2018/11/28

Wednesday, November 28, 2018 11:30 AM

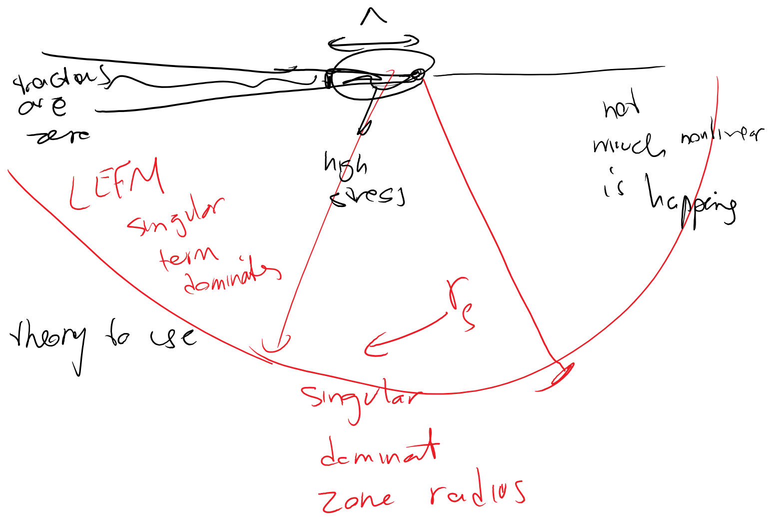
SSY in dynamic fracture

Λ is fracture process zone size

$\Lambda \ll L_i$

L_i relevant length scales

→ LEFM is a good theory to use



$\Lambda(\hat{v})$

one relevant length scale (L_i) is r_s

$$\frac{\Lambda(\hat{v})}{r_s(\hat{v})}$$

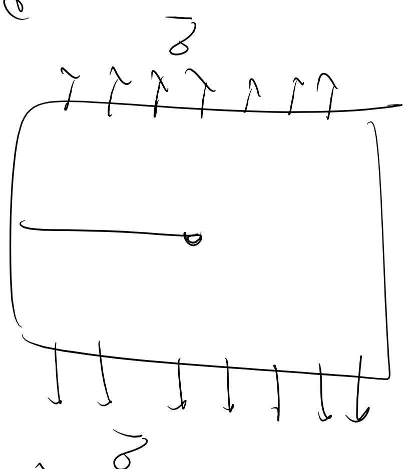
how this changes by crack tip

applied load

$$\frac{\Lambda(\hat{v})}{r_s(\hat{v})} \propto \left(\frac{\sigma}{\sigma_y}\right)^2$$

very insensitive to crack speed

material strength (e.g yield stress



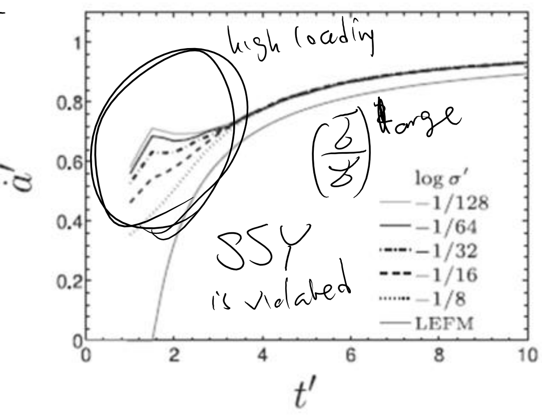
~~al~~although both Λ & r_s are highly dependent on crack speed

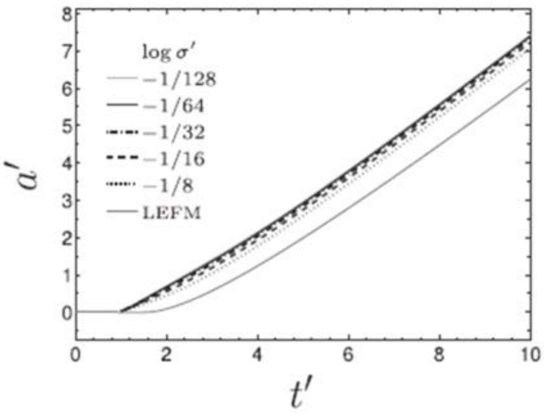
ki

although both ... & r_s are ...

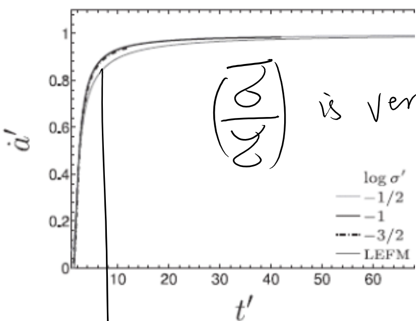
crack speed

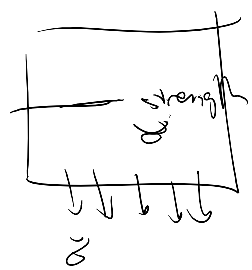
$\dot{a}/c_R$



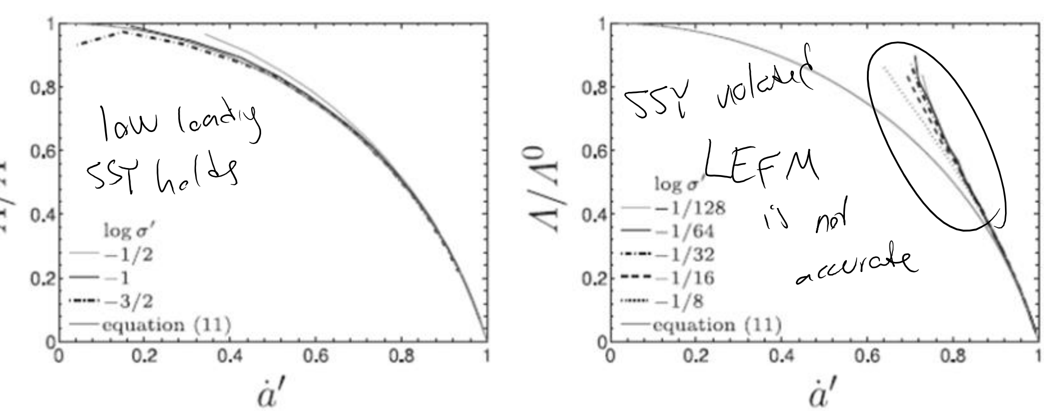


time $\frac{t}{\tau_0}$





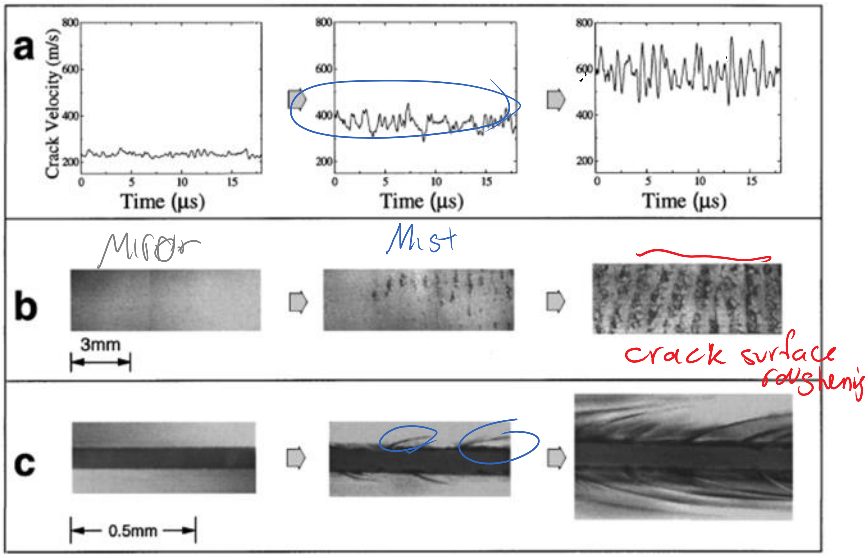
LEFM solution very close to nonlinear solutions



(a) Low-amplitude loading, $\bar{t}_\infty \ll \tilde{\sigma}$. (b) High-amplitude loading, $\bar{t}_\infty \to \tilde{\sigma}^-$.

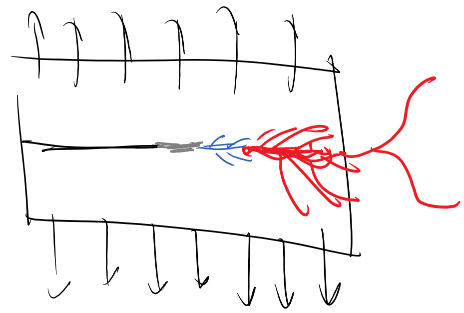
nonlinear responses for high loading

Why we don't get to Rayleigh wave speed limit in practice (mode I in homogeneous material)



$v_C = 340$m/s

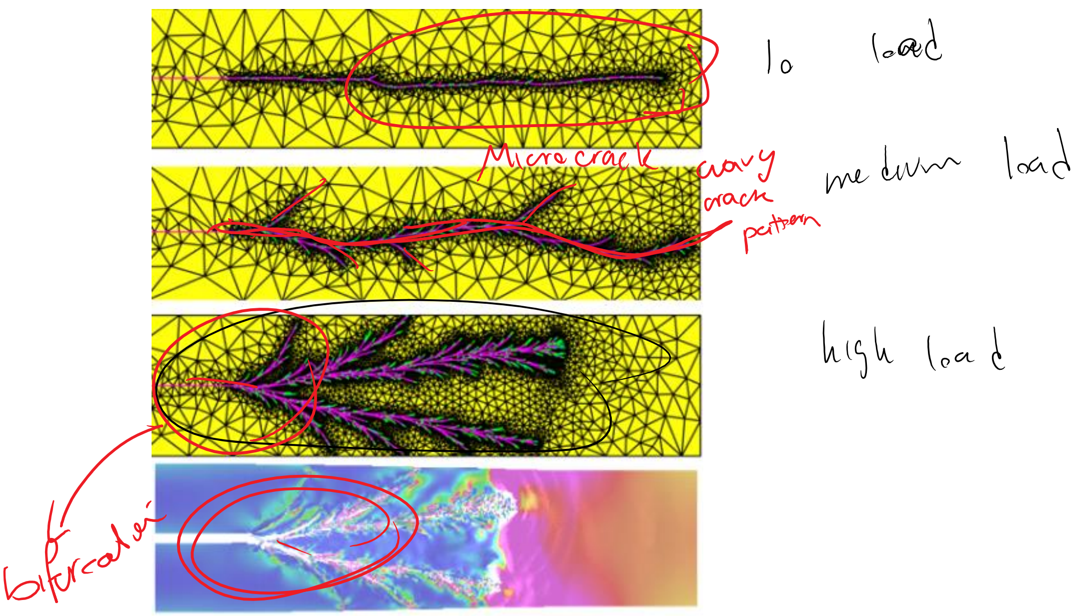
$\boxed{(\text{or } 0.36V_R)}$



V<Vc V≥Vc V>Vc

early stag

crack surface roughening



Gao: 1993: Wavy crack path to have a higher energy release rate

J. Mech. Phys. Solids Vol. 41, No. 3, pp. 457–486, 1993.
Printed in Great Britain.

0022–5096/93 $6.00+0.00

SURFACE ROUGHENING AND BRANCHING INSTABILITIES IN DYNAMIC FRACTURE

HUAJIAN GAO
Division of Applied Mechanics, Stanford University, Stanford, CA 94305, U.S.A.

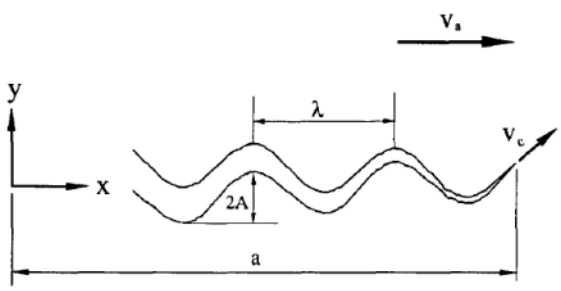


FIG. 1. A cosine wave crack propagating at local velocity v_c and apparent velocity v_a. The fracture surface is roughened with parameters A and λ.

wavy pattern is energetically more favorable at higher crack speeds

Int J Fract (2007) 143:245–271
DOI 10.1007/s10704-007-9061-x

ORIGINAL PAPER

Theory of dynamic crack branching in brittle materials

E. Katzav · M. Adda-Bedia · R. Arias

optai !

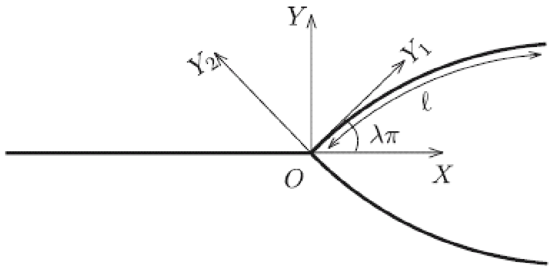
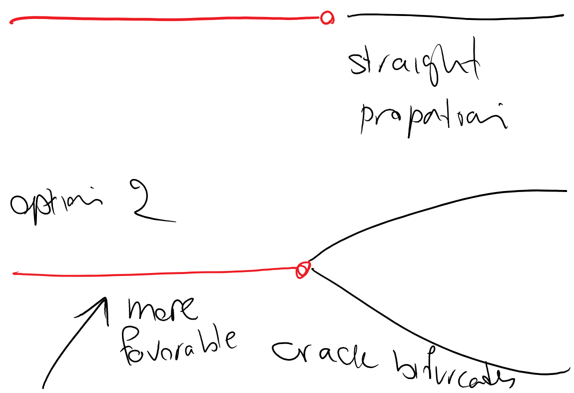


Fig. 1 Schematic representation of a straight crack with two symmetrically branched curved extensions

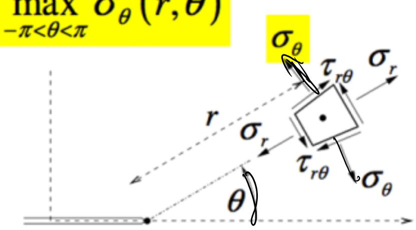


$\hat{V} \geq 30\% - 40\%\ C_R$

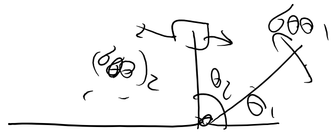
Explanation 3: why crack path becomes more unstable?

Yoffe's instability

$$\sigma_\theta(r,\theta_c) = \max_{-\pi<\theta<\pi} \sigma_\theta(r,\theta)$$



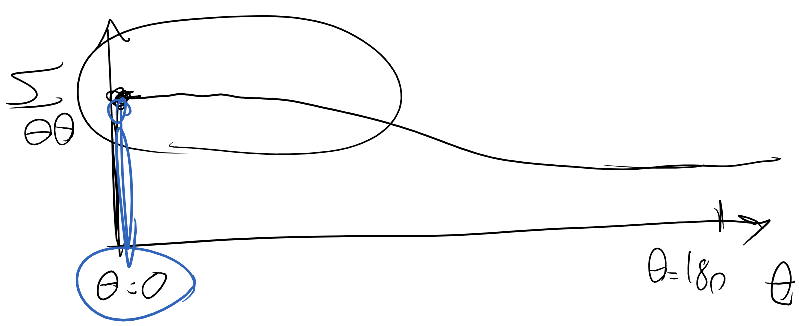
Maximum circumferential criterion

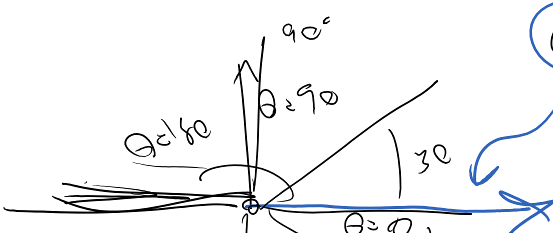


crack propagates in an angle θ where $\sigma_{\theta\theta}$ is maximum

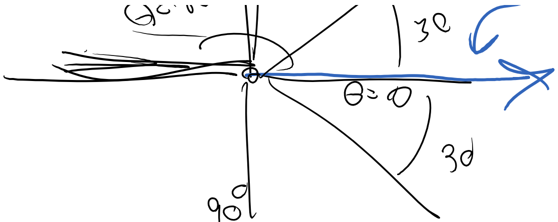
$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \Sigma_{\theta\theta}(\theta, \hat{V})$$

$\hat{V} = 0$

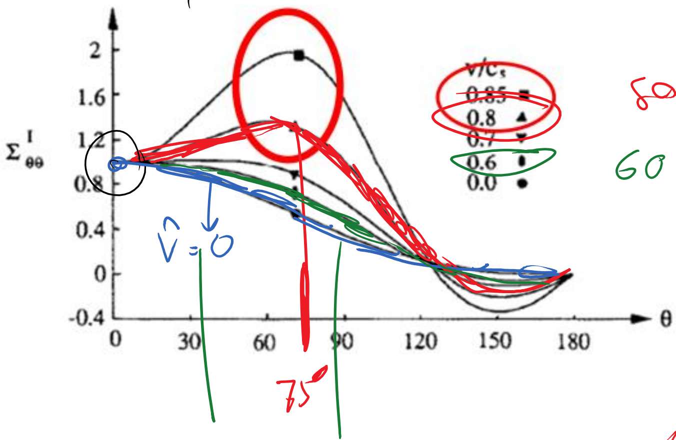




crack prefers to go @ zero

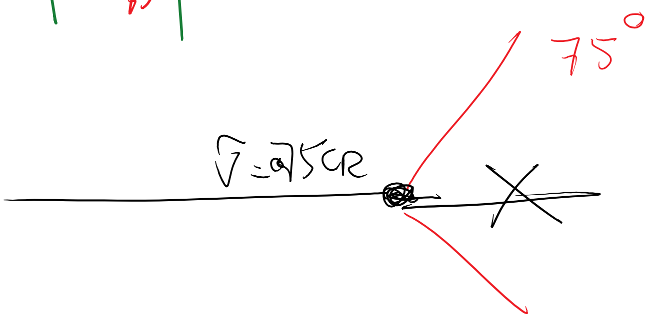


crack prefers to go @ zero degree

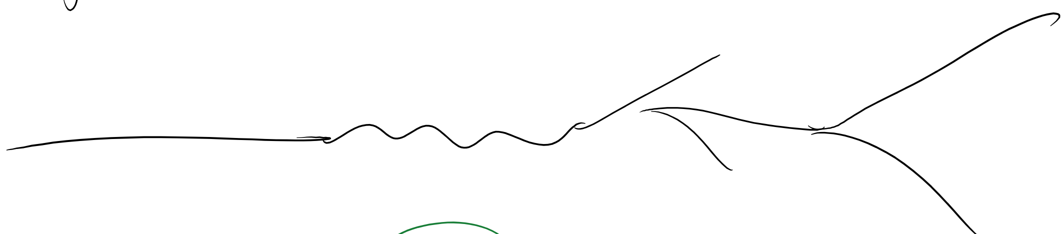


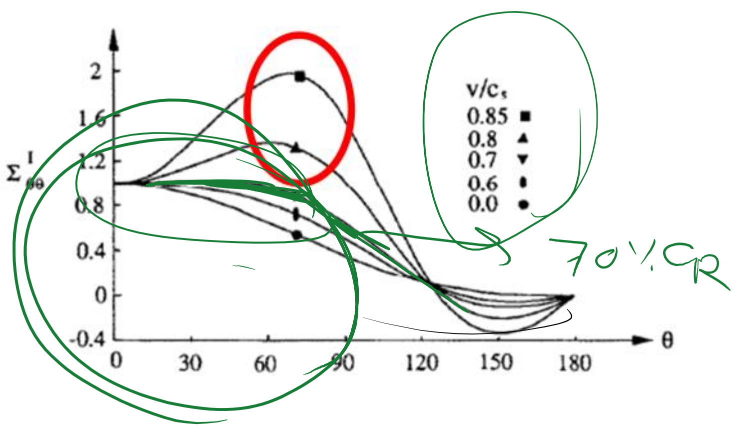
80% Rayleigh speed

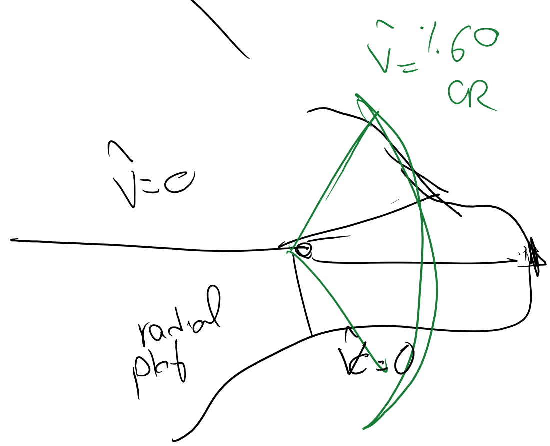
60% Rayleigh wave speed



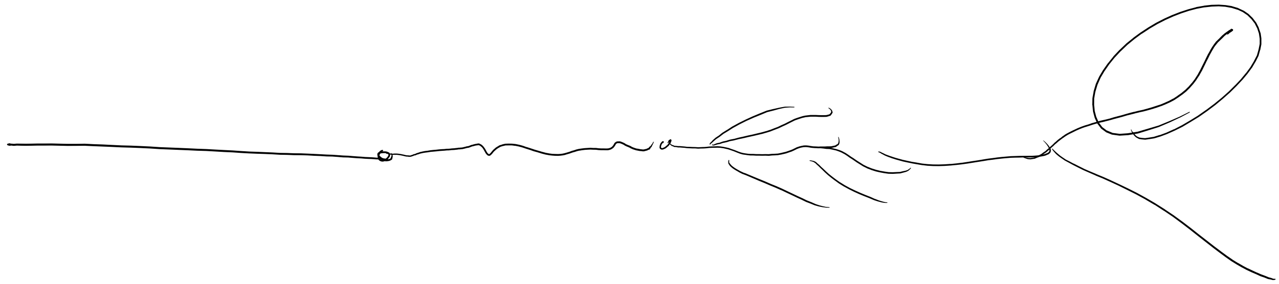
why defects & small variations become more important @ higher speeds ($\hat{V} \to c_R$)



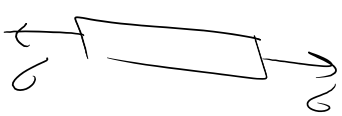


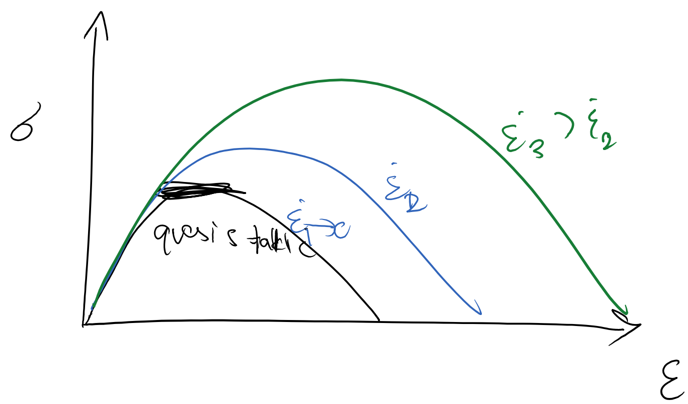


The stress field (sigma_theta,theta) is getting relatively uniform as we get to about 30 -> 70% of cR -> any defect or variation can easily change the crack path (we even start to see crack path oscillations, microcracking, ...) at lower crack speeds.



Rate effect:





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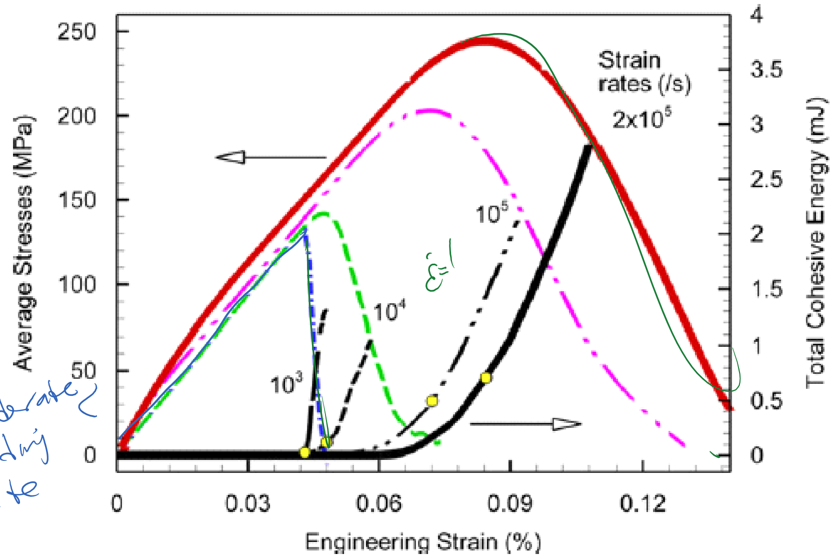
Predicting variability in the dynamic failure strength of brittle materials considering pre-existing flaws

Nitin P. Daphalapurkar [a,*], K.T. Ramesh [a], Lori Graham-Brady [b], Jean-Francois Molinari [c]

[a] The Johns Hopkins University, Department of Mechanical Engineering, Baltimore, MD 21218, USA
[b] The Johns Hopkins University, Department of Civil Engineering, Baltimore, MD 21218, USA
[c] LSMS.—IIS.—E.N.A.C., Ecole Polytechnique Fédérale de Lausanne, 1015, Lausanne, Switzerland



moderate loading rate

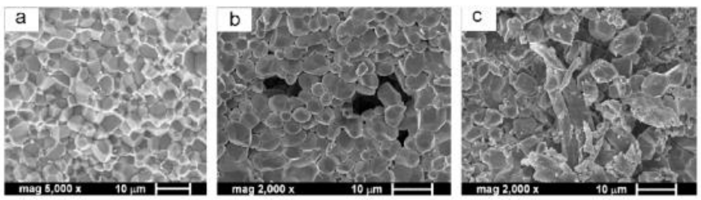
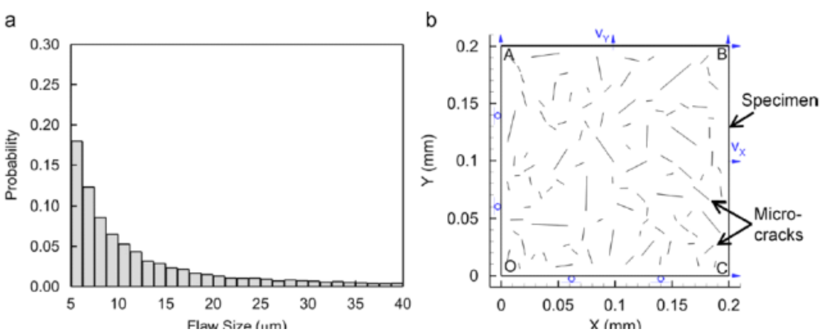
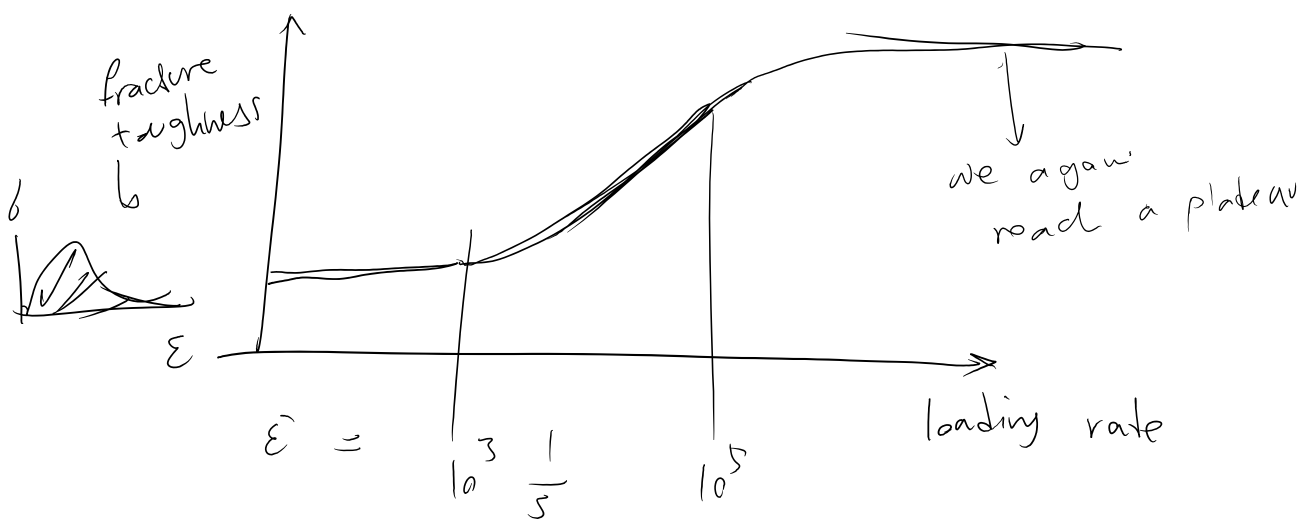


Fig. 1. Scanning electron micrographs of aluminum nitride (average grain size of 6 µm) showing microstructural flaws: (a) interface flaws, (b) pores, and (c) inclusions.





New topic:

Damage models

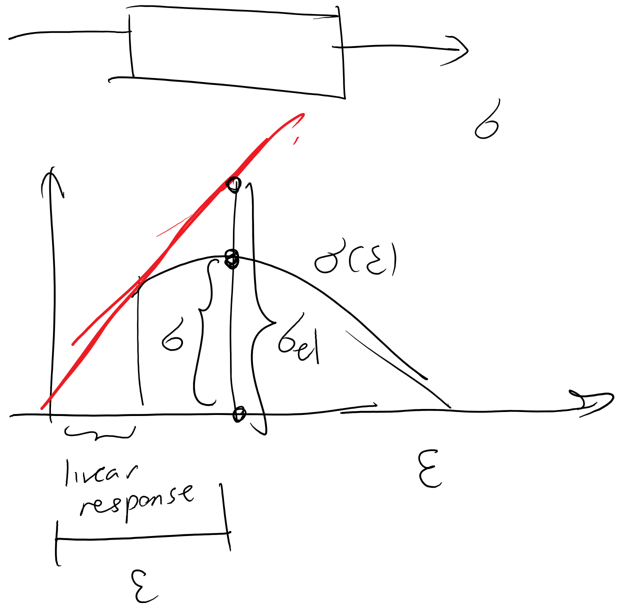
$$\sigma = E\varepsilon$$

$$\sigma(\varepsilon)$$

$$\sigma = (1-D)\,\sigma_{el}$$

stress that we'd have obtained with no dame

$$\sigma_{el} = E\varepsilon$$



$D = 0 \quad \sigma = \sigma_{el}$ no damage

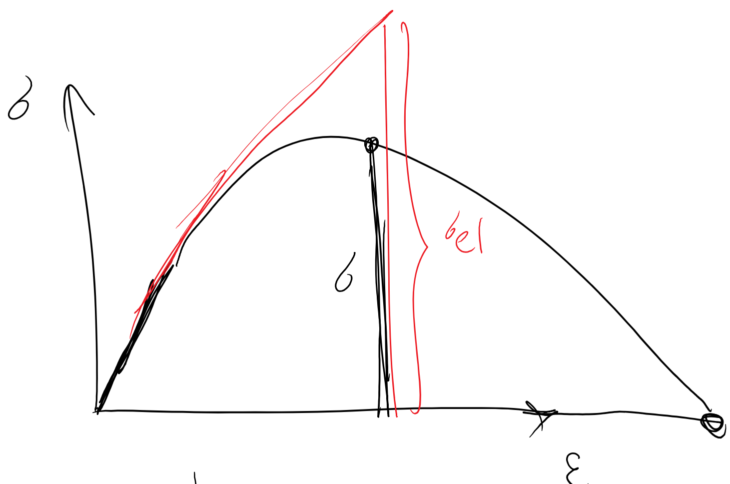
$D = 1 \quad \sigma = (1-1)\,\sigma_{el} = 0$ @ full damage $\sigma = 0$

How to calibrate a Damage model from $\sigma(\varepsilon)$

$$\sigma = (1-D)\,\sigma_{el}$$

$$\Rightarrow D = 1 - \frac{\sigma}{\sigma_{el}}$$

$$D(\varepsilon) = 1 - \frac{\sigma(\varepsilon)}{E\varepsilon}$$



How damage model is used

$$\nabla \cdot \sigma + \rho b = \rho \ddot{u}$$ dynamic

$$\sigma = (1-D)\,\sigma_{el}$$

$$\sigma = (1-D)\,\sigma_{el}$$

$$\sigma_{el} = \mathbb{C}\,\varepsilon \rightarrow \text{strain}$$

$\mathbb{C}$: 4th order elasticity tensor

$u = \bar{u}$