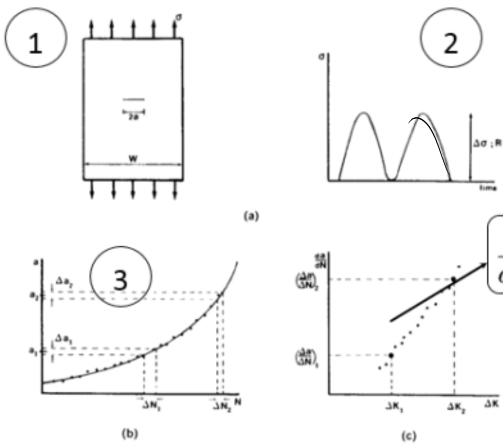
2018/11/12

Monday, November 12, 2018 11:29 AM

# Crack growth data

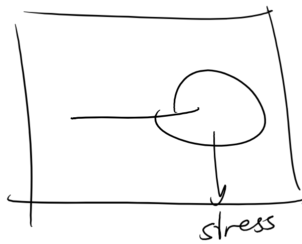
$K = \sigma\sqrt{\pi a}$




$$\frac{da}{dN} = f_1(\Delta K, R)$$

$\Delta\sigma$ fixed

$\Delta K = \Delta\sigma\sqrt{\pi a}$ ↗ because crack grows



stress field around the crack tip is a function of $K$

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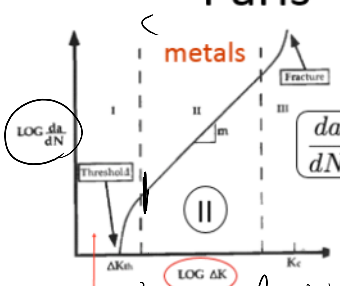
# Paris' law (fatigue)




log ΔK

Paris' law $\boxed{2 \le m \le 7}$

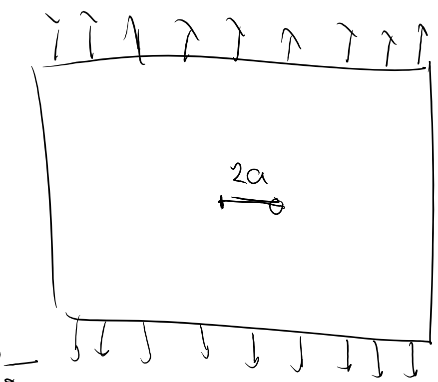
$$\frac{da}{dN} = C(\Delta K)^m, \quad \Delta K = K_{\max} - K_{\min}$$

(Power law relationship for fatigue crack growth in region II)

if $\Delta K < \Delta K_{th}$ fatigue fracture does not occur

slow crack growth regime

$\sigma(t)$




$\Delta K = \Delta\sigma\sqrt{\pi a}$

$\sigma(t)$

$R = 0$

$$\frac{da}{dN} = C(\Delta K)^m$$

Paris law for medium crack growth regime (zone II)

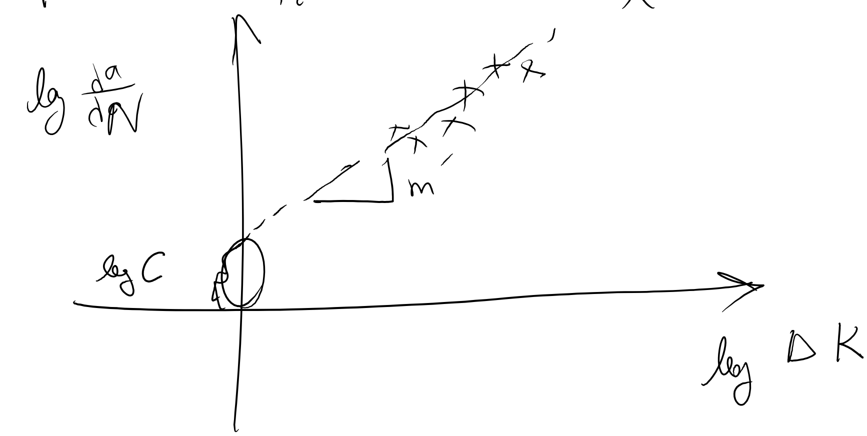
$$\log \frac{da}{dN} = \log C(\Delta K)^m$$

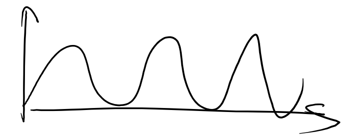
(Zone II)

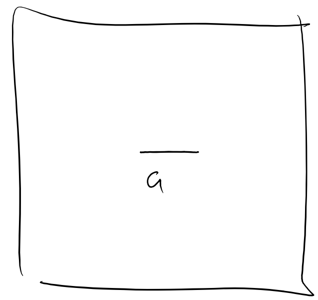
$\log xy = \log x + \log y$

$\log x^m = m \log x$

$$\rightarrow \boxed{\underbrace{\log \frac{da}{dN}}_{Y} = \underbrace{\log C}_{Y_c} + m \underbrace{\log \Delta K}_{X}}$$






$\Delta K = \Delta\sigma\sqrt{\pi a}$

fixed, changing

$\Delta K \nearrow$ as $a \nearrow$

MUST be careful about the units

$$\frac{da}{dN} = C(\Delta K)^m$$

no a dimensionally consistent equation.

C : depends on units used for example in $MPa\sqrt{m}$ is used of $\Delta K$

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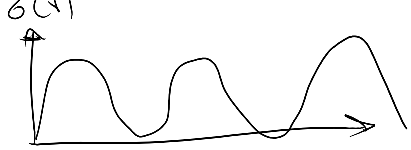
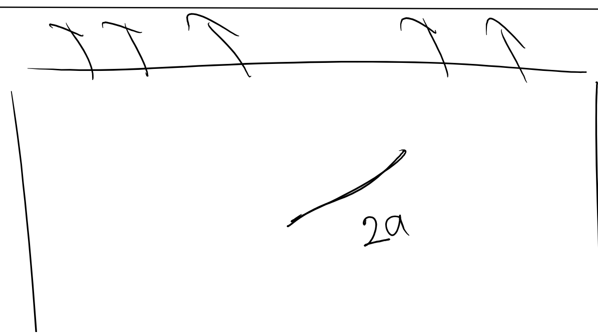
Example

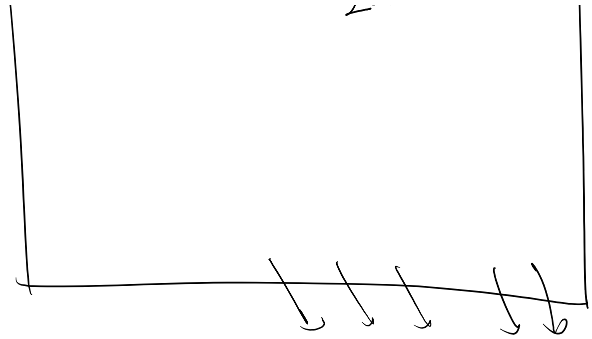


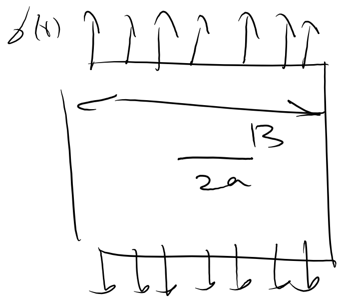

$\sigma(t)$



$K = 0$

$$K = \sqrt{\pi a}\,\sigma\,\underbrace{Y(a)}_{\text{geometry factor}}$$






$$Y(a) = \sec\left(\frac{a}{B}\right) = \frac{1}{\cos\left(\frac{a}{B}\right)}$$

$\to \infty$

as $a \to B$

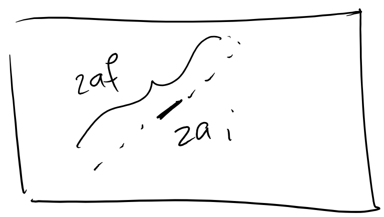
$$\boxed{\begin{aligned} \Delta K &= \sqrt{\pi a}\, Y(a)\, \Delta\sigma \\ \frac{da}{dN} &= C\, \Delta K^m \end{aligned}}$$

Differential equation on $N$

for $N = 0$

$a = a_i$

Initial Condition (IC)




we solve this equation, until crack length reaches the unstable length for which the crack can propagate under $\sigma_{max}$


$a_f$:

$$K(t) = \sqrt{\pi a}\, Y(a)\, \sigma(t)$$

max in a cycle is for $\sigma_{max}$

$$\underline{K_{max}} = \sqrt{\pi a}\; Y(a)\, \sigma_{max}$$

$$\boxed{K_{IC} = \sqrt{\pi a_f}\; Y(a_f)\, \sigma_{max}}$$

for unstable crack growth

get $a_f$ from here

Summary

$$\frac{dN}{da} = C\Delta K^m$$

$$\Delta K = \sqrt{\pi a}\, Y(a)\, \Delta\sigma$$

IC $a = a_i$

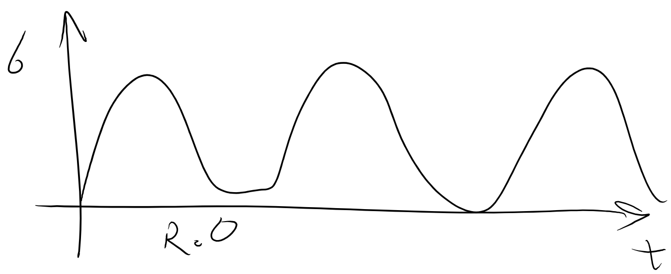
Final condition $a = a_f$ found from

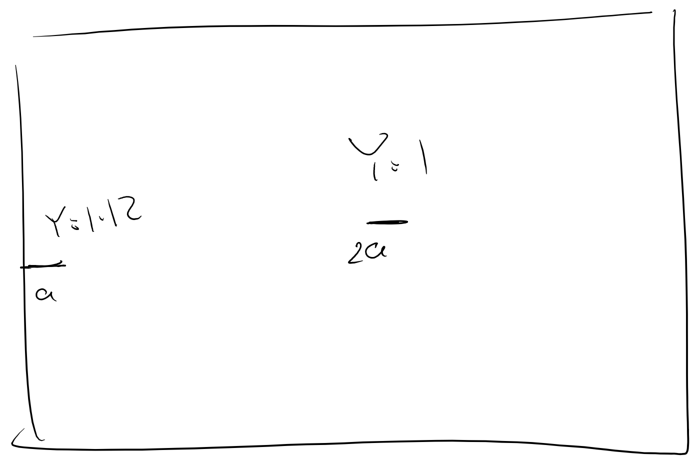
$$K_{IC} = \sqrt{\pi a_f}\, Y(a_f)\, \sigma_{max}$$

→ $N_f$ is the # cycles this specimen can take

In general, we need to solve this numerically because of $Y(a)$

Do the case where $Y(a)$ can be assumed to be constant:




constant assumption

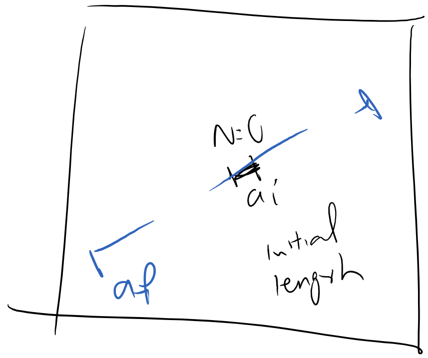
$$\left.\begin{array}{l}\Delta K = \sqrt{\pi a}\, Y\, \Delta\sigma \\ \dfrac{da}{dN} = C(\Delta K)^m\end{array}\right\} \Rightarrow \frac{da}{dN} = C\left(\sqrt{\pi a}\, Y \Delta\sigma\right)^m = \underbrace{\left(C Y^m \pi^{m/2} \Delta\sigma^m\right)}_{A} a^{m/2}$$

$$\rightarrow \boxed{\frac{da}{dN} = A\, a^{m/2}, \quad A = C Y^m \pi^{m/2} \Delta\sigma^{m/2}}$$

$$\rightarrow \boxed{\frac{da}{dN} = A\, a^{m/2} \quad , \quad A = C\, Y^{m}\, \pi^{m/2}\, \Delta\sigma^{m/2}}$$

$$\rightarrow \frac{da}{a^{m/2}} = A\, dN \longrightarrow$$ integrate this

$$\int_{a_i}^{a} \frac{da}{a^{m/2}}\, da = \int_{0}^{N} A\, dN$$

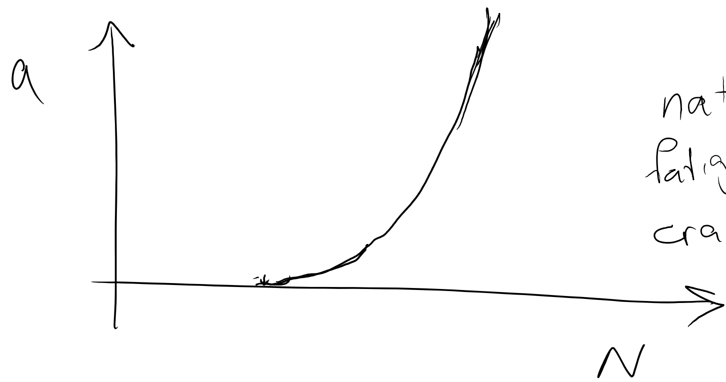



$\underline{m > 2}$ assumed

$$\rightarrow \frac{1}{\left(1 - \frac{m}{2}\right)} a^{1-\frac{m}{2}} \Bigg|_{a_i}^{a} = AN \longrightarrow$$

$$\frac{1}{\left(\frac{m}{2} - 1\right)} \left( \frac{1}{a^{\frac{m}{2}-1}} - \frac{1}{a_i^{\frac{m}{2}-1}} \right) = AN(a)$$

$$\boxed{N(a) = \frac{1}{\left(\frac{m}{2} - 1\right) C\, \pi^{m/2}\, Y^{m}\, \Delta\sigma^{m}} \left( \frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a_f^{\frac{m}{2}-1}} \right)}$$




nature of fatigue crack growth

$m > 2$

$Y$ constant

to get the life of specimen ($N_f$) we plug $a = a_f$

$$N_{f} = \frac{1}{\quad} \left( \frac{1}{\quad} - \frac{1}{\quad} \right)$$

$$N_f = \frac{1}{\left(\frac{m}{2}-1\right) C \pi^{\frac{m}{2}} Y^m \Delta\sigma^m}\left(\frac{1}{a_i^{m/2}} - \frac{1}{a_f^{m/2}}\right)$$

$$K_{IC} = Y\sqrt{\pi a_f}\,\sigma_{max} \longrightarrow a_f = \frac{1}{\pi}\left(\frac{K_{IC}}{Y\sigma_{max}}\right)^2$$

For $m > 2$:

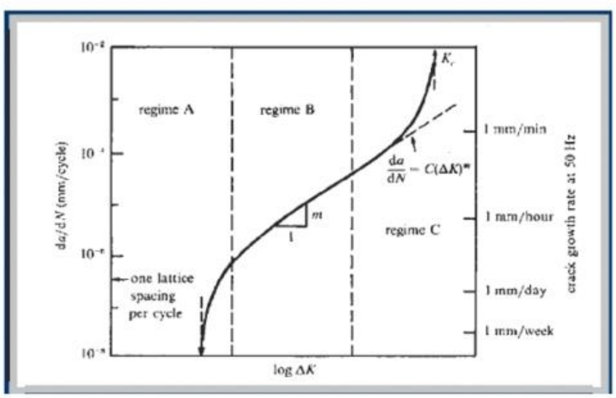
$$N_f = \frac{2}{(m-2)\,CY^m\,(\Delta\sigma)^m\,\pi^{m/2}}\left[\frac{1}{(a_0)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}}\right]$$

For $m = 2$:

$$N_f = \frac{1}{CY^2\,(\Delta\sigma)^2\,\pi}\ln\frac{a_f}{a_0}$$

(source Course presentation S. Suresh MIT)

# Fatigue crack growth stages




Question:

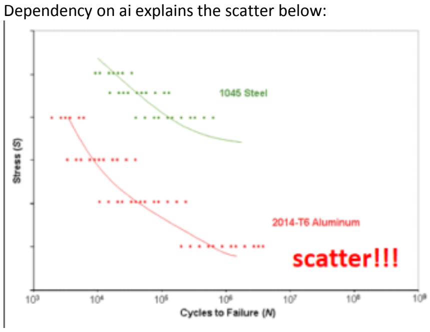
How do we ai?


$$N_f = \frac{1}{\left(\frac{m}{2}-1\right) C \pi^{\frac{m}{2}} Y^m \Delta\sigma^m}\left(\frac{1}{a_i^{m/2}} - \frac{1}{a_f^{m/2}}\right)$$

$$N_f = (\tfrac{m}{2}-1)\, C \pi^{m/2} Y^m \Delta\sigma^m \left( a_i^{\cdots} \qquad a_f^{m/2} \right)$$

$$K_{Ic} = Y\sqrt{\pi a_f}\,\sigma_{max} \longrightarrow a_f = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma_{max}}\right)^2$$

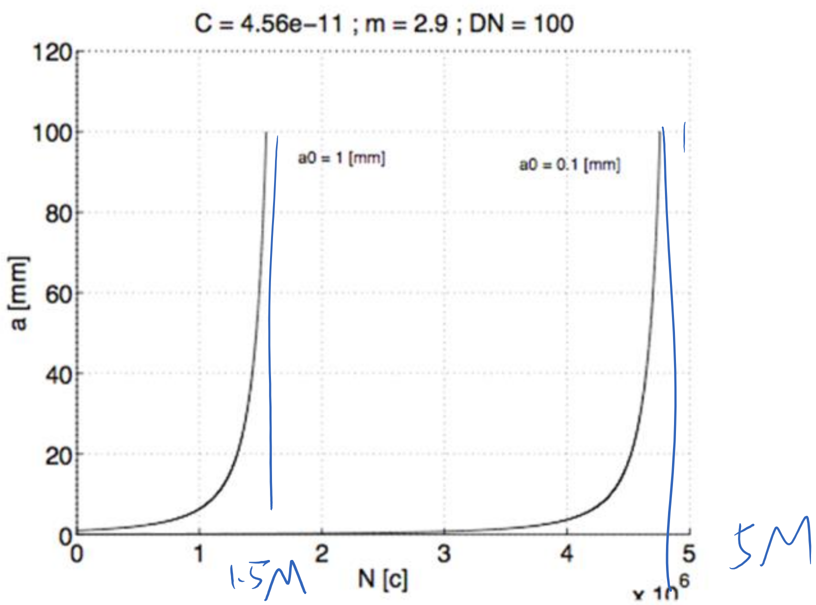
Dependency on ai explains the scatter below:




$a_i =$
→ Case I: we know the initial crack length "it's visible & we can measure it"
→ Case 2: we cannot observe it

$a_i =$ TOLERANCE of Measurement (SHM, NDE) "the worst case scenario

Example of the sensitivity to the size of initial cracks:




aluminum ; $\Delta\sigma = 50$ [MPa]

factor of ~3 difference in N

How do we build factor of safety into our designs

$$N(a) = \frac{1}{(\frac{m}{2}-1) C \pi^{m/2} Y^m \Delta\sigma^m} \left( \frac{1}{a_i^{m/2-1}} - \frac{1}{a_f^{m/2-1}} \right)$$

— Modify $N \rightarrow \frac{N}{FS}$

— Another way $a_f \xrightarrow{\text{modified}}$ use $a_f (F.S.)^{.6}$

we only only allow the crack to go to 60% of $a_f$

# Nondestructive testing (NDT)

Nondestructive Evaluation (NDE), nondestructive Inspection (NDI)

NDT is a wide group of analysis techniques used in science and industry to evaluate the properties of a material, component or system without causing damage

**NDT: provides input (e.g. crack size) to fracture analysis**

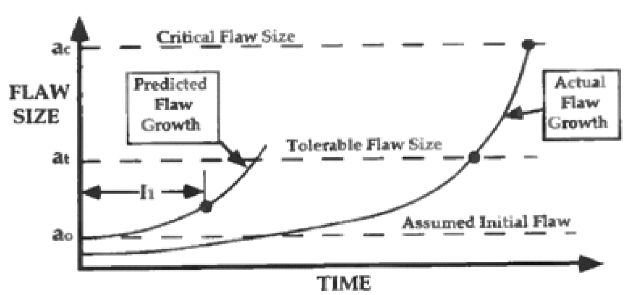
safety factor s

$K(a,\sigma) = K_c \rightarrow a_c -> a_t * s$

$\mathrm{NDT} \rightarrow a_o$

$t: \quad a_o \rightarrow a_t$ (Paris)

inspection time




374 (a) Determination of first inspection interval, $I_1$.

m denotes the sensitivity of fatigue to $\Delta K$

| alloy | $m$ | $A$ |
|---|---|---|
| Steel | 3 | $10^{-11}$ |
| Aluminum | 3 | $10^{-12}$ |
| Nickel | 3.3 | $4 \times 10^{-12}$ |
| Titanium | 5 | $10^{-11}$ |

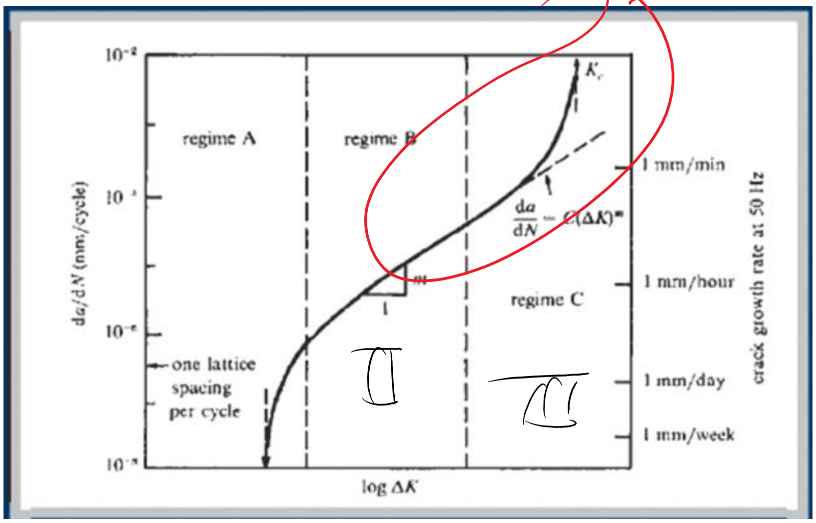
$C, m$

are material properties that must be determined experimentally from a log(delta K)-log(da/dN) plot.

m
2-4 metals
4-100 ceramics/ polymers

Some notes on fatigue crack growth:

- How to improve Paris law to better model transition between regimes II and III fatigue crack growth?




crack starts to grow even faster

there are models that unify responses in regimes II & III

**Forman's model (stage II-III)**

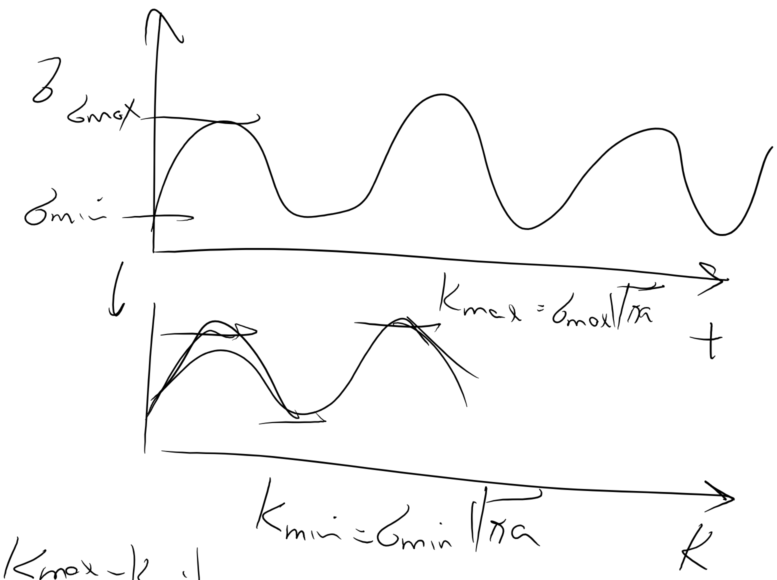
$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K}$$

$$R = K_{min}/K_{max}$$

**Paris' model**

$$\frac{da}{dN} = C(\Delta K)^m$$

Forman's correction




Denominator $= (1-R)K_{IC} - \Delta K$

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{K_{min}}{K_{max}}$$

Denominator $= \left(1 - \frac{K_{min}}{K_{max}}\right) K_{IC} - (K_{max} - K_{min})$

$$= \underbrace{(K_{max} - K_{min})}_{\Delta K}\left(\frac{K_{IC}}{K_{max}} - 1\right)$$

Forman's correction

Basically smoothens

$$\frac{da}{dN} = \frac{C\Delta K^m}{\Delta K\left(\frac{K_{IC}}{K_{max}} - 1\right)} \to \infty, \quad \left(\frac{K_{IC}}{K_{max}} - 1\right) \to 0$$

Basically smoothens transition of $\frac{da}{dN} \sim \Delta K$ relation from zone II (Paris law) to " III (fast growth rate)

$$\underbrace{\Delta K \left(\left(\frac{K_{IC}}{K_{max}} - 1\right)\right)}_{\text{correction}} \to 0$$

$$K_{max} = \sigma_{max} \sqrt{\pi a}\, Y$$

$$a \to a_f \quad \left( K_{IC} = \sigma_{max} \sqrt{\pi a_f}\, Y \right)$$

$$\frac{K_{IC}}{K_{max}} \to 1$$

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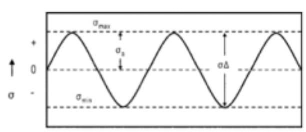
---- Do not consider negative K when using Paris law:

## Tension/compression cyclic loads

$$R = \frac{\sigma_{min}}{\sigma_{max}} < 0$$

$$\frac{da}{dN} = C(\Delta K)^m, \quad \Delta K = K_{max} - \cancel{K_{min}}$$

$$\boxed{\frac{da}{dN} = C(K_{max})^m}$$




Fully Reversed Loading

369

$$\Delta K = K_{max} - \underbrace{\max(0, K_{min})}_{\text{no negative value for } K_{min}}$$

## Miner's rule for variable load amplitudes

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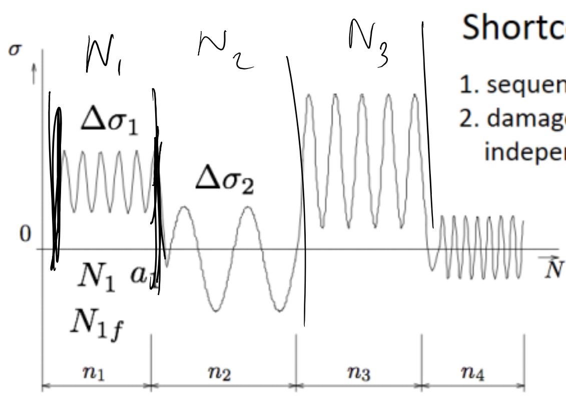


**Shortcomings:**

1. sequence effect not considered
2. damage accumulation is independent of stress level

**$N_i/N_{if}$ : damage**

$$\text{Damage} = \frac{\#\ \text{cycles}}{\#\ \text{" that it can take under that loading condition}}$$

$$\boxed{\sum_{i=1}^{n} \frac{N_i}{N_{if}} = 1}$$

$\Delta\sigma_i$

$N_i$ number of cycles $a_0$ to $a_i$

$N_{if}$ number of cycles $a_0$ to $a_c$