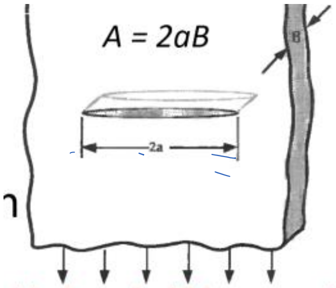
2020/09/02

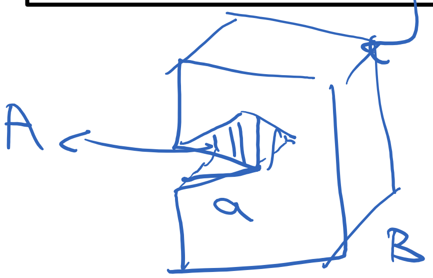
Wednesday, September 2, 2020 2:48 PM

From last time

$$\boxed{2\gamma_s = -\underbrace{\frac{1}{B}\frac{\partial \Pi}{\partial a}} = -\frac{\partial \Pi}{\partial A}}$$



th's through-thickness crack

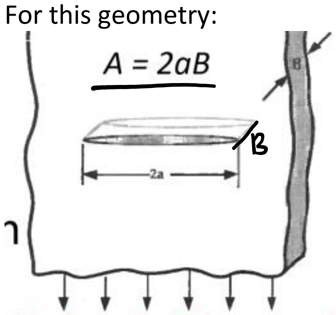


potential energy

$$-\underbrace{\frac{\partial \Pi}{\partial A}}_{\text{crack surface}} = \underbrace{2\gamma_s}_{\text{how much energy is required per unit area of crack}}$$

How much energy released per unit surface of crack advance

For this geometry:



th's through-thickness crack

From theory of elasticity

Inglis

$$\Pi(a) - \Pi(0) = -\frac{\pi a^2 \sigma^2 B}{E}$$

$$\frac{d\Pi}{dA} = \frac{d\left(-\frac{\pi a^2 \sigma^2 B}{E}\right)}{d(2aB)} = -\frac{\pi a \sigma^2}{E}$$

constant

$$-\frac{d\Pi}{dA} = 2\gamma_s$$

σ_f

$$\frac{\pi a \sigma^2}{E} = 2\gamma_s$$

failure strength (causes crack propagation) for an infinite domain containing a mid-crack of length 2a is:

$$\frac{\pi a \sigma_f^2}{E} = 2\gamma_s \longrightarrow \sigma_f = \sqrt{\frac{E\gamma_s}{\frac{\pi}{2} a}}$$

Last time

$$\sigma_f = \sqrt{\frac{E\gamma_s}{4a}}$$

stress concentration argument

$$\approx .5\sqrt{\frac{E\gamma_s}{a}}$$

This time

$$\sigma_f = \sqrt{\frac{E\gamma_s}{\frac{\pi}{2}a}}$$

$$\simeq .8\sqrt{\frac{E\gamma_s}{a}}$$

compare this with atomistic-based strength which was:

$$\sigma_c = \sqrt{\frac{E\gamma_s}{x_0}} \gg \sigma_f$$

because $x_0 \ll a$

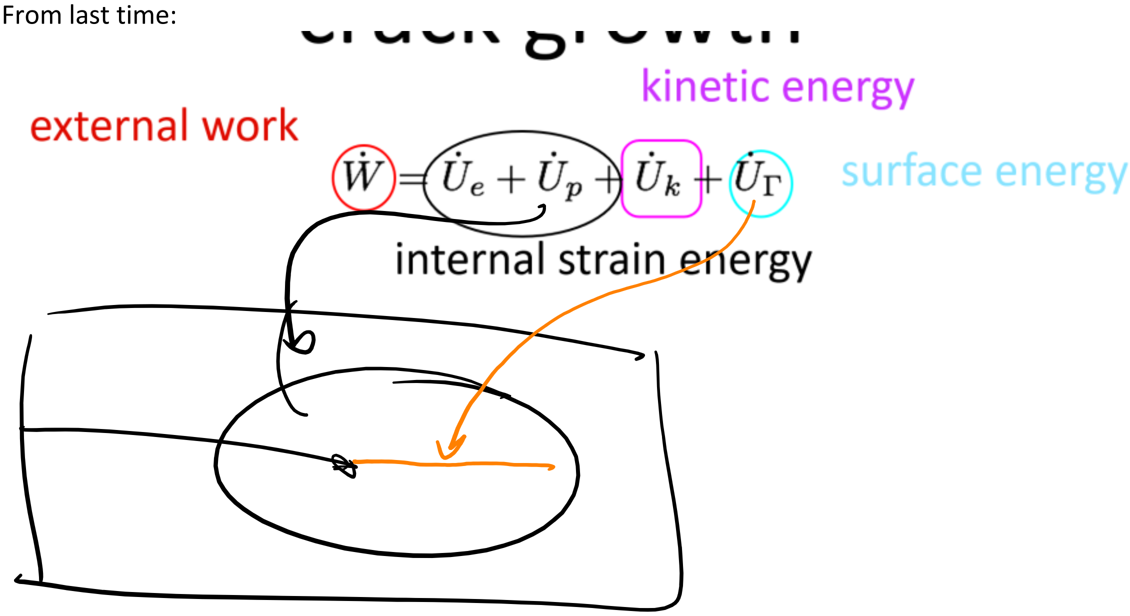
Stress approach:
Stress Concentration

$$\sigma_f = 0.5\sqrt{\frac{E\gamma_s}{a}}$$

Energy approach:
Griffith

$$\sigma_f = \sqrt{\frac{2}{\pi}}\sqrt{\frac{E\gamma_s}{a}} \approx 0.8\sqrt{\frac{E\gamma_s}{a}}$$

From last time:



~~ductile materials~~

Plane stress

$$\sigma_c = \sqrt{\frac{2E\gamma_s}{\pi a}}$$ Griffith (1921), ideally brittle solids

$$\boxed{\sigma_c = \sqrt{\frac{2E(\gamma_s + \gamma_p)}{\pi a}}}$$ Irwin, Orowan (1948), metals

$$\gamma_p \gg \gamma_s$$

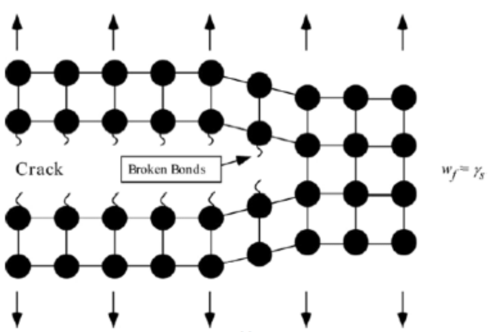
$$\gamma_p \approx 10^3 \gamma_s \quad \text{(metals)}$$

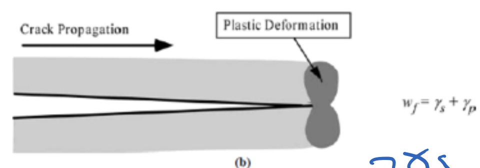
general form of the equation

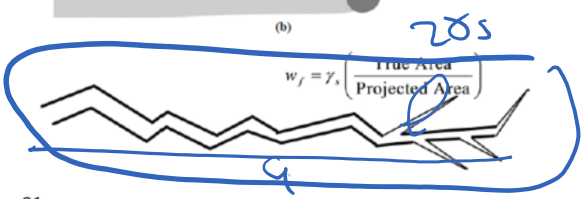
$$\sigma_f = \sqrt{\frac{2E w_f}{\pi a}}$$

energy needed to advance the crack per unit area

- w_f: Fracture energy from plastic, viscoelastic, or viscoplastic effects
- w_f can also be influenced by crack meandering and branching
- Caution: If nonlinear displacement regions are large enough this equation is not accurate as it is based on linear elastic solution ($\Pi = \Pi_0 - \frac{\pi\sigma^2 a^2 B}{E}$)







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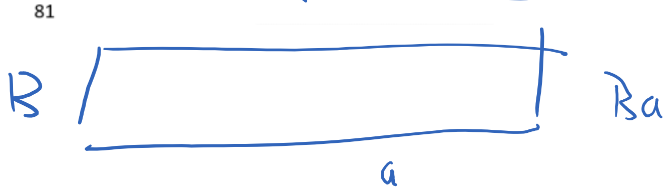


effective area

Energy release rate

Irwin 1956 $\boxed{G \equiv -\frac{d\Pi}{dA}}$ a.k.a Crack extension force
Crack driving force

$$G = -\frac{d\Pi}{dA}$$

potential energy

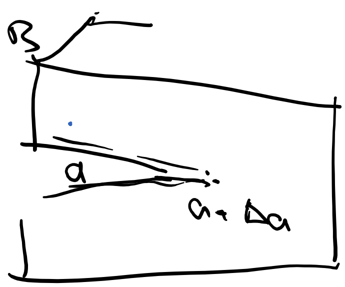
crack surface

how much energy is going to be released per surface of crack (not time rate)

$$\underset{\text{resistance}}{R} = \underbrace{2w_f}$$

How much energy is required to create a unit surface of crack

- material property, also depend on crack length



$A = aB$

$\Pi(a)$

$\Pi(a+\Delta a)$

$$G(a) = -\frac{d\Pi}{dA}$$

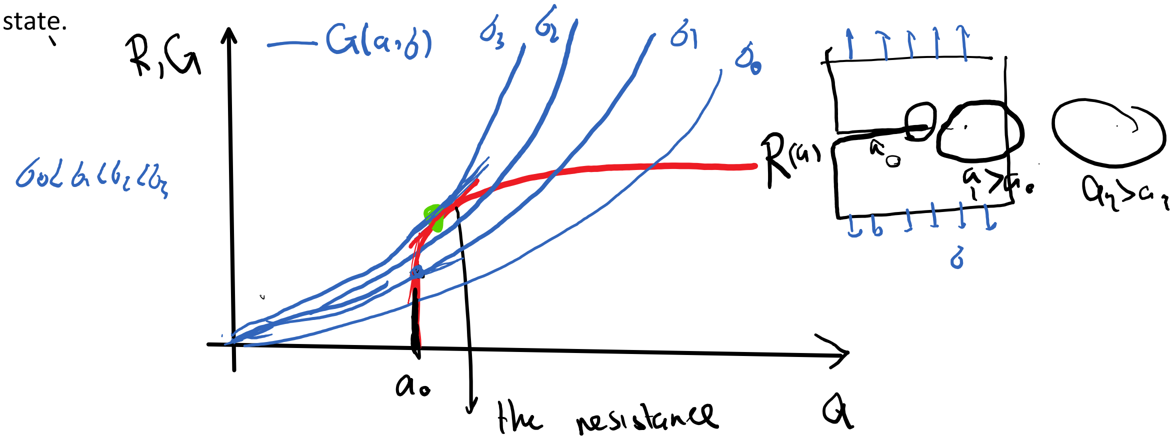
$$= -\frac{\Pi(a+\Delta a) - \Pi(a)}{\Delta a B}$$

- $G < R$ crack does not propagate (not enough energy released)

$G = R$ crack advance.

$G > R$ → extra energy goes to dynamic effects, heating of the crack tip, etc.

Stable vs. unstable crack growth:
For both crack will grow, but for unstable crack growth it will not stop propagating with the current "loading" state.



the resistance sometimes grows as crack propagates since plastic field around crack tip fully develops

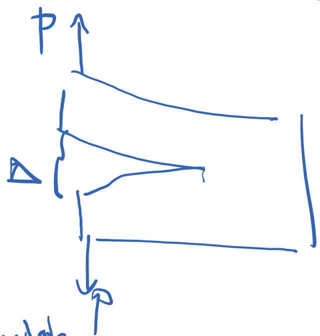
- crack does not grow until σ_1 for which $G(a_0, \sigma_1) = R(a_0)$ for lower loads, $G < R$.
- between σ_1 & σ_3 we need to keep increasing the load σ for the crack to keep propagating. This is because $\underbrace{\frac{dG}{da} < \frac{dR}{da}}_{\text{stable crack propagation}}$ in this range.
- at the critical load σ_3 we have $G = R$ & $\frac{dG}{da}(\sigma_3) = \frac{dR}{da}$ based on the curvatures the crack propagates for this load σ_3 (with no need to increase it) past this point.

$G = R$ crack propagation → $\frac{dG}{da} < \frac{dR}{da}$ stable propagation

→ $\frac{dG}{da} \geq \frac{dR}{da}$

unstable propagation

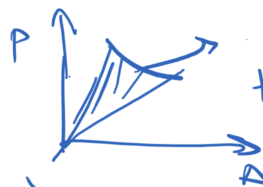
A key question is how to calculate G



Different ways to calculate G:

- G from P vs Δ

today



used to calculate G

we'll derive analytical expression

$G = \frac{P^2}{2B}\frac{dC}{da}$ compliance

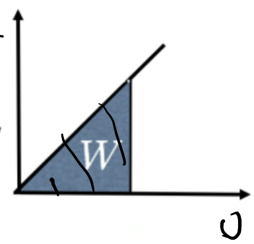
stress intensity factors

$G = f(?)\frac{K_I^2 + K_{II}^2}{E'}$

will learn this later

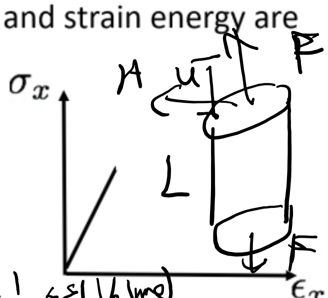
Consider a linear elastic bar of stiffness k, length L, area A, subjected to a force F, the work is

$$W = \int_0^u F du = \int_0^u kudu = \frac{1}{2}ku^2 = \frac{1}{2}Fu$$



This work will be completely stored in the structure in the form of strain energy. Therefore, the external work and strain energy are equal to one another

$$\boxed{U = W = \frac{1}{2}Fu}$$



In terms of stress/strain

$$\boxed{U = \frac{1}{2}Fu = \frac{1}{2}\frac{F}{A}\frac{u}{L}AL}$$

$= \frac{1}{2}\sigma\varepsilon$ (volume)

σ ε

Strain energy density $u = \frac{1}{2}\sigma_x\epsilon_x \longrightarrow \boxed{u = \int \sigma_x d\epsilon_x}$

$[\mathrm{J/m^3}]$

83

$$W = \int \underbrace{\tfrac{1}{2}\sigma\cdot\varepsilon}_{u}\, dV = \underbrace{U_e}_{\text{internal energy}}$$

strain energy density

$= U_e$

strain energy as crack grows — internal energy

For a quasi-static problem with no plasticity and fracture all external work goes to internal energy.

Evaluation of G

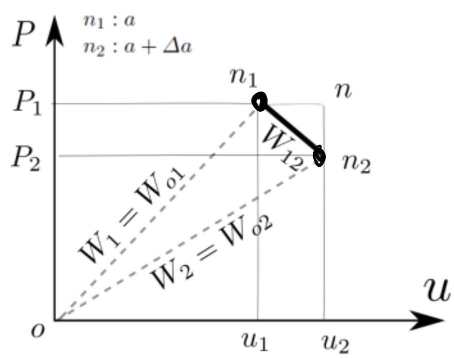
Given: A point load - displacement system with a crack and two data points:

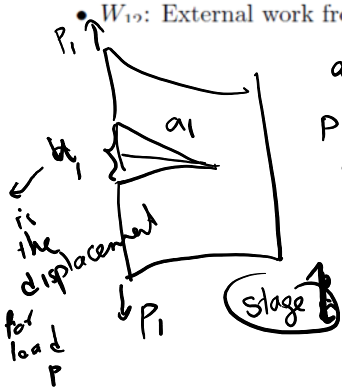
- Load P_1, displacement u_1, & crack length a_1
- Load P_2, displacement u_2, & crack length $a_2 = a_1 + \Delta a$ (small Δa)

Goal: Compute G

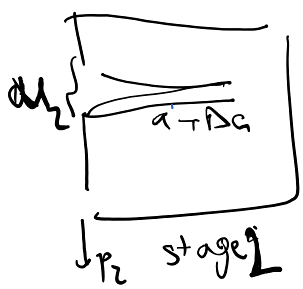
Notation:

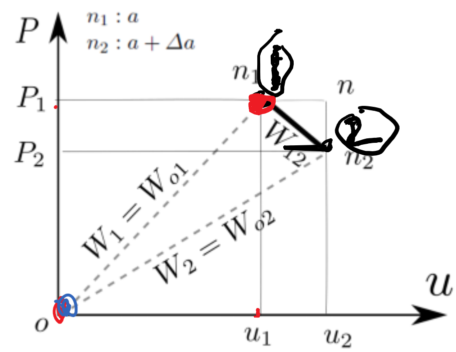
- W_{12}: External work from n_1 to n_2





as a grows P decreases and or u increases





$$\Pi = U_e - W$$

$$\Delta\Pi = \Delta U_e - \Delta W$$

$$G = -\frac{d\Pi}{dA} \approx -\frac{\Delta\Pi}{B\,\Delta a}$$

$$\Delta\Pi = \Delta U_e - \Delta W$$

$$\Delta U_e = \underbrace{U_e(a+\Delta a)}_{\frac{1}{2}P_2 u_2} - \underbrace{U_e(a)}_{U_e(\text{stage 1})}$$

$= \frac{1}{2} P_1 u_1$

$= W_{o2}$ external work

$= W_{o1}$ external work from 0

external work from 0 to stage 2

$-W_{o1}$
external work from 0 to stage 1

ΔW = external work from stage 1 where crack length is a to stage 2 where " " " $a + \Delta a$