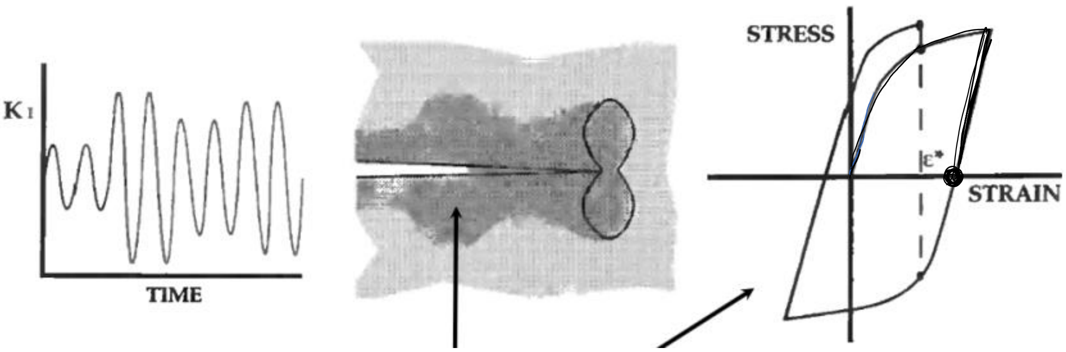
2018/11/19

Monday, November 19, 2018 11:36 AM

Fatigue under random vibration (stress values)

Variable amplitude cyclic loadings



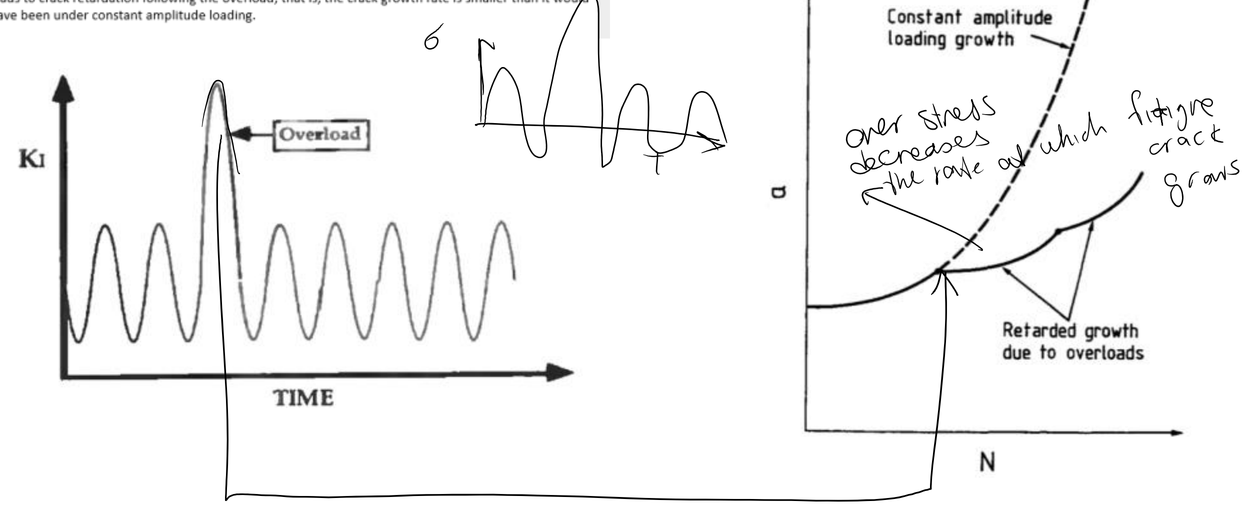
$$\boxed{\frac{da}{dN} = f_2(\Delta K, R, H)}$$

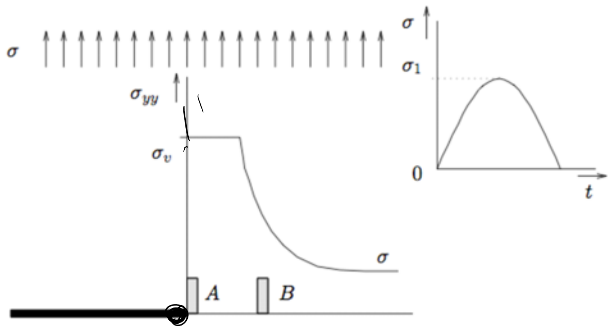
keeps history of the loading

How can we increase fatigue life?

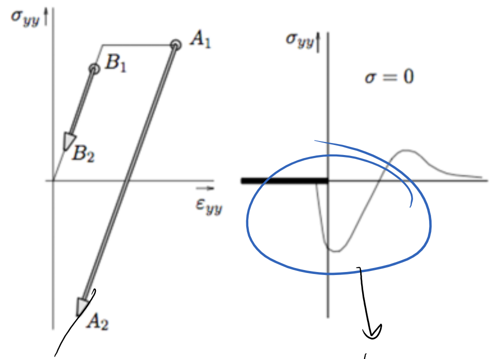
Overload and crack retardation

It was recognized empirically that the application of a tensile overload in a constant amplitude cyclic load leads to crack retardation following the overload; that is, the crack growth rate is smaller than it would have been under constant amplitude loading.





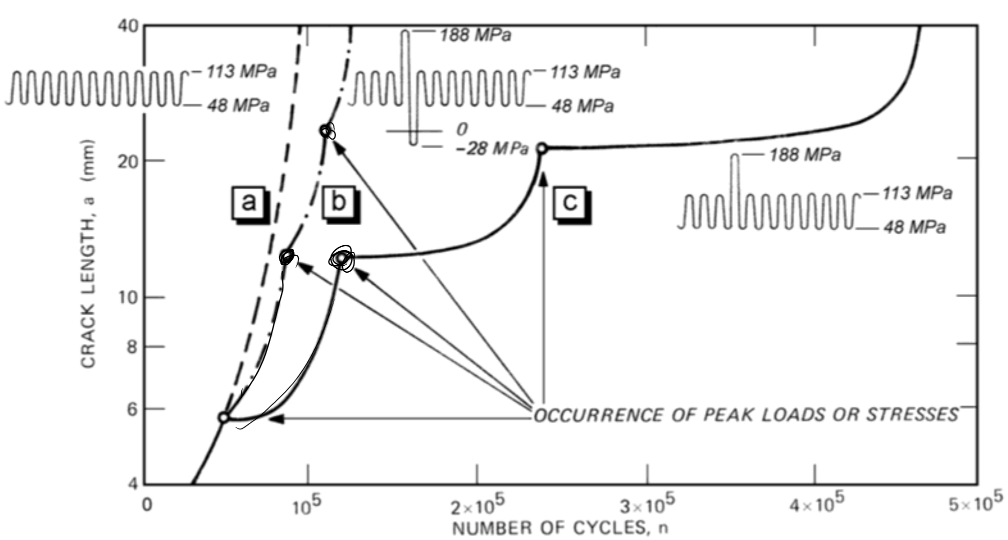
Point A: plastic
point B: elastic



After unloading: point A and B has more or less the same strain -> point A : compressive stress.

by overstressing & unloading after that we create a zone under compressive stress

It was recognized empirically that the application of a tensile overload in a constant amplitude cyclic load leads to crack retardation following the overload; that is, the crack growth rate is smaller than it would have been under constant amplitude loading.



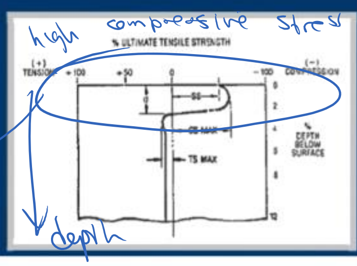
Putting material in compressive stress state is a good idea to retard fatigue crack growth

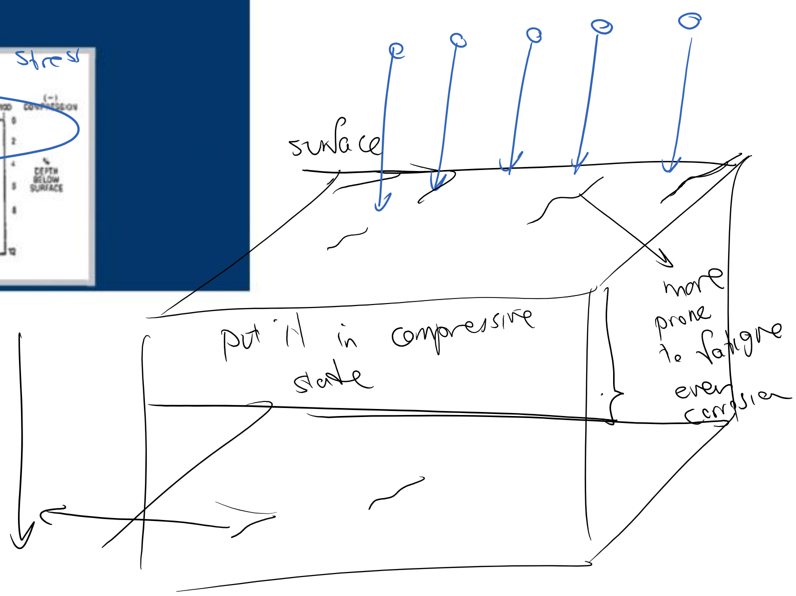
Shot peening is a cold working process in which the surface of a part is bombarded with small spherical media called *shot*. Each piece of shot striking the surface acts as a tiny peening hammer, imparting to the surface a small indentation or dimple. The net result is a layer of material in a state of residual compression. It is well established that cracks will not initiate or propagate in a compressively stressed zone.

Shot-peening

→ a techique used to increase fatigue life

A typical residual stress profile created by shot peening is shown below:



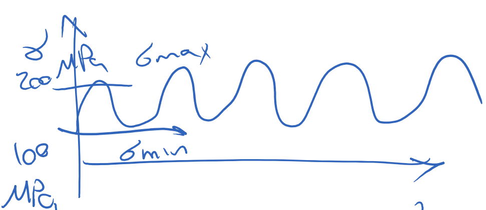


fatigue crack grows becomes slower

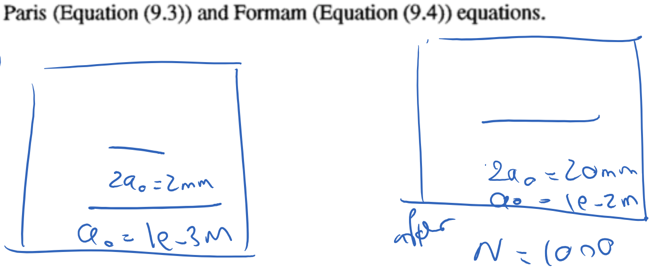
the Compressive stress on the surface can be as high as

$\approx 50\% \ \sigma_y$

A large plate contains a crack of length $2a_0$ and is subjected to a constant-amplitude tensile cyclic stress normal to the crack which varies between 100 MPa and 200 MPa. The following data were obtained: for $2a_0 = 2$ mm it was found that $N = 20,000$ cycles were required to grow the crack to $2a_f = 2.2$ mm, while for $2a = 20$ mm it was found that $N = 1000$ cycles were required to grow the crack to $2a_f = 22$ mm. The critical stress intensity factor is $K_c = 60$ MPa $\sqrt{m}$. Determine the constants in the Paris (Equation (9.3)) and Formam (Equation (9.4)) equations.



(a)



after $N = 20000$ cycles

$2a_f = 2.2$ mm

we go to fast crack growth mode

after $N = 1000$

we reach at

$2a_f = 22$ mm

$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{100}{200} = .5$

$K_{Ic} = 60 MPa\sqrt{m}$

Q: What are Paris law parameters $\frac{da}{dN} = C(\Delta K)^m \longrightarrow$

$$\log \frac{da}{dN} = \log C + m \log \Delta K$$

$$K = \sigma\sqrt{\pi a} \longrightarrow \Delta K = \underbrace{\Delta\sigma}_{(200-100)\,MPa}\sqrt{\pi a} \longrightarrow$$

$\Delta K_i = 100\sqrt{\pi(1e\text{-}3)\,m} = 5.6\ MPa\sqrt{m}$ case (1)

case 2

$\Delta K = 100\sqrt{\pi(1e\text{-}2)\,m} = 17.72\ MPa\sqrt{m}$

short fatigue life, we can use Finite Difference for $\frac{da}{dN}$

$$\frac{da}{dN} \approx \begin{cases} \frac{a_f - a_i}{N} = \frac{(1.1-1)e\text{-}3}{20{,}000} \text{ case (1)} = 5e\text{-}9 \text{ case (1)} \\ = \frac{(1.1-1)\,e\text{-}2}{1000} \text{ case (2)} = 10^{-6} \text{ case (2)} \end{cases}$$

Summary

For $a_0 = 1\,mm$: $\Delta K = 5.6\,MPa\sqrt{m}$ $\quad \frac{da}{dN} = 5\times10^{-9}\ \frac{m}{cycle}$

For $a_0 = 10\,mm$ $\quad \Delta K = 17.72\,MPa\sqrt{m}$ $\quad \frac{da}{dN} = 10^{-6}\ \frac{m}{cycle}$

$$\log \frac{da}{dN} = \log C + m \log \Delta K \longleftarrow$$

plug these values in

$$\begin{aligned} -8.3 &= \log C + .748\,m \quad \text{case (1)} \\ -6 &= \log C + 1.248\,m \quad \text{" (2)} \end{aligned} \Longrightarrow$$

$$\boxed{m = 4.6\ , \quad C = 1.82\times10^{-12}\ \frac{m}{(MPa\sqrt{m})^{4.6}}}$$

What Forman's correction

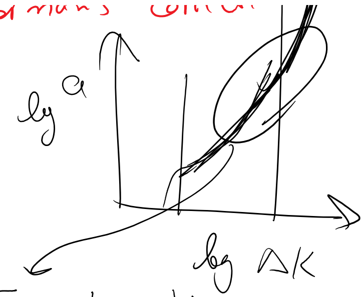
What if we used Forman's correction

$$\frac{da}{dN} = \frac{C\,\Delta K^m}{[(1-R)K_c - \Delta K]} \longrightarrow \text{Forman's correction}$$

Again to get m & C we need to

Forman's correction

Again to get m & C we need to equations



Forman's model makes transition between medium & fast crack growth regions smoother

$$\underbrace{[(1-R)\, K_c - \Delta K]}\frac{da}{dN} = C \Delta K^m$$

Case 1

$$[(1-.5)\times 60 - 5.6]\; \underbrace{5e\text{-}9} = C \Delta K^m$$

Case 2

$$[(1-.5)\times 60 - 17.72] \times 10^{-6} = C \Delta K^m$$

take the log of two equations to get

$$\left.\begin{aligned} -6.914 &= \log C + 0.748m \\ -4.911 &= \log C + 1.248m \end{aligned}\right\} \Longrightarrow \boxed{\begin{aligned} C &= 1.22 \\ m &= 4.006 \end{aligned}}$$

< 4.6 without Forman's correction

Probabilistic Fatigue models:

$$N_f = \frac{2}{(m-2)\, C Y^m (\Delta\sigma)^m \pi^{m/2}} \left[\frac{1}{(a_i)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}}\right]$$

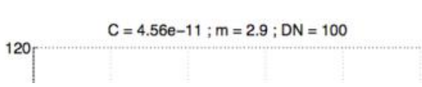
D

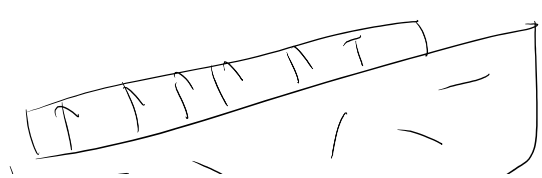
Fatigue life for $m > 2$

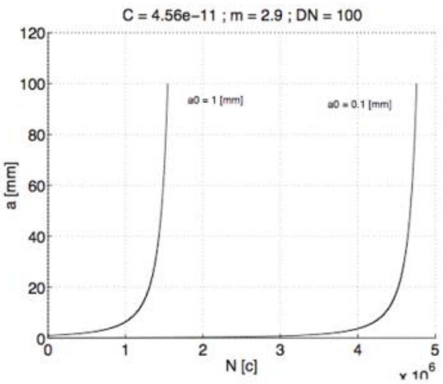
$K_{IC} = \sigma_{max} Y \sqrt{\pi a_f} \longrightarrow$ we get a_f from here

$$\boxed{N_f(a_i) = D\left[\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a_f^{\frac{m}{2}-1}}\right]}$$

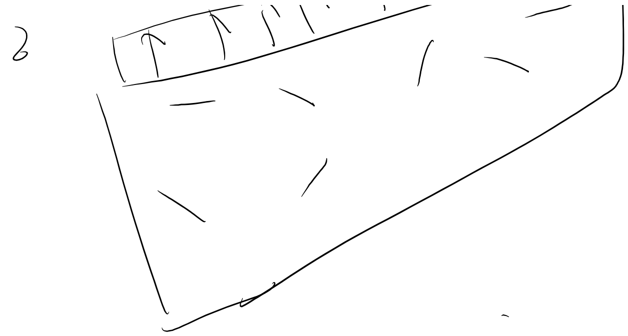
Before we saw significance dependency of fatigue life on initial crack length:







2

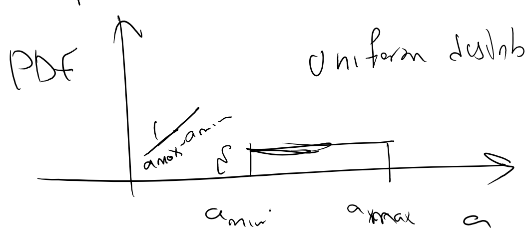


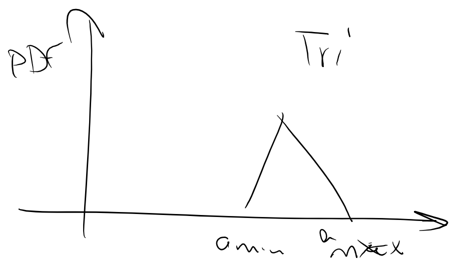
- initial crack distribution is given
- Assume they do not interact with each other
- All cracks experience the same $\Delta\sigma$

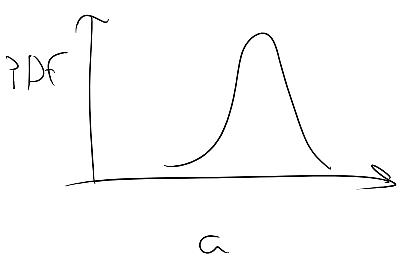
Under these assumptions fatigue life for an initial crack length of ai is given by:

$$\boxed{N_f(a_i) = D\left[\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a_f^{\frac{m}{2}-1}}\right]}$$

Sample Distributions for initial crack







Q1: What is the expected value of fatigue life

$$\underbrace{E(f(a_i))}_{\text{"mean value of fatigue life"}} = \int_{-\infty}^{+\infty} \underbrace{\rho(a_i)}_{\text{Probability density function}} f(a_i)\, da_i$$

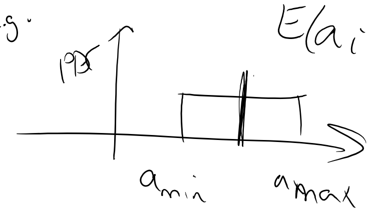
fatigue life"

Probability density function

$$\underbrace{E(a_i)}_{\text{average crack length}} = \int_{-\infty}^{+\infty} \rho(a_i)\, a_i \, da_i$$

$f(a_i) \cdot a_i$

e.g.



$$E(a_i) = \frac{a_{min} + a_{max}}{2}$$

$$E(N_f) = E(N_f(a_i)) = \int_{-\infty}^{+\infty} \rho(a_i)\, \underbrace{N_f(a_i)}\, da_i$$

$$= \int \underbrace{\left(\frac{1}{a_{max} - a_{min}}\right)}_{\rho(a_i) \text{ for uniform distribution}} \left\{ D\left(\frac{1}{a_i^{m/2-1}} - \frac{1}{a_f^{m/2-1}} \right) \right\} da_i$$

$$\underbrace{E(N_f)}_{\text{average life}} \overset{?}{\neq} \underbrace{N_f(E(a_i))}_{\text{life corresponding to average initial crack length}}$$

not the case

this point & the need to come up with confidence levels in fatigue life prediction $\Longrightarrow$ Probabilistic approaches are the right models for fatigue analysis (& ~~for~~ many failure analysis)

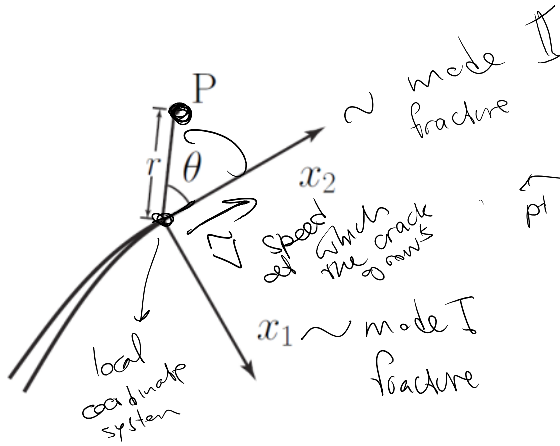
↓

quasi-brittle failure analysis

↓ e.g. quasi-brittle failure analysis

9. Dynamic fracture mechanics and rate effects

9.1. LEFM solution fields

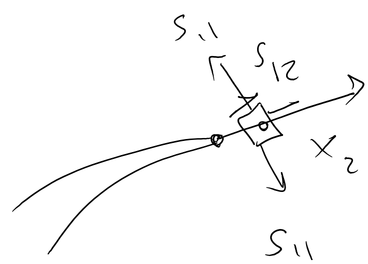


$$\underbrace{S^{ij}}_{\text{at pt } P}(r,\theta) = \frac{K_I(t)}{\underbrace{\sqrt{2\pi r}}_{r \text{ dependency}}} \underbrace{\Sigma_I^{ij}}_{\text{angle dependency}}(\theta, \underbrace{\hat{V}}_{\text{crack speed}}) + \frac{K_{II}(t)}{\sqrt{2\pi r}} \Sigma_{II}^{ij}(\theta, \hat{V})$$

in Dynamic LEFM Σ^{ij} depend on θ & $\hat{V}$

$$K_I(t) = \lim_{x_2 \to 0} \sqrt{2\pi x_2}\, S^{11}(x_2, \underset{\theta}{0}, t)$$

$$K_{II}(t) = \lim_{x_2 \to 0} \sqrt{2\pi x_2}\, S^{12}(x_2, 0, t)$$



Expressions for angle and crack speed dependent functions:

$$\Sigma_I^{11} = -\frac{1}{D}\left\{(1+\alpha_{II}^2)^2\frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - 4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_I^{12} = \frac{2\alpha_I(1+\alpha_{II}^2)}{D}\left\{\frac{\sin\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - \frac{\sin\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_I^{22} = \frac{1}{D}\left\{(1+\alpha_{II}^2)(1+2\alpha_I^2-\alpha_{II}^2)\frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - 4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_{II}^{11} = \frac{2\alpha_{II}(1+\alpha_{II}^2)}{D}\left\{\frac{\sin\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - \frac{\sin\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_{II}^{12} = \frac{1}{D}\left\{4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - (1+\alpha_{II}^2)^2\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_{II}^{22} = -\frac{2\alpha_{II}}{D}\left\{(1+2\alpha_I^2-\alpha_{II}^2)\frac{\sin\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - (1+\alpha_{II}^2)\frac{\sin\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\}$$

$$\underset{1 \text{ or } 2}{\alpha_{(k)}} = \sqrt{1 - \left(\frac{\hat{V}}{c_{(k)}}\right)^2}$$

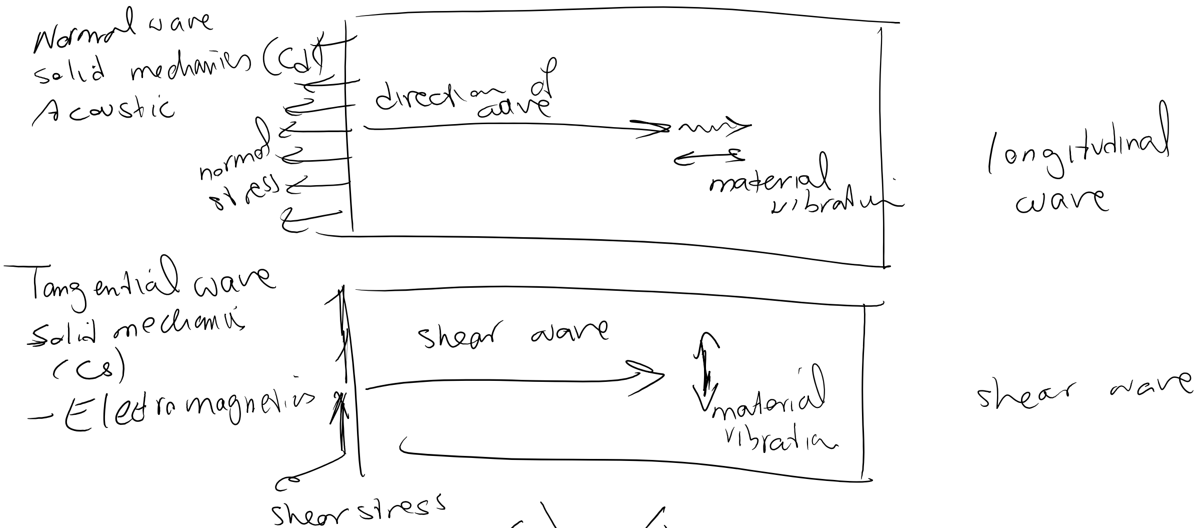
$$\gamma_{(k)} = \sqrt{1 - \left(\frac{\hat{V}\sin\theta}{c(k)}\right)^2}$$

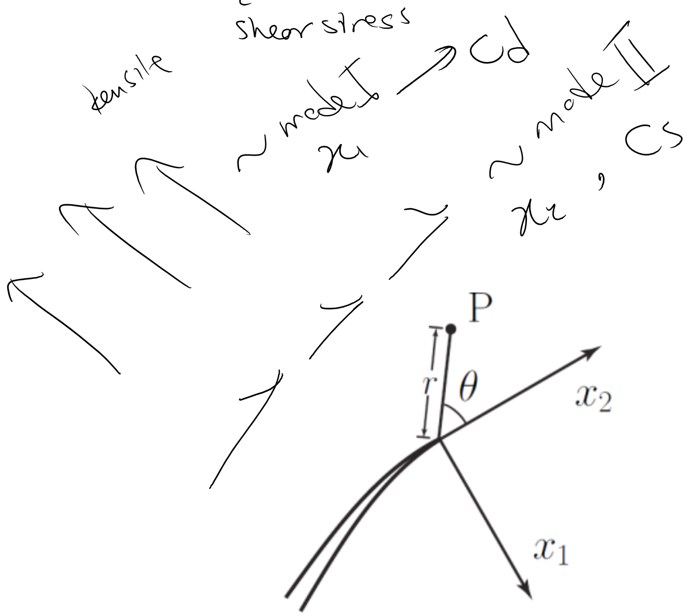
$$\operatorname{tg}\theta_{(k)} = \alpha_{(k)} \operatorname{tg}\theta \qquad k = 1, 2$$

$$D = 4\alpha_I\alpha_{II} - (1+\alpha_{II}^2)^2$$

$$c_{(1)} = C_d = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$ longitudinal wave speed

$$c_{(2)} = C_s = \sqrt{\frac{\mu}{\rho}}$$ shear wave speed





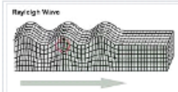
$$D(\hat{v}) = 4\alpha_{I}(\hat{v})\alpha_{II}(\hat{v}) - (1+\alpha_{II}^2)^2$$

$$D(\hat{v}) = 0$$ what does it correspond to?

$D \to 0$ $\quad s^{ij} \to \infty$!

For what crack speed $D \to 0$?

Rayleigh wave speed



Picture of a Rayleigh wave.

shear wave speed

$$c_R / c_S = \frac{0.862 + 1.14\nu}{1 + \nu}.$$

Approximate eqn for Rayleigh wave speed

[Rayleigh wave]

Rayleigh wave speed