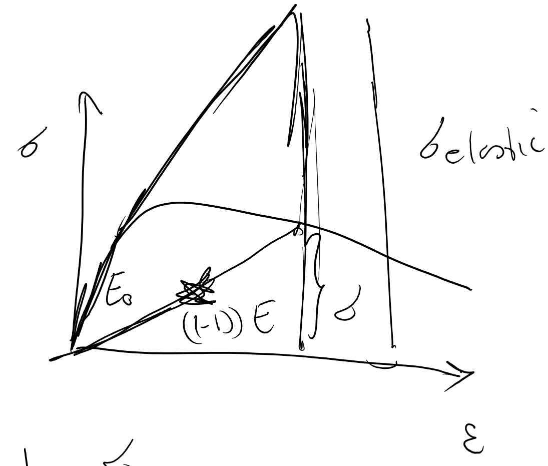
Continue the damage model:

$$\sigma = (1-D)\,\sigma_{el}$$

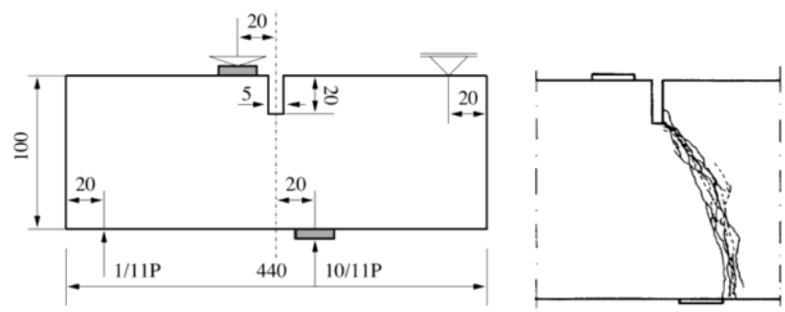
$$L \quad = \left((1-D)\,E\right)\varepsilon$$

Damage parameter degrades the stiffness of material

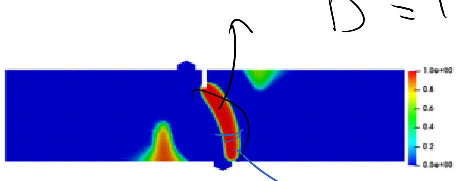


$$D = 1 - \frac{\sigma}{\sigma_E}$$

$$D = D(\varepsilon) \quad \text{or} \quad D(\sigma_{el}) = D(C\varepsilon)$$

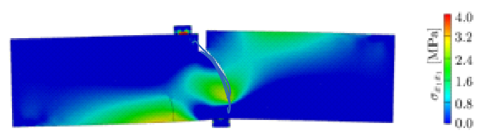


$D = 1$



(a) GED

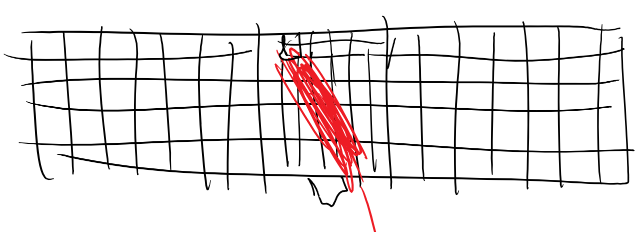
disadvantage
"damage is spread"



sharp crack model

Advantage of damage model:

we don't need to track fractures

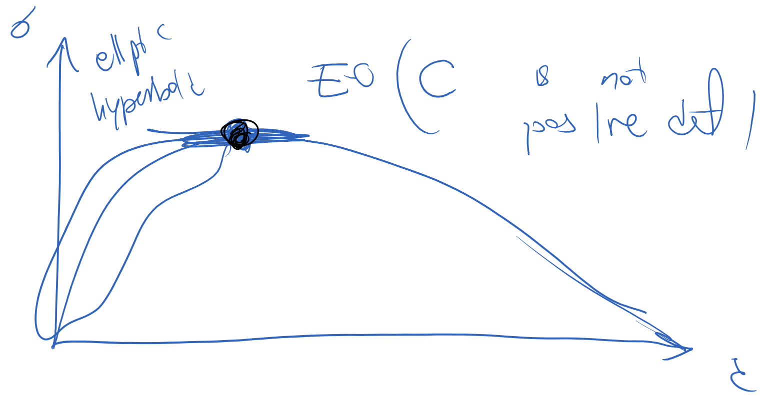


A uniform mesh
" or refined in area of failure"

D gets close to 1!

The more serious problem is mesh dependency!

problem:
is loss of ellipticity (static) or hyperbolicity (dynamic)
fracture problems

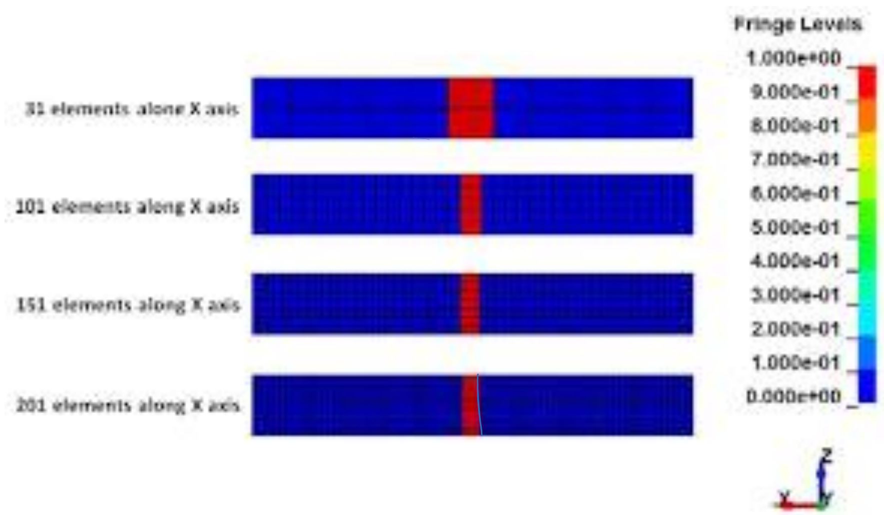


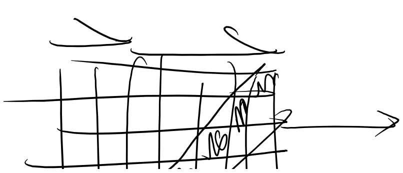
E→0 (C is not pos. def.)

$$\underbrace{\nabla\cdot(C\varepsilon)}_{\nabla\cdot\sigma} + \rho b = \rho\ddot{u} \rightarrow \text{dynamic} \quad \text{hyperbolic}$$

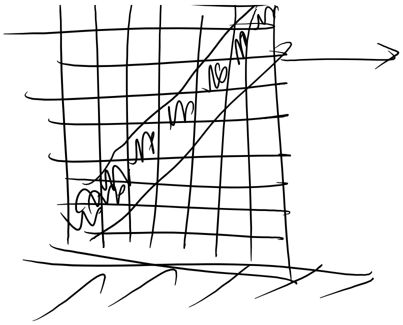
$$0 \quad \text{static} \quad \text{elliptic}$$

Impacts of loss of ellipticity (hyperbolicity) in numerical results when LOCAL DAMAGE models are used:

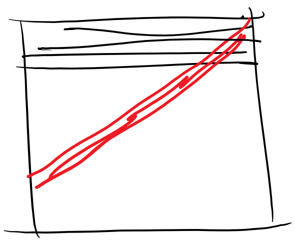




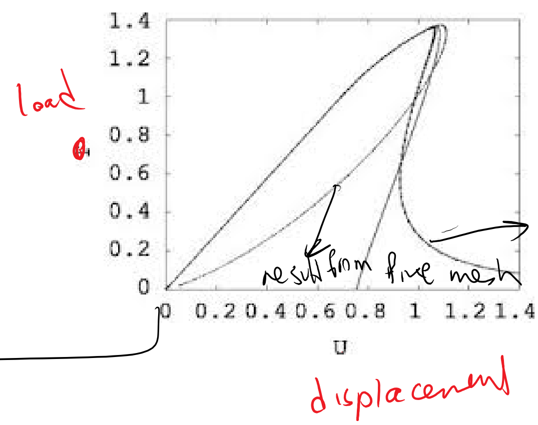
the # elements in the band of



the # elements in the band of localization is almost insensitive to mesh size



sample results of local damage models



coarse mesh

we want to fix this issue & ideally make the thickness of zone of failure mesh independent

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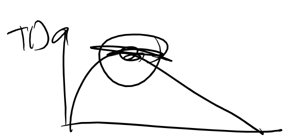
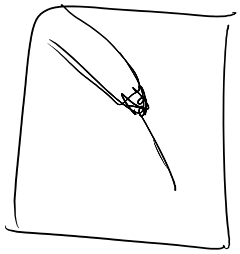
Solutions:

I. Whenever C (elasticity tensor) is no longer positive definite



insert a crack



II. We fix the damage model

- Introduce a length scale to the model

II. i Non-local approach

$$\sigma = (1-D) C \varepsilon$$

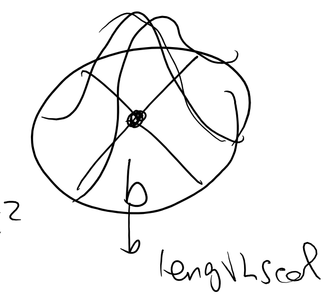
$$D = f(\varepsilon) \longrightarrow D = f(\varepsilon_{eq})$$

local approach

$$\varepsilon_{eq}(x) = \frac{1}{S} \int \varepsilon(y)\, w(y)\, dS$$

we're computing an average of ε in a region of length scale (b)

$S = b\pi^2$



this fixes the problem: Non locallity causes a lot of issues computationally

II. ii Higher order models

$$D = f(\varepsilon) \longrightarrow \left\{ D + \underbrace{b^2 \Delta D} = f(\varepsilon) \right.$$

$$D = f(\varepsilon) \longrightarrow \begin{cases} D + \underbrace{b^2 \Delta D}_{\text{we add higher order derivative}} = f(\varepsilon) & \text{1 way} \\ \varepsilon_{eq} + b^2 \Delta \varepsilon_{eq} = \varepsilon & \end{cases}$$

2nd way

$$D = f(\varepsilon_{eq})$$

II. iii for dynamic problems a length scale can be introduced through a time scale

$$D = f(\varepsilon) \longrightarrow \dot{D} = \frac{1}{\tau}\left[1 - e^{-a\langle f(\varepsilon) - D\rangle_+}\right]$$

$$\mathrm{Max}(\dot{D}) = \frac{1}{\tau}$$