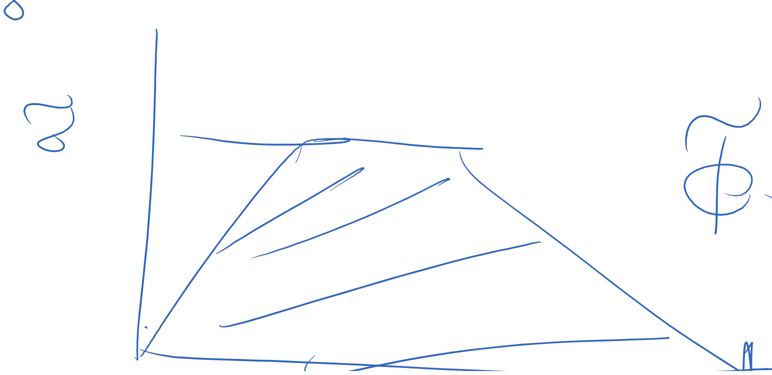
Intrinsic



$\tilde{\phi} = \frac{\tilde{\sigma}\,\tilde{\delta}}{2}$

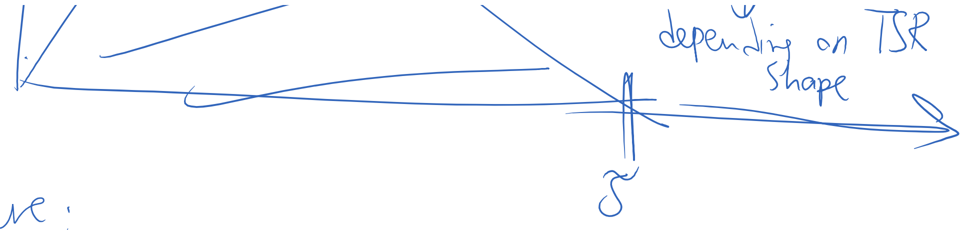


$\tilde{\phi} = \frac{\tilde{\sigma}\tilde{\delta}}{2}$



$\tilde{\phi} = m\,\tilde{\sigma}\tilde{\delta}$

m depending on TSR shape



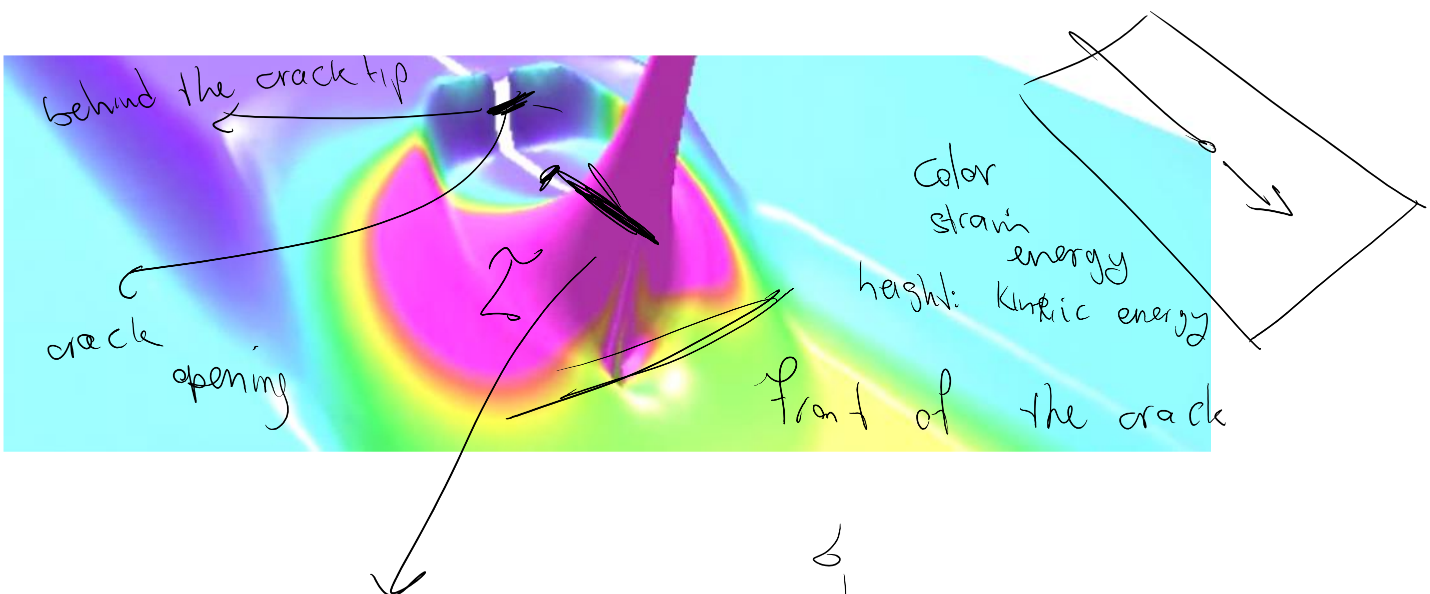
We often measure:

- $\tilde{\delta}$

& often

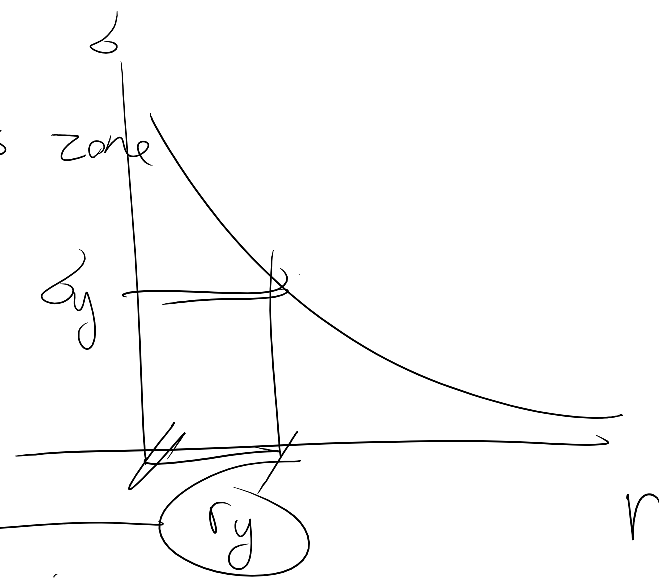
- $\tilde{\Phi}$ (related to fracture toughness)

$\Phi, \delta \longrightarrow \tilde{\delta}$ depending on the shape of TSR



fracture process zone

PFM

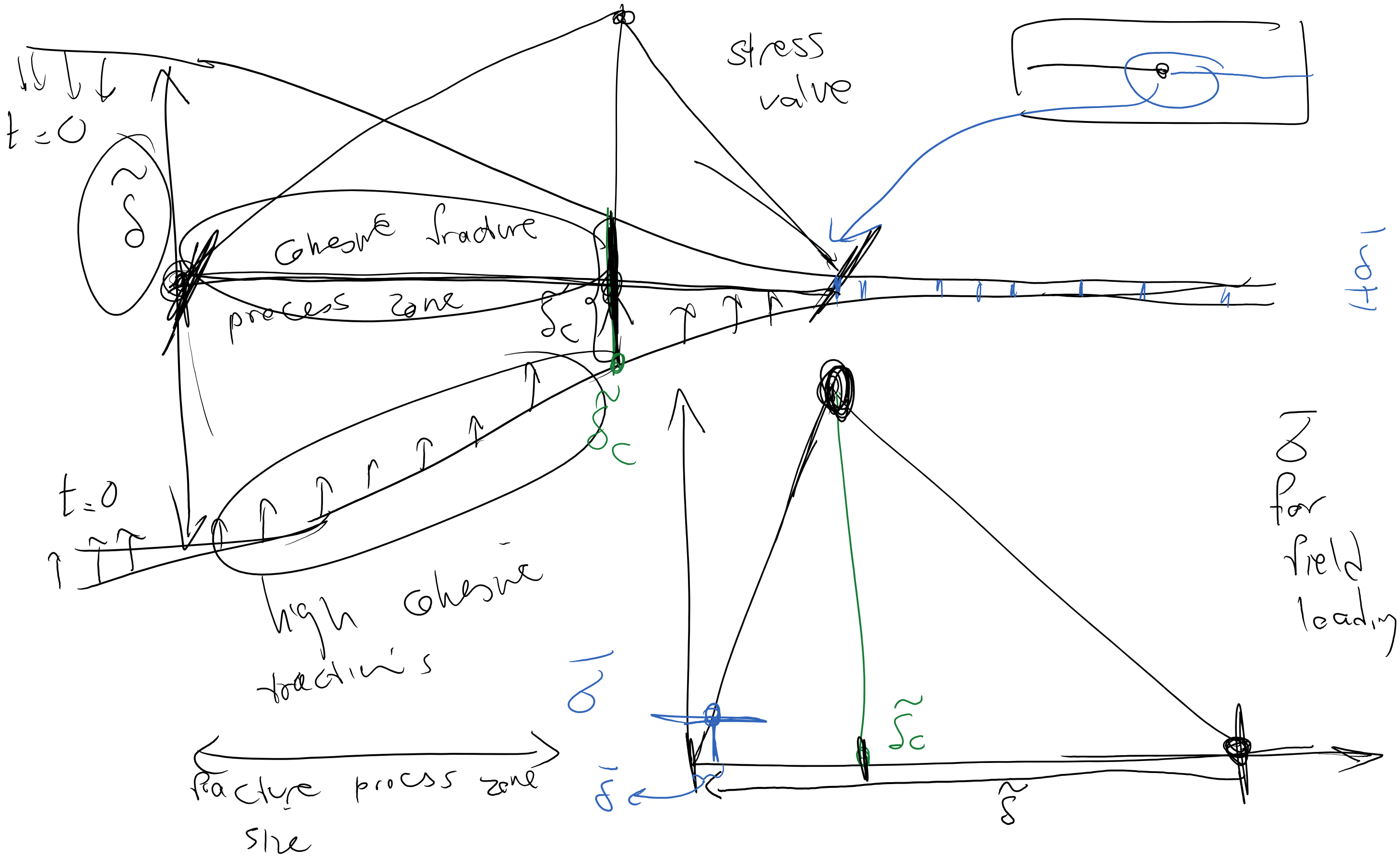


plastic process zone ← r_y

" a length scale where material

nonlinearity yields"

How to compute cohesive fracture process zone size?

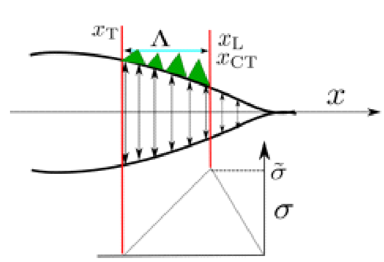


Typically fracture process zone size is much larger than $\tilde{\delta}$

- Importance of process zone size Λ
  - Static estimate:

$$\Lambda = \varsigma\pi \frac{\mu}{1-\nu}\frac{\bar{\phi}}{\bar{\sigma}^2} \propto \bar{L}$$

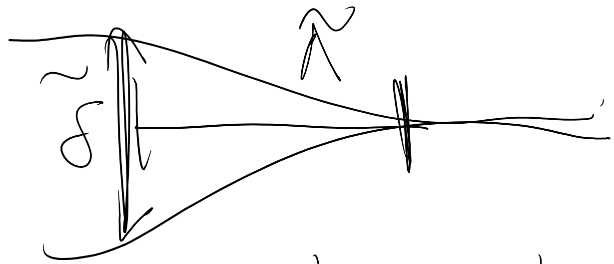
$$\varsigma = \begin{cases} \frac{1}{4} & \text{Dugdale model} \\ \frac{9}{16} & \text{Potential-based TSRs} \end{cases}$$



$$\tilde{\Lambda} \propto \frac{E\tilde{\phi}}{\tilde{\sigma}^2} = \frac{E}{\tilde{\sigma}}\left(\frac{\tilde{\phi}}{\tilde{\sigma}}\right) \propto \frac{E}{\tilde{\sigma}}\tilde{\delta}$$

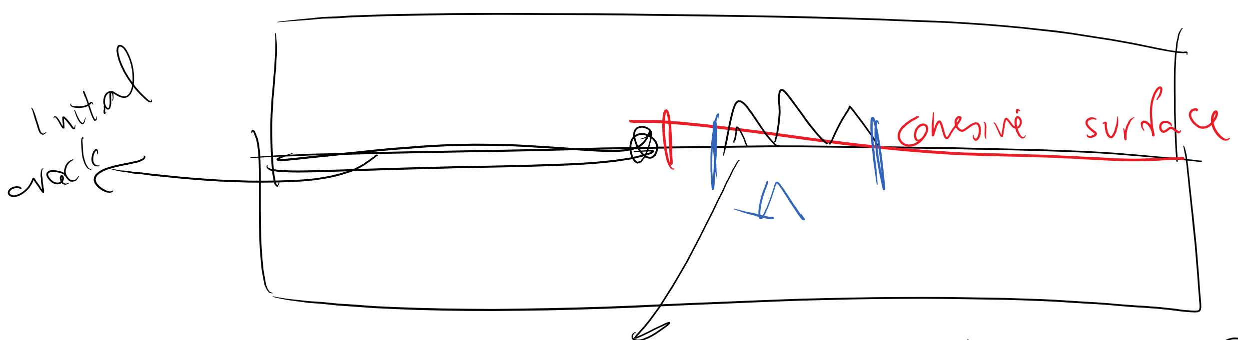
$$\frac{1}{\tilde{\delta}^2} \simeq \frac{\tilde{E}}{\tilde{\delta}}\left(\frac{\tilde{\varphi}}{\tilde{\delta}}\right) \propto \frac{\tilde{E}}{\tilde{\delta}}\,\delta$$

$$\boxed{\frac{\tilde{\delta}}{\tilde{\Lambda}} \propto \frac{\tilde{\sigma}}{E}}$$



small number $\frac{1}{100}$ — $\frac{1}{1000}$

Why we care about this Λ



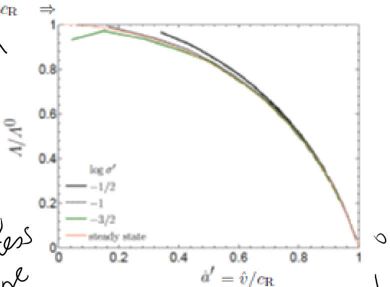
have at least 3 to 6 elements discretizing cohesive process zone

- is process zone size fixed?

– Dynamic estimate: PZS decreases as crack speed $\hat{v}$ approaches Rayleigh wave speed c_R

$\Lambda(\hat{v}) = \frac{\Lambda}{A(\hat{v})}, \quad A(\hat{v}) \to 0$ as $\hat{v} \to c_R \quad \Rightarrow$

Smaller elements are needed in PZT as crack accelerates!



process zone size

static process zone size

321

crack speed

Rayleigh wave speed

Rayleigh w. speed

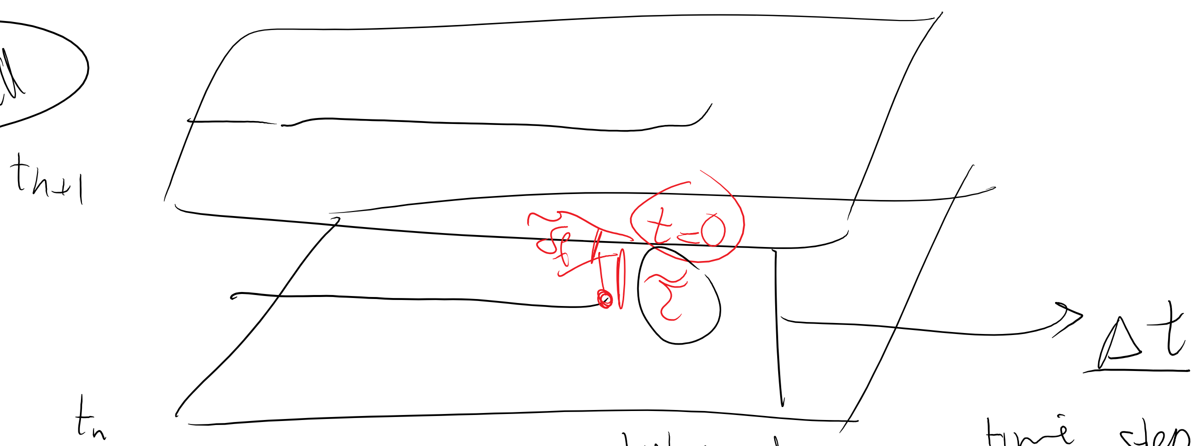
$$\tilde{L} = \frac{E\tilde{\delta}}{\tilde{\sigma}}$$

is a length scale for static cohesive process zone size

→ Make sure h (element size) ahead of the crack is at least $\frac{1}{5}$ to $\frac{1}{6}$ of this

$\frac{\Delta t}{\tilde{\tau}}$ small #

$\frac{1}{5}$ ~ 1



$$\tilde{\tau} = \frac{\tilde{\delta}}{\tilde{V}} = \frac{\tilde{\delta}}{\tilde{\sigma}/C_d\rho} = \frac{\tilde{\delta}\, C_d \rho}{\tilde{\sigma}}$$

$\tilde{\tau}$: cohesive time scale

$\tilde{V}$: cohesive velocity scale

$\tilde{\delta}$: displacement scale

C_d: wave speed

ρ: mass density

$\tilde{\sigma}$: strength

↳ The time that takes for a point to go from fully bonded to fully debonded $\propto \tilde{\tau}$