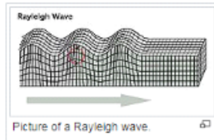
2018/11/26

Monday, November 26, 2018 11:33 AM

$$c_R/c_S \approx \frac{0.862+1.14\nu}{1+\nu}.$$



Picture of a Rayleigh wave.

$$\Sigma_I^{11} = \frac{1}{D}\left\{(1+\alpha_{II}^2)^2\frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - 4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

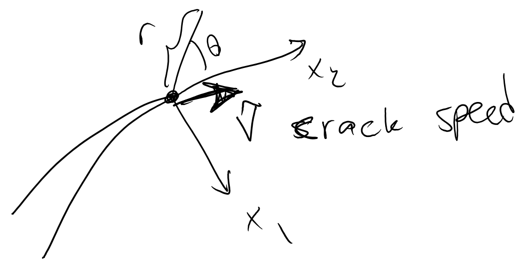
$$s^{ij} = \frac{K_I}{\sqrt{2\pi r}}\,\tilde{\Sigma}^{ij}(\theta, \hat{v})$$

Σ's are divided by $D(\hat{v})$

and $D(\hat{v}) \rightarrow 0$

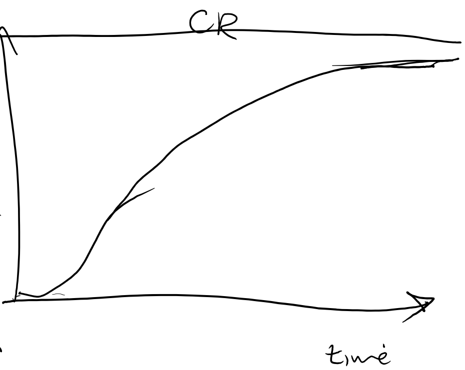
as $\hat{v} \rightarrow c_R$

Rayleigh wave speed

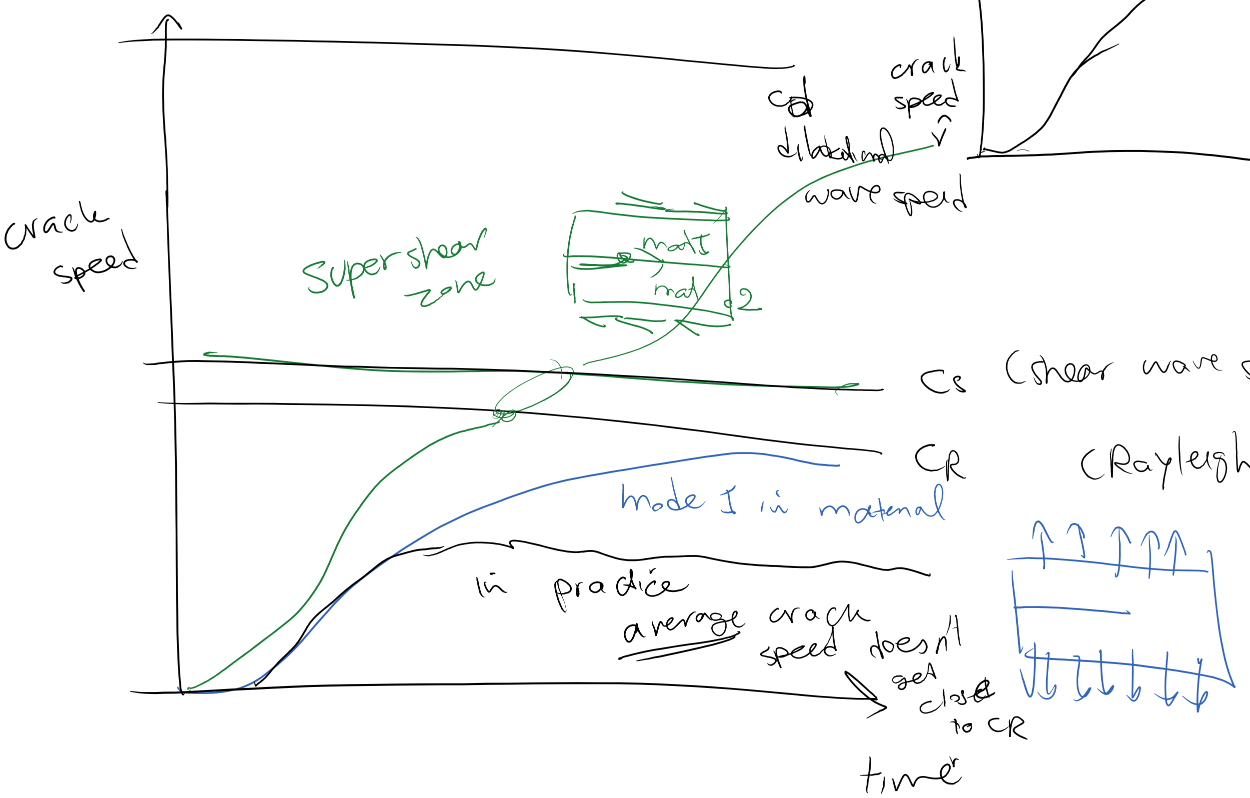


so $\tilde{\Sigma}^{ij}(\theta, \hat{v}) \rightarrow \infty$

as $\hat{v} \rightarrow C_R$

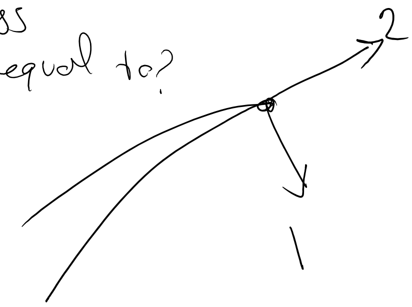


For one material under mode I loading, maximum possible crack speed is Rayleigh wave speed when then angular functions tend t o infinity.



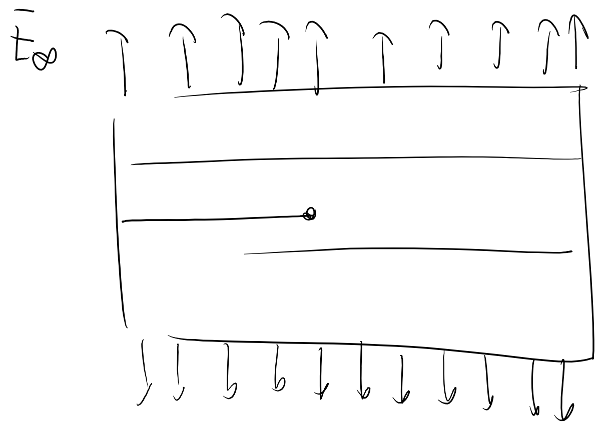
Steps for solving a dynamic LEFM problem

1. $K_{(k)} = ?$ what is stress intensity factor equal to?



(k): I or 1 for mode I

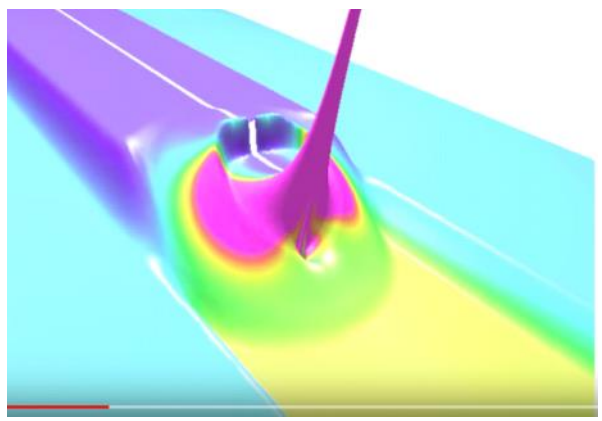
II or 2 " " II

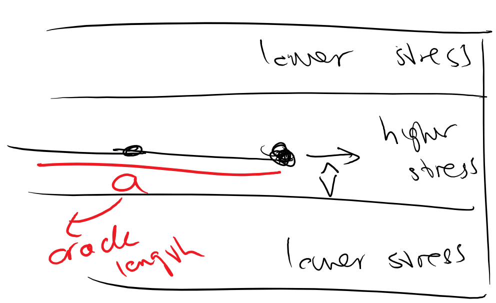


example

infinite domain

for example we want to calculate K at this time





$$K_{(k)}(t, a, \hat{v}) = k_{(k)}(\hat{v})\, \underbrace{K_{(k)}(t, a, \underset{\downarrow}{0})}_{\text{quasi\_static problem!}}$$

0 = zero crack speed

Basically get $K_{(k)}(t, a, 0)$

from quasi-static & multiply it by $K_{(k)}(V)$

$K_{(k)}(\hat{V})$ = stationary -to dynamic SIF factor

Universal function (independent of loading, geometry, ...)

$$K_{(k)}(\hat{V}) \approx \frac{\left(1 - \frac{\hat{V}}{c_R}\right)}{\sqrt{1 - \frac{\hat{V}}{c_{(k)}}}}$$

$c_{(1)} = c_d$

$c_{(2)} = c_s$

$$K_I(\hat{V}) \approx \frac{\left(1 - \frac{\hat{V}}{c_R}\right)}{\sqrt{1 - \frac{\hat{V}}{c_d}}}$$

$$K_{II}(\hat{V}) \approx \frac{\left(1 - \frac{\hat{V}}{c_R}\right)}{\sqrt{1 - \frac{\hat{V}}{c_s}}}$$

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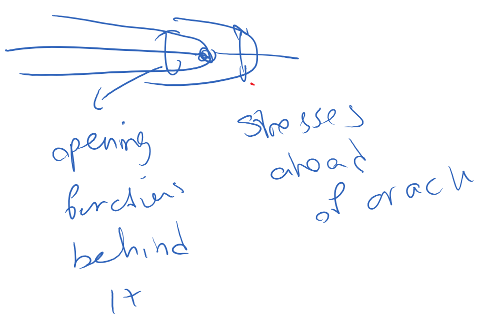
2. Energy release rate in dynamic fracture:

Recall $G \sim K$



in static case we obtained

$$G = \frac{1-\nu}{2\mu}\left[K_I^2 + K_{II}^2\right]$$

In dynamic case for the same K's more energy is release ($\Sigma \to \infty$ as $\hat{V} \to c_R$)

$$G = \frac{1-\nu}{2\mu}\left[A_I(\hat{V})\, K_I^2 + A_{II}(\hat{V})\, K_{II}^2\right]$$

$$G = \frac{1-\nu}{2\mu}\left[A_I(V)\,K_I + A_{II}(V)\,K_{II}\right]$$

$A_I(\hat{V})$, $A_{II}(\hat{V})$ are universal functions depending on $\hat{V}$

$\hat{V} \to 0 \qquad A_{I}, A_{II}(\hat{V}) \to 1$

Rayleigh speed limit $\quad (\hat{V} \to c_R)$

$$A_{(K)} = O\left(\left(c_R - \hat{V}\right)^{-1}\right)$$

$$A_{(I)} \;\&\; A_{(II)} \propto \frac{1}{c_R - \hat{V}}$$

as $\hat{V} \to c_R \qquad A_I \;\&\; A_{II} \to \infty$

Dynamic <u>Griffith criterion</u>:

When does a crack propagate?

$G = R$ (resistance) crack can grow

also called Γ_0 (similar to K_{IC} is called fracture toughness)

$G(\hat{V}) = \Gamma_0(\hat{V})$ typically $\Gamma_0(\hat{V}) \nearrow$ as $\hat{V} \nearrow$

but for many applications $\Gamma_0(\hat{V}) \approx \Gamma_0(\hat{V}=0)$ is used(some error is involved)

$$\boxed{\begin{array}{l} K_{(k)}(t, a, \hat{v}) = k_{(K)}(\hat{v})\, K_{(K)}(t, a, 0) \\ G = \dfrac{1-\nu}{2\mu}\left[A_I(\hat{v})\, K_I^2 + A_{II}(\hat{v})\, K_{II}^2 \right] \\ G = \Gamma_0(\hat{v}) \quad \text{for crack propagation} \\ \text{Equations for dynamic crack propagation} \end{array}}$$

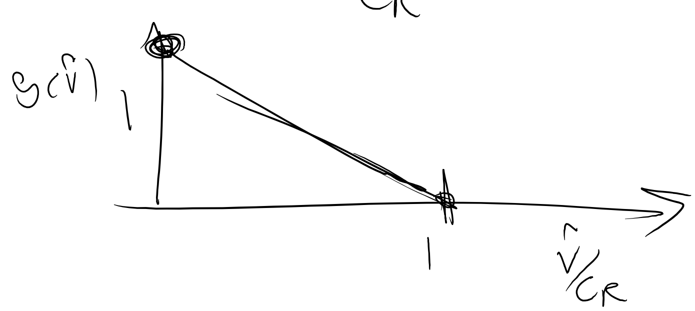
For mode I

independent from $\hat{v}$

$$\frac{1-\nu}{2\mu}\underbrace{\left(\overset{A_I}{A(\hat{v})}\ \overset{k_I}{k^2(\hat{v})}\right)}\ \underbrace{K^2(t,a,0)}_{\text{quasi-static } K} = \Gamma_0$$

$g(\hat{v}) = A(\hat{v})\, k^2(\hat{v})$ is a universal function that is very accurately approximated by,

$$g(\hat{v}) \approx 1 - \frac{\hat{v}}{C_R} \quad \text{for} \quad 0 \leq \hat{v} \leq c_R$$

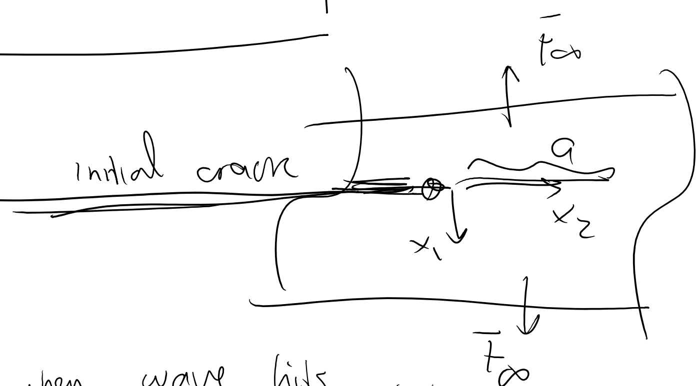


(**)
$$\boxed{\begin{gathered}\underline{\frac{1-\nu}{2\mu}\, g(\hat{v})\, K^2(t,a,0) = \Gamma_0} \\ g(\hat{v}) \approx 1 - \frac{\hat{v}}{c_R}\end{gathered}}$$

mode I

crack propagation

Sample problem



$$\bar{\sigma} = 2\,\bar{t}_\infty$$

experienced on crack line

relative to when wave hits crack line)

crack speed

$$K(t, \underbrace{a}_{\text{(new) crack length}}, 0) = C\sqrt{2\pi \underbrace{c_d}_{\text{dilatational wave speed}}\, t}\;\bar{\sigma}$$

$$C = \frac{\sqrt{2(1-2\nu)}}{\pi(1-\nu)}$$

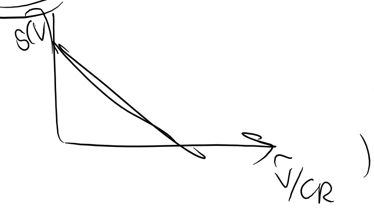
plugging K in ** we get

$$g(\hat{v}) = \frac{\mu\,\Gamma_0}{(1-\nu)\,\pi c_d\,(C\bar{\sigma})^2\, t}$$

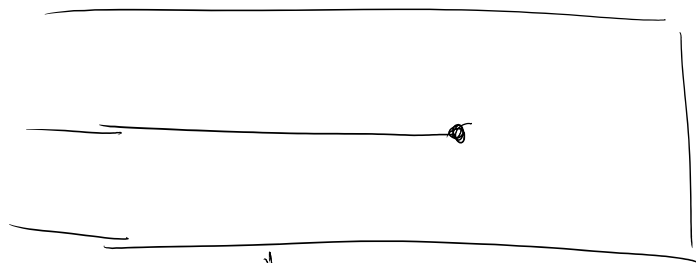
time scale

$$\boxed{\underbrace{g(\hat{v})}_{\in[0,1]} = \frac{\tau_0}{t} \qquad \tau_0 = \frac{\mu \Gamma_0}{(1-\nu)\,\pi c_d (C\bar{\sigma})^2} = \frac{\pi \mu \Gamma_0 c_d}{4C(C_s\bar{\sigma})^2}}$$

$$g(\hat{v}) \approx \underbrace{1 - \frac{\hat{v}}{c_R}}$$

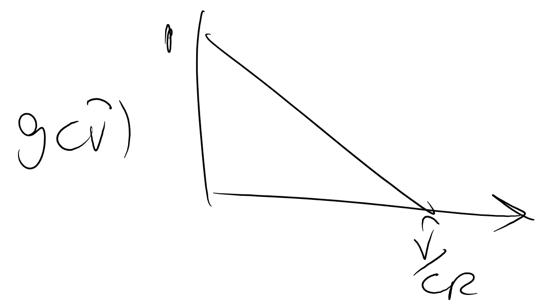


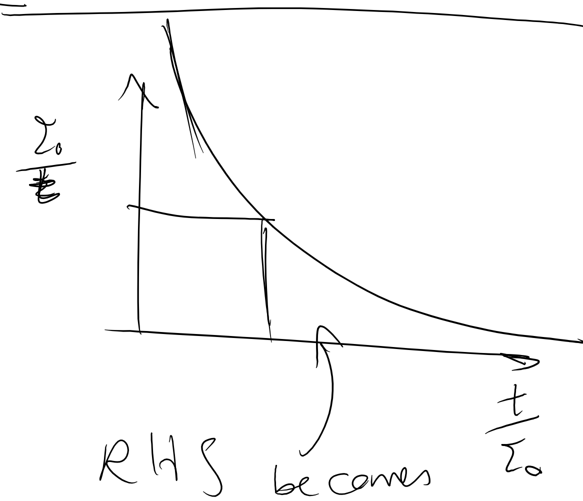
$\hat{v} = 0 \quad g(\hat{v}) = 1$



$$\frac{\tau_0}{t} \in [\infty, 1]$$

$t = 0^+, \ \tau_0$





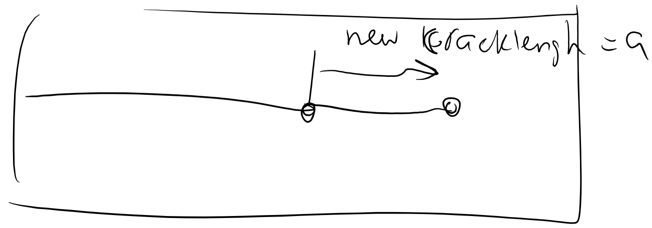
Crack can propagate when RHS becomes $\underline{\le 1}$

$$\frac{\tau_0}{t} = 1 \implies \boxed{t_{\text{initiation}} = \tau_0}$$

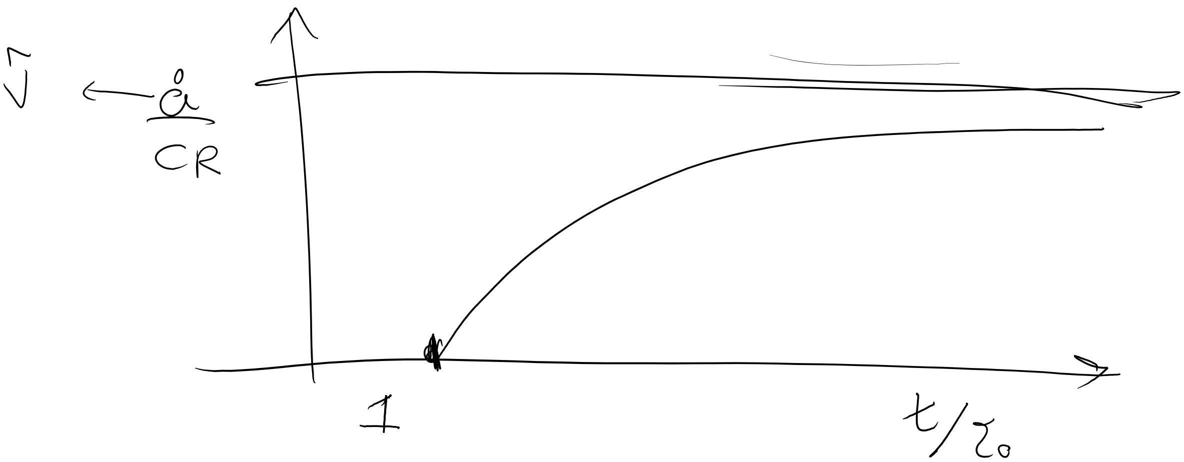
After that

$$g(\hat{v}) \approx 1 - \frac{\hat{v}}{c_R} = \frac{\tau_0}{t}$$

$$\hat{V} = \frac{da}{dt} = \dot{a}$$



$$1 - \frac{\dot{a}}{c_R} = \frac{\tau_0}{t} \Longrightarrow \boxed{\frac{\dot{a}}{c_R} = 1 - \frac{\tau_0}{t}}$$



$$1 - \frac{\dot{a}}{c_R} = \frac{\tau_0}{t}$$

Integrate this to get a,

$$\boxed{\frac{a}{c_R \tau_0} = \frac{t}{\tau_0} - 1 - \ln\left(\frac{t}{\tau_0}\right)}$$

Some other features of Dynamic fracture

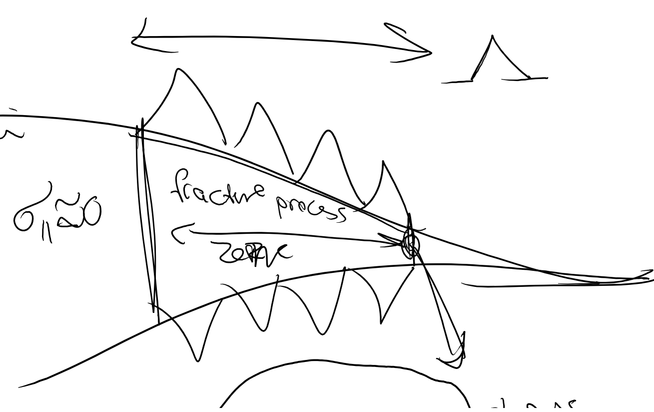
- Static estimates

$$\Lambda = S\pi \frac{\mu}{1-\nu} \frac{\tilde{\Phi}}{\tilde{\sigma}^2}$$

$\tilde{\Phi}$ → fracture toughness or work of separation

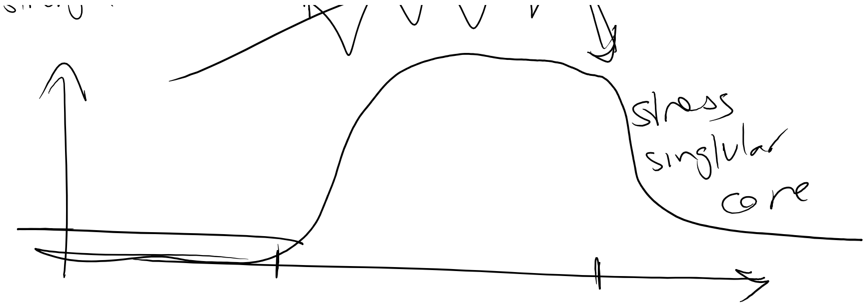
$\tilde{\sigma}$ → material strength

$$S = \begin{cases} \frac{1}{4} & \text{Dugdale model} \\ 1 & \end{cases}$$



$S = \begin{cases} 4 & \text{model} \\ \frac{1}{16} & \text{Potential based cohesive model} \end{cases}$

stress

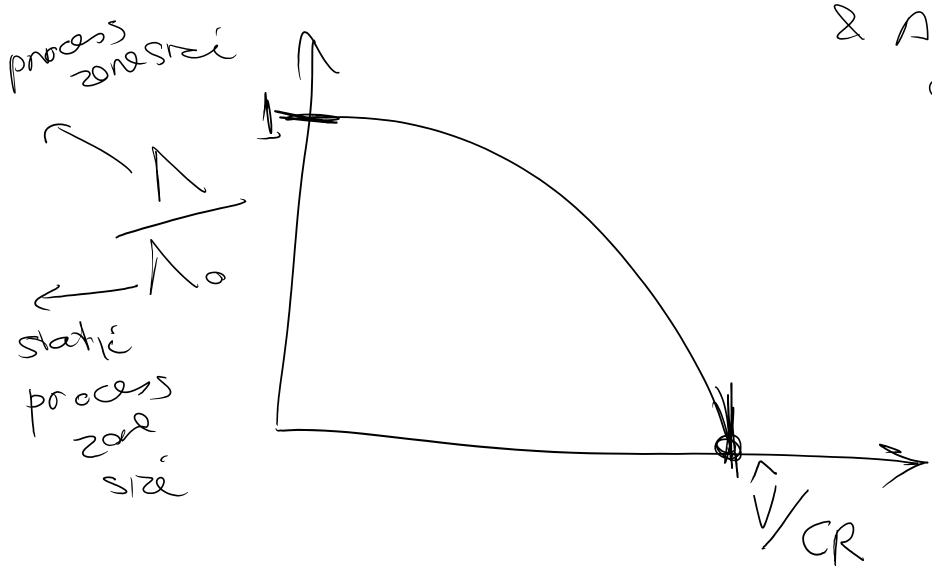


$\Lambda \longrightarrow O \quad \text{as} \quad \hat{V} \longrightarrow C_R$

static process zone size

$$\Lambda(\hat{V}) = \frac{\Lambda_0}{A(\hat{V})}$$

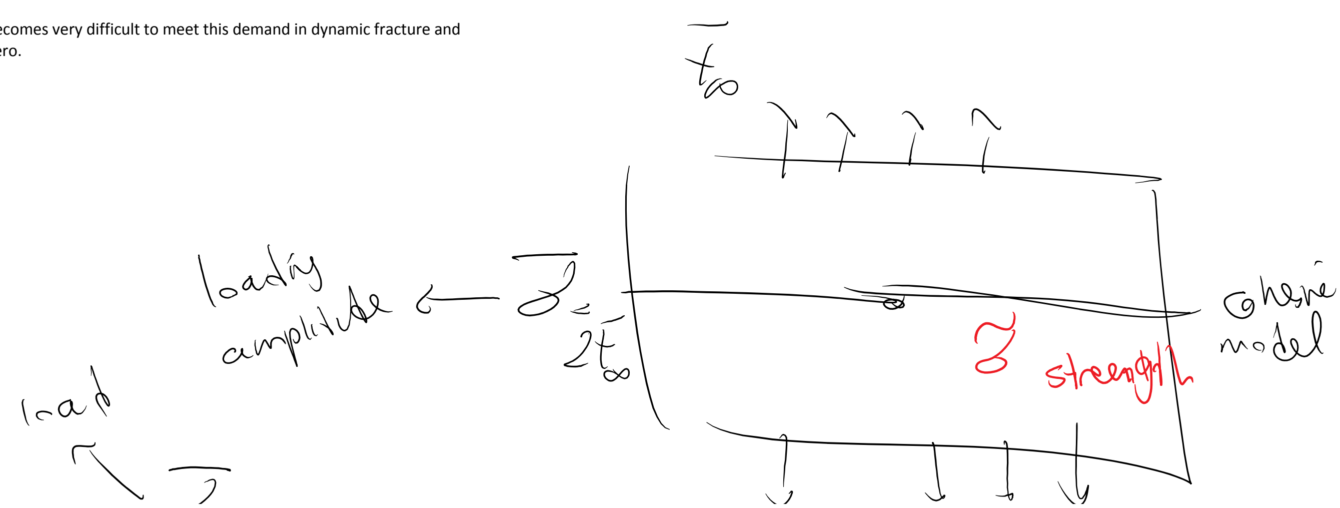
$A(\hat{V}) \to \infty$ as $\hat{V} \to C_R$ & $A(\hat{V}) = 1$ at $\hat{V} = 0$



Suggested value: Use at least 4 to 10 elements in fracture process zone.

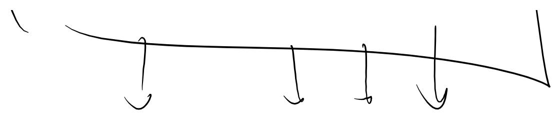
Challenge: It becomes very difficult to meet this demand in dynamic fracture and FP shrinks to zero.

Question:



low

$$\sigma' = \frac{\bar{\sigma}}{\tilde{\sigma}} \rightarrow \text{small}$$



strength

LEFM is a good theory

normalized load

static LEFM

process zone size

$$\frac{\Lambda}{r_s} \propto \left(\frac{\bar{\sigma}}{\tilde{\sigma}}\right)^2$$

singular dominant zone

Same thing holds in dynamics