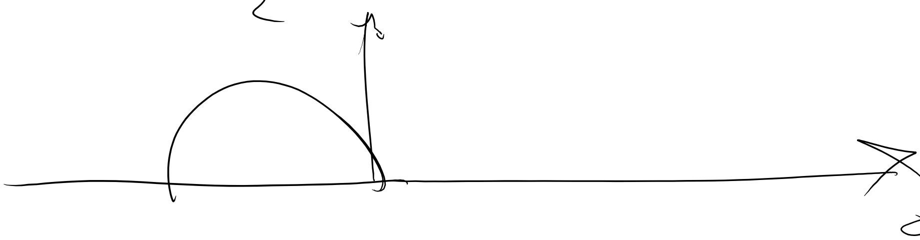
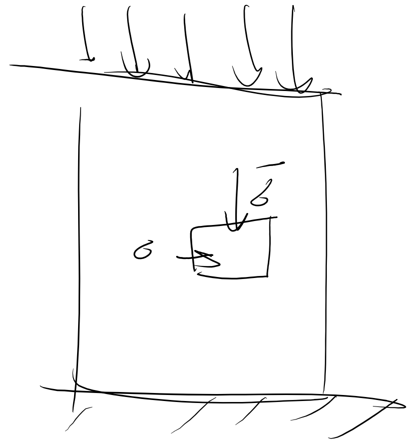
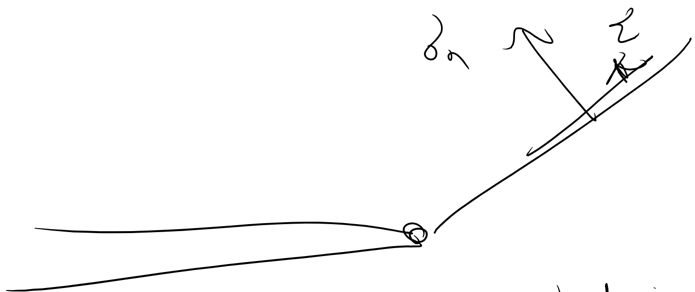
2018/11/07

Wednesday, November 07, 2018 11:24 AM

Modifications to maximum circumferential stress criterion

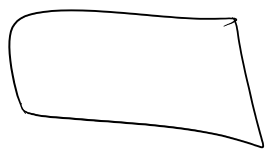






for a lot of rock applications $\sigma_n < 0$

(σ_n, τ) ⟶



determine if a crack can propagate in this direction

search of all angles, chose the one that has the most favorable condition for crack propagation

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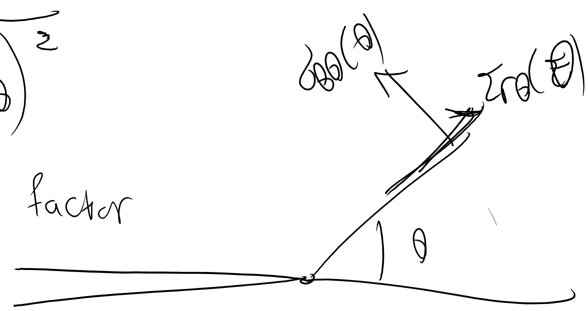
$$s_{eff}(\theta) = \sqrt{\left(\langle\sigma_{\theta\theta}\rangle_+\right)^2 + \left(\alpha\tau_{r\theta}\right)^2}$$

mode mixity factor

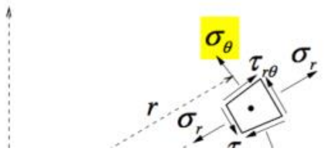


what if $\alpha = 0$

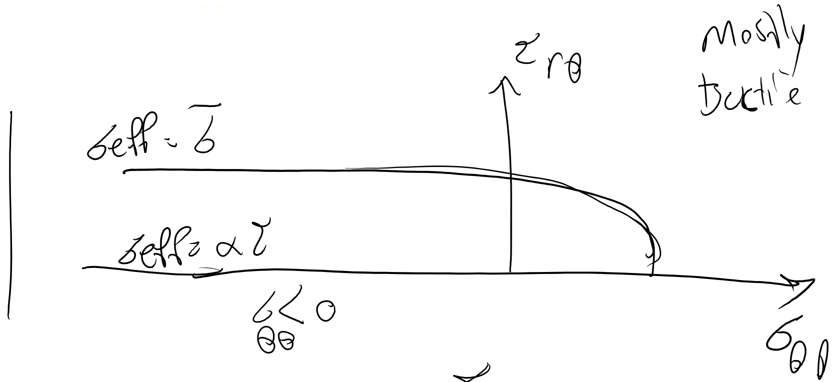
$s_{eff} = \langle\sigma_{\theta\theta}\rangle_+$

For alpha = 0, this is the same max circumferential criterion we used before:

Maximum circumferential stress criterion



$$\sigma_\theta(r,\theta_c) = \max_{-\pi<\theta<\pi} \sigma_\theta(r,\theta)$$

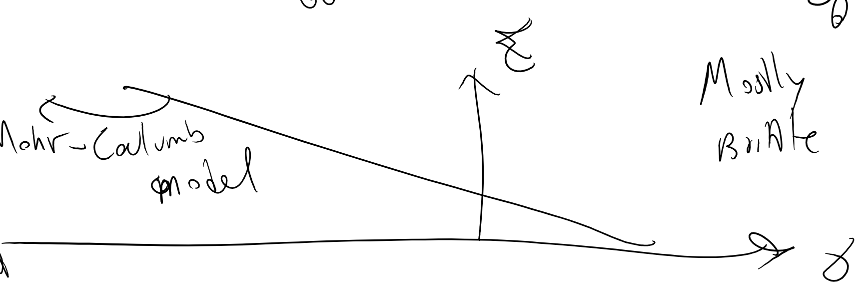


Mostly Ductile

Mohr-Coulomb condition

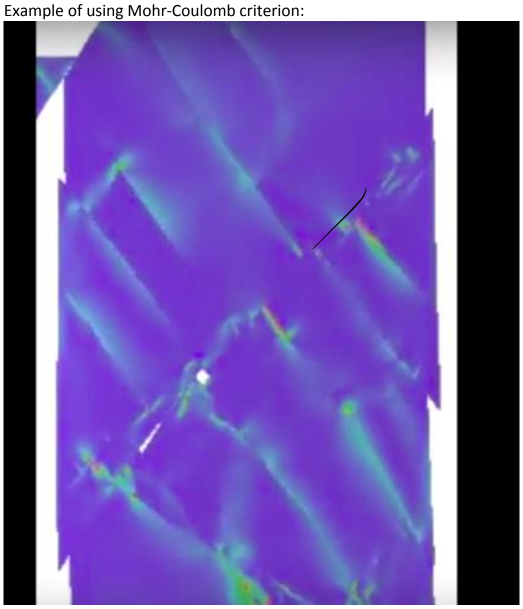
$\sigma_{eff} = \tau + k\sigma$

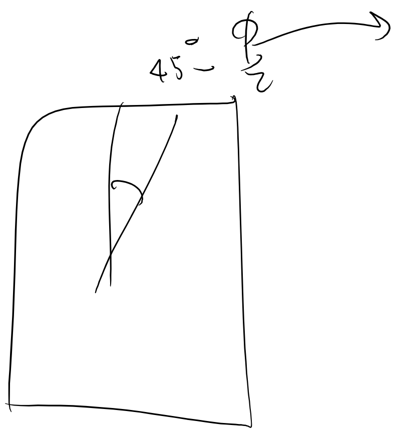
friction coefficient



Mostly Brittle

Example of using Mohr-Coulomb criterion:





$\phi = tg^{-1} k$

friction coeff

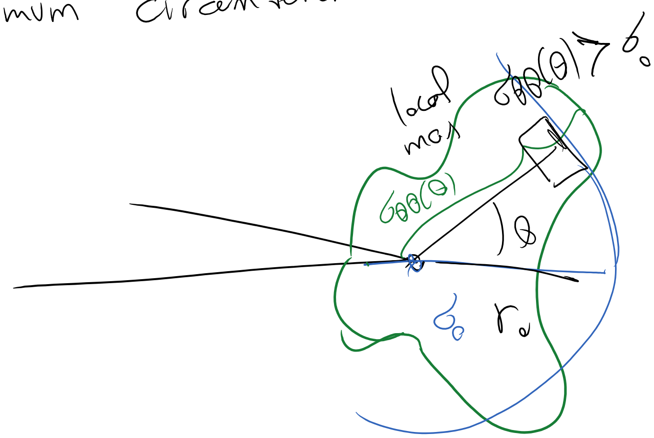
Nucleation of cracks

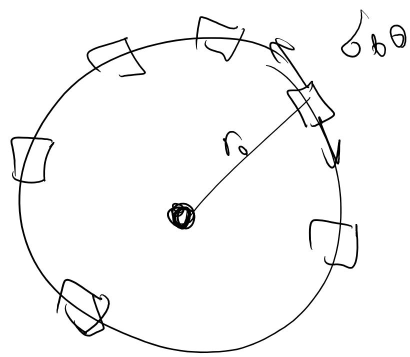
Nucleation of cracks

This should be consistent with propagation criterion

Example

→ Maximum circumferential stress



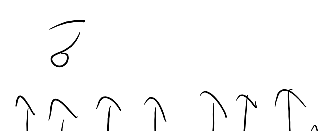


if for any point $\sigma_{\theta\theta}(\theta) > \sigma_0$ a crack is nucleated here

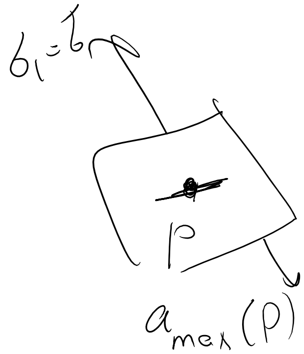
Example 2 Maximum energy release rate

$G(\theta)$ is local maximum & $G(\theta) \geq G_c$

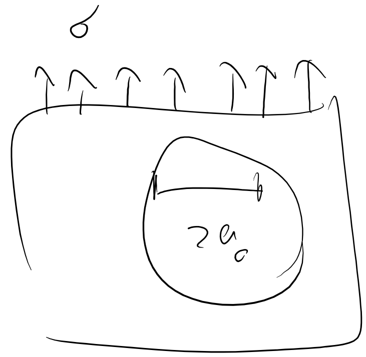
How about nucleation model?



How about nucle...



$G(P) \geq G_c$



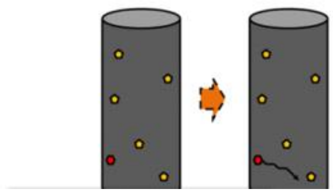
$K_I = \sigma\sqrt{\pi a_0}$

$G = \frac{K_I^2}{E'}$

$$G(P) = \frac{\left(\sigma\sqrt{\pi a_{max}(P)}\right)^2}{E'} \geq G_c$$

- Same concept applies to modified maximum circumferential tensile stress criteria:

$$\max_{-\pi<\theta<\pi}\sigma_{\text{eff}}(r \to 0^+, \theta) = \sigma_0, \quad \text{crack nucleates}$$



Crack nucleation criterion

- For Maximum Energy Release Rate Criterion if we assume there are no defects, there will be no crack nucleation. However, assuming that local stress field generates a tensile maximum principal stress of σ_1 a "microscopic" initial crack (defect) of length a_{ini} perpendicular to σ_1 direction generates,

$$G = \frac{K_I^2 + K_{II}^2}{E'} = \pi a_{\text{ini}}\sigma_1^2$$

so the microcrack propagates (*i.e.*, a "macroscopic" crack nucleates) when,

$$G = G_c \quad \Leftrightarrow \quad \sigma_1 = \sqrt{\frac{G_c}{\pi a_{\text{ini}}}}$$

  - Initial crack direction perpendicular to σ_1 is chosen to maximize G.
  - We have assumed the initial crack to be small enough to use the infinite domain SIF formula of $K_I = \sqrt{\pi a}\bar{\sigma}$.

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8. Fatigue

8.1. Fatigue regimes

8.2. S-N, P-S-N curves

8.3. Fatigue crack growth models (Paris law)

- Fatigue life prediction

8.4. Variable and random load

- Crack retardation due to overload

Fatigue fracture is prevalent!

- Deliberately applied load reversals (e.g. rotating systems)
- Vibrations (machine parts)
- Repeated pressurization and depressurization (airplanes)
- Thermal cycling (switching off electronic devices)
- Random forces (ships, vehicles, planes)

(source: Schreurs fracture notes 2012)

Fatigue occurs always and everywhere and is a major source of mechanical failure

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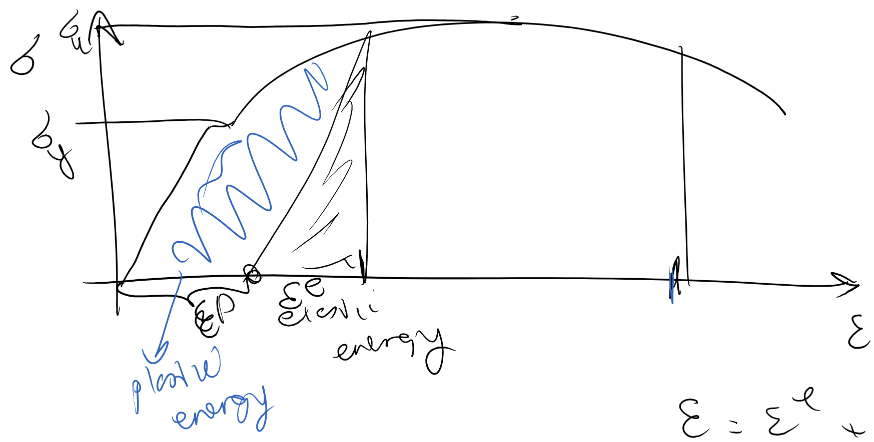
Fatigue Regimes

Table 7.1 Classification of fatigue damage

Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$
Very high cycle fatigue	$> 10^7$	$< \sigma_F$	≈ 0	≈ 0
High cycle fatigue	10^5 to 10^6	$< \sigma_Y$	≈ 0	≈ 0
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

Source: Dufailly and Lemaitre (1995)



$$\varepsilon = \varepsilon^e + \varepsilon^p$$

for low cycle fatigue — plastic deformation is important

— PFM is the appropriate set-up

— Strain rather than stress is used for fatigue analysis

for high cycle fatigue:

— Plastic deformation is not important

∴ energy ≈ 0

— LEFM is used

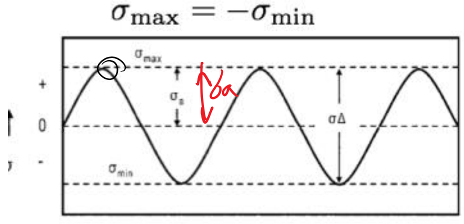
— Analyzed by stress

- Very high cycle and high cycle fatigue:
  - Stresses are well below yield/ultimate strength.
  - There is almost no plastic deformation (in terms of strain and energy ratios)
  - Fatigue models based on LEFM theory (e.g. SIF K) are applicable.
  - Stress-life approaches are used (stress-centered criteria)
- Low cycle and very low cycle fatigue:
  - Stresses are in the order of yield/ultimate strength.
  - There is considerable plastic deformation.
  - Fatigue models based on PFM theory (e.g. J integral) are applicable.
  - Strain-life approaches are used (strain-centered criteria)

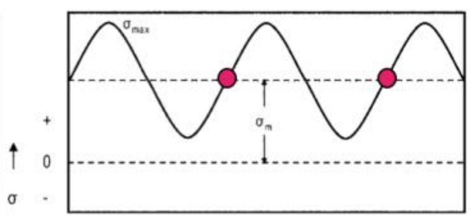
Focus in this course is only on High-cycle fatigue

LEFM is appropriate for this regime

$$\sigma_{\max} = -\sigma_{\min}$$



Fully Reversed Loading



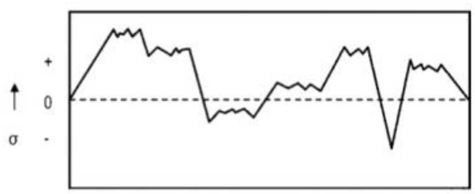
Tension-Tension with Applied Stress

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

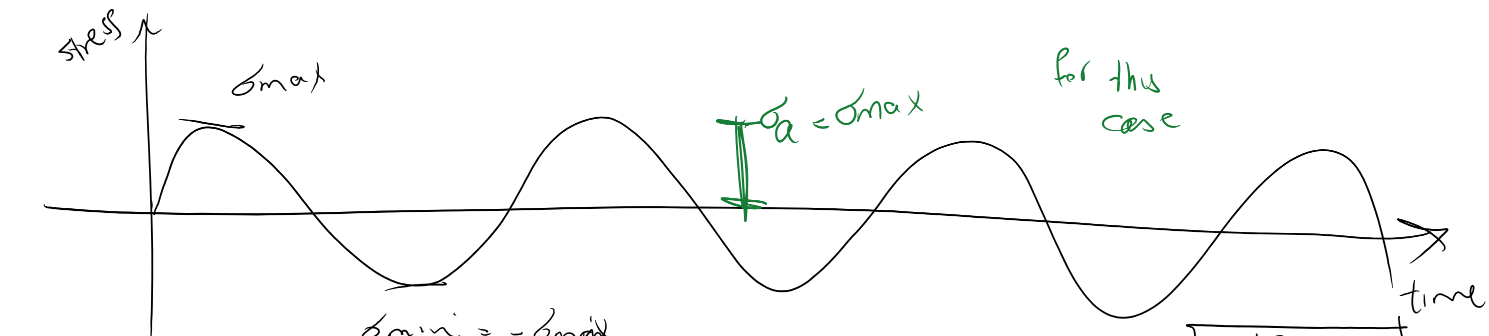
$$\boxed{\sigma_a = 0.5(\sigma_{\max} - \sigma_{\min})}$$

mean $\sigma_m = 0.5(\sigma_{\max} + \sigma_{\min})$

$$\boxed{R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad \text{load ratio}}$$



Random or Spectrum Loading



standard fatigue condition $\boxed{R = -1}$

$R : (-\infty, 1)$ not fixed

range of R

$R = \frac{\sigma_{\min}}{\sigma_{\max}}$

$(-\infty, -1)$ $(-1, 0)$ $(0, 1)$

R ... range of R $R = \frac{\sigma_{min}}{\sigma_{max}}$

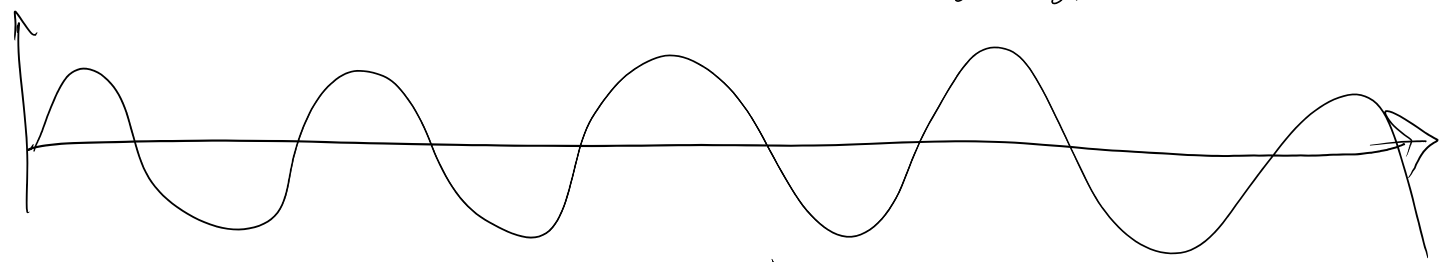
σ fixed

$(\infty, -1)$ $(-1, 0)$ $(0, 1)$ all tensile

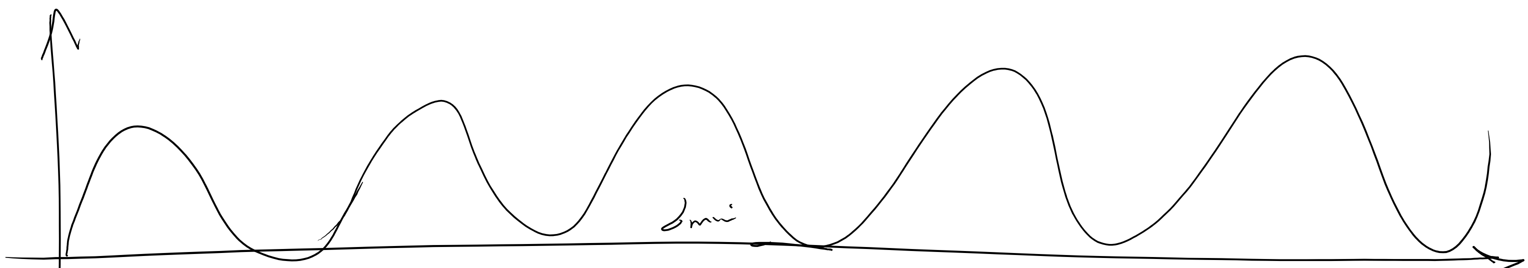
compressive dominant tensile dominant

$R = \frac{\sigma_{min}}{\sigma_{max}} \to -\infty$

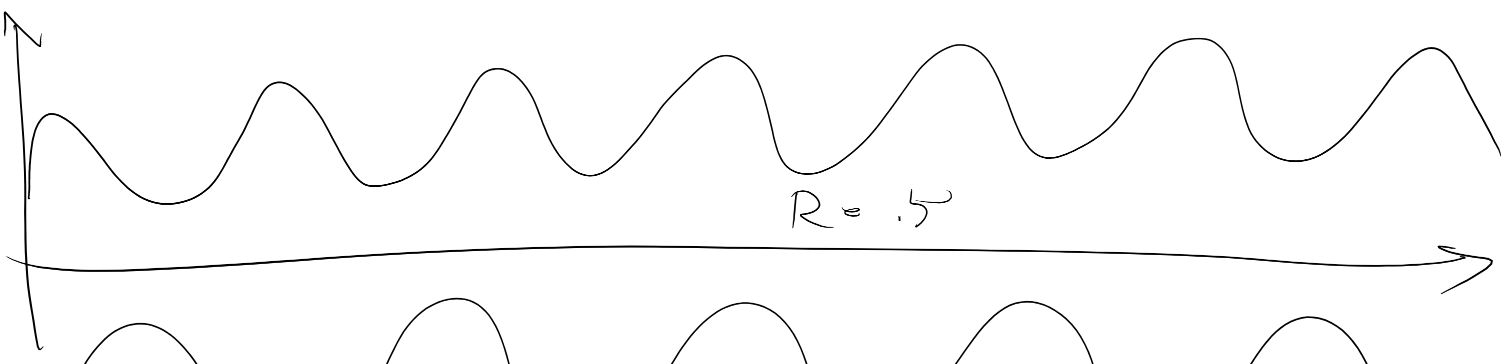
as $\sigma_{max} \to 0$



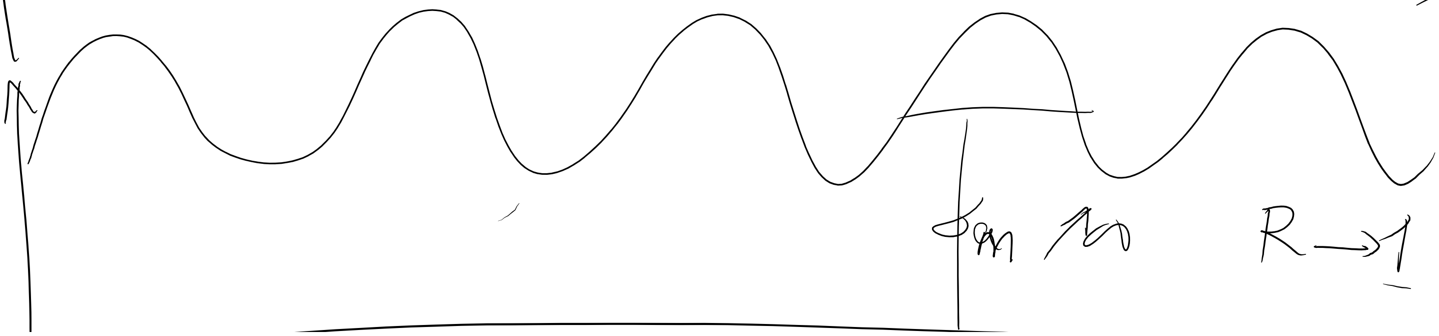
$R = -1$



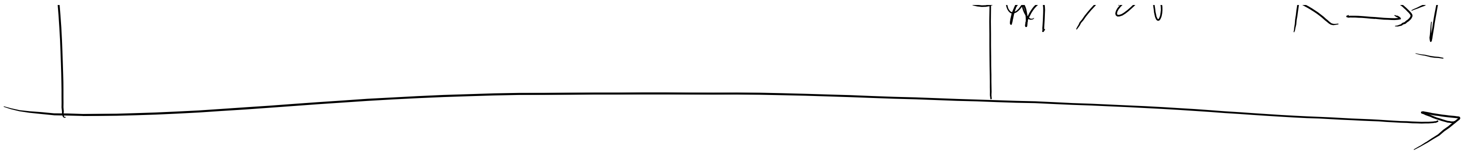
$R = 0$



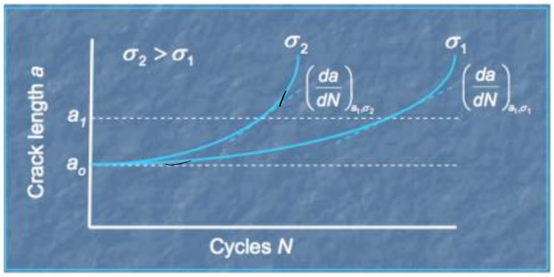
$R = .5$

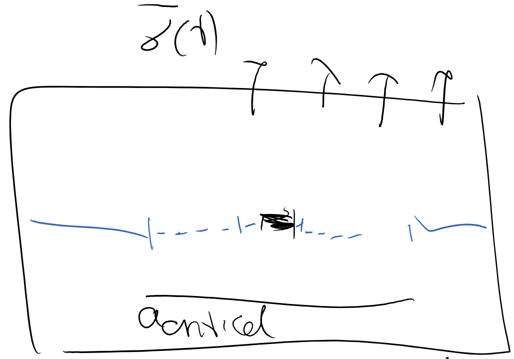


σ_m ... $R \to 1$



Typical fatigue growth:





1. Initially, crack growth rate is small
2. Crack growth rate increases rapidly when a is large
3. Crack growth rate increases as the applied stress increases

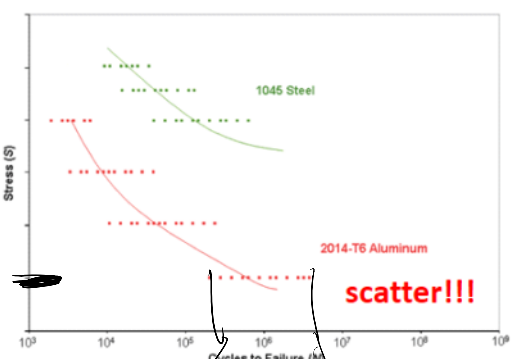
$K_I = \bar{\sigma}_{max}\sqrt{\pi a} = K_{IC}$

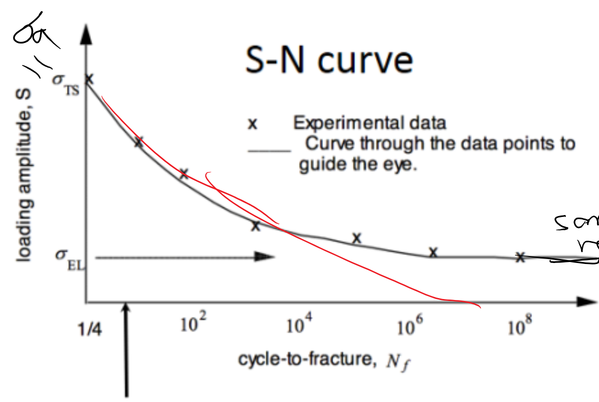
$a_{critical} = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_{max}}\right)^2$

S-N curve

Reminder: ASTM defines *fatigue life*, N_f, as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs.

- Stress->Nf
- Nf->allowable S

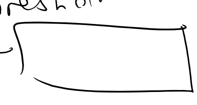




endurance limit (g.han keo dai)

e.g. steel

some materials reach a threshold

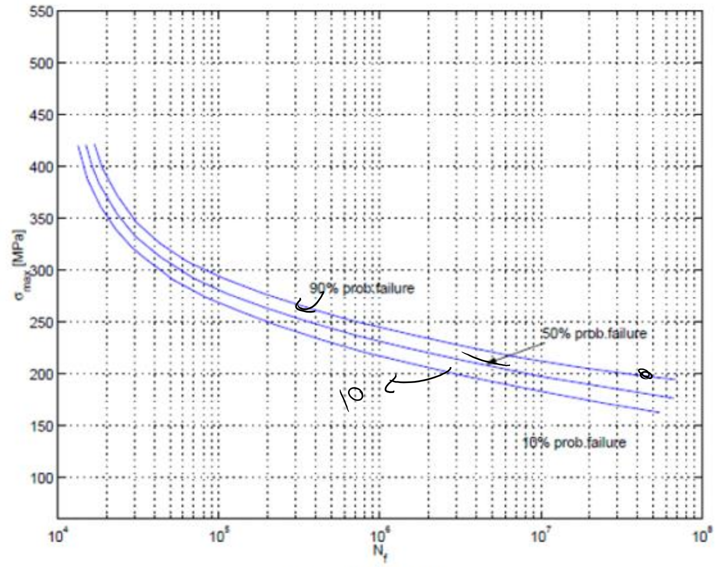


5e4

7e6

> 1 order of magnitude difference in N (# of cycles)

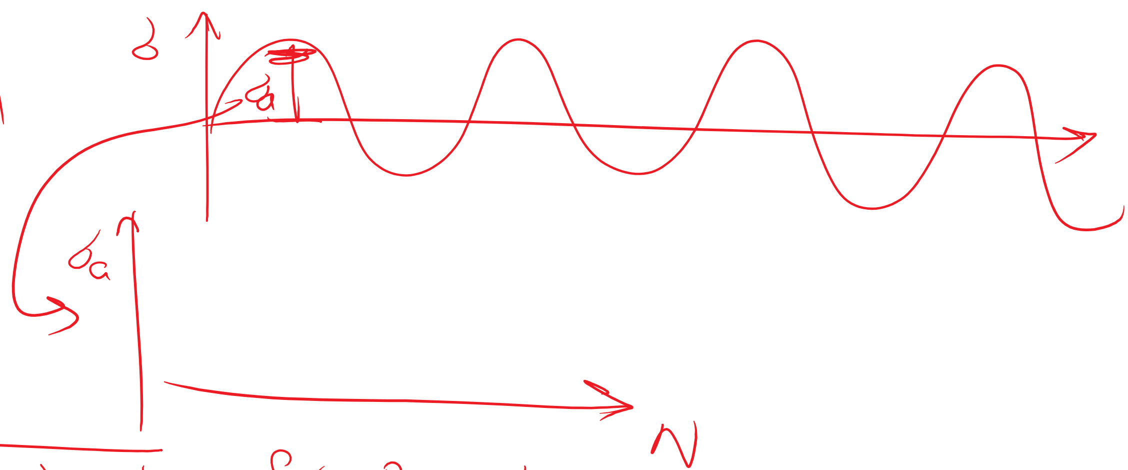
S-N-P curve: scatter effects



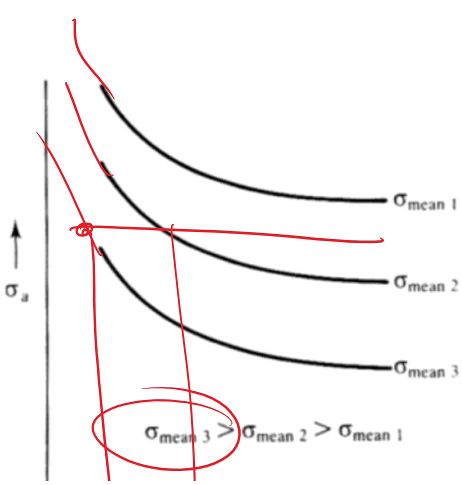
P-S-N-curves

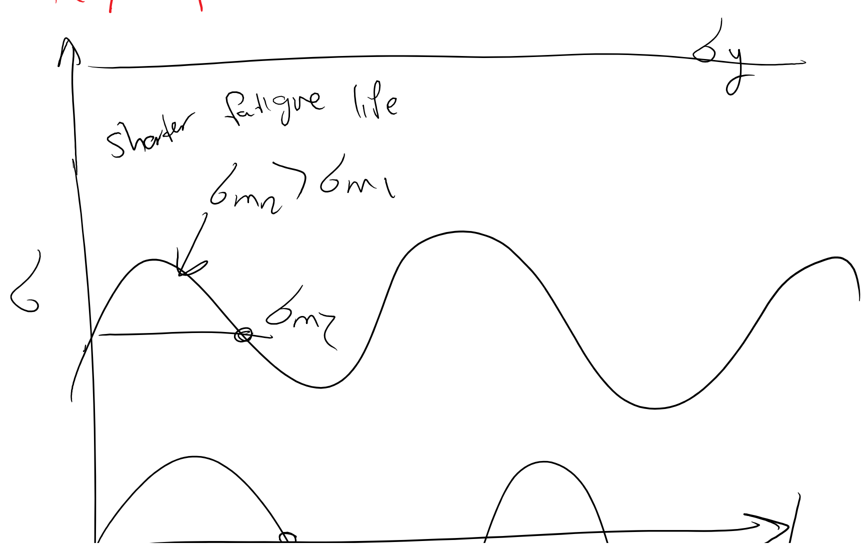
All those curves are calibrated for

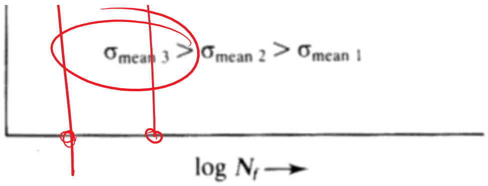
R = -1

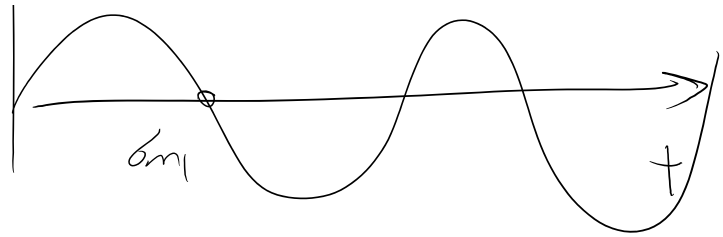


What should be done for $R \neq -1$









$$\sigma_a = \sigma_{f0}\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^r\right]$$

N is given

where σ_a is the amplitude of allowable stress (alternating stress).

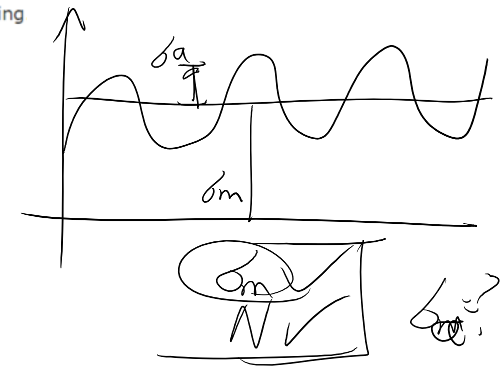
σ_{f0} is the stress at fatigue fracture when the material under zero mean stress cycled loading

σ_m is the mean stress of the actual loading.

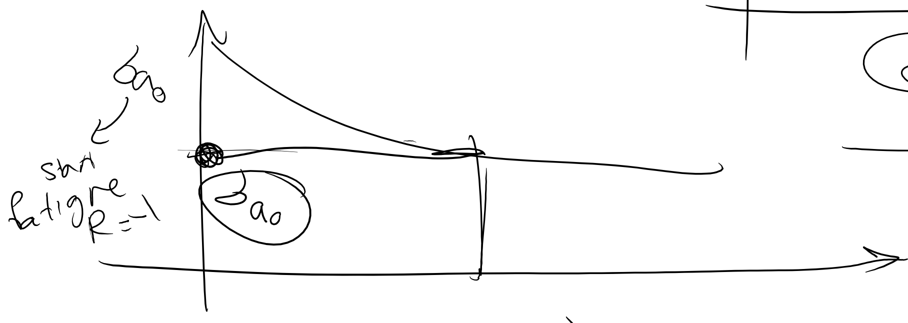
σ_u is the tensile strength of the material.

r = 1 is called Goodman line which is close to the results of notched specimens.

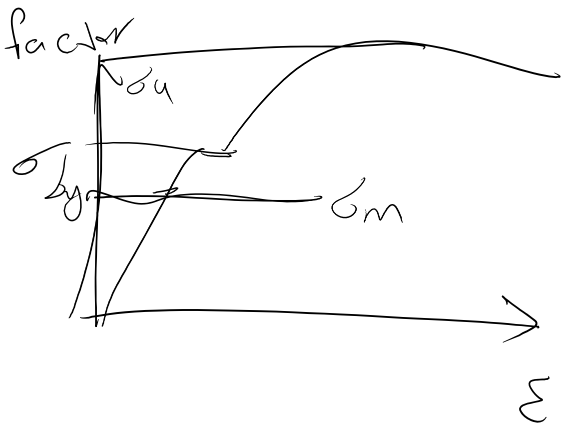
r = 2 is the Gerber parabola which better represents ductile metals.



σ_a is



$$\sigma_a = \sigma_{ao} \underbrace{\left(1 - \left(\frac{\sigma_m}{\sigma_{uy}}\right)^{\gamma}\right)}_{\text{reduction factor}}$$



Gerber (1874)	$\dfrac{\sigma_a^*}{\sigma_a} = 1 - \left(\dfrac{\sigma_m}{\sigma_u}\right)^2$
Goodman (1899)	$\dfrac{\sigma_a^*}{\sigma_a} = 1 - \dfrac{\sigma_m}{\sigma_u}$
Soderberg (1939)	$\dfrac{\sigma_a^*}{\sigma_a} = 1 - \dfrac{\sigma_m}{\sigma_{y0}}$

$\sigma_m = \sigma_y$ for Sodrberg model

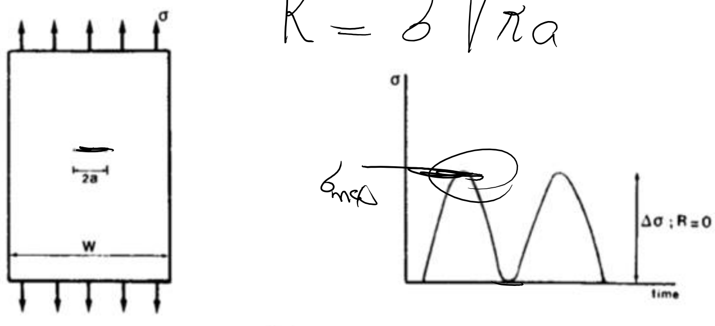
Reduction factor $= 1 - \frac{1}{1} = 0$

$\rightarrow \sigma_a = \sigma_{ao} \times 0$

as $\sigma_m \rightarrow \sigma_{y_0}$

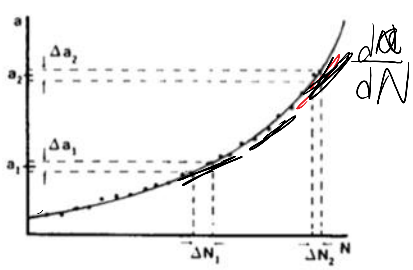
S-N-P (modifying by simga_m) all this is a very crude model for fatigue, without considering a particular "realization" (sample) of a given material

$$K = \sigma\sqrt{\pi a}$$

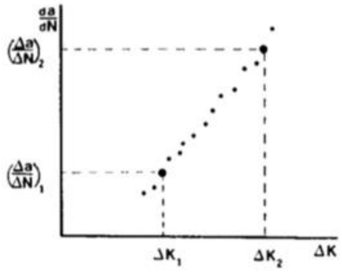


(a)

$a(\sigma)$ doesn't explain why $\frac{da}{dN}$ grows as a grows



(b)



(c)

$\frac{da}{dN}$ (rate of crack growth) is a function of K $K_{max} = \sigma_{max}\sqrt{\pi a}$

$$\frac{da}{dN} = C\,\Delta K^{m}$$