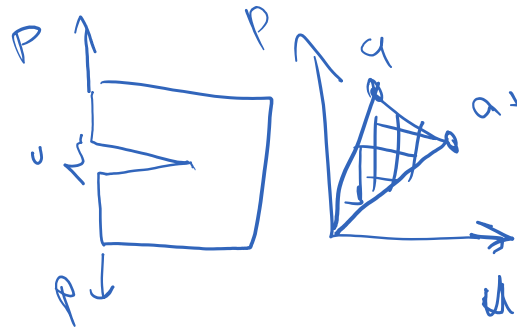
2020/09/07

Monday, September 7, 2020 2:50 PM

Continuing from the last time

- The condition G (energy release rate) = R (resistance) is needed for crack propagation.
- There are several methods to calculate G based on given data.
- One of those is when we have a P versus u (displacement) system.

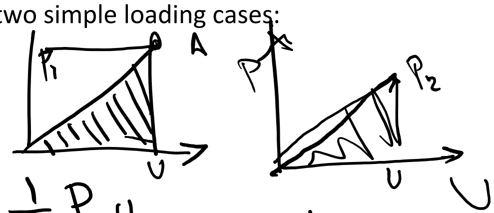


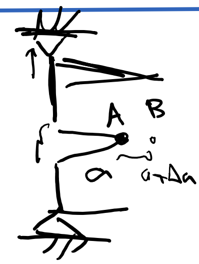
want to show

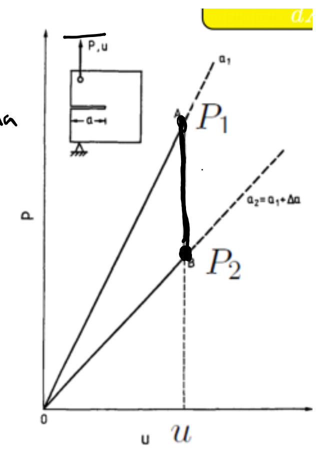
$$G = \frac{\text{area under } P\text{-}u}{B\Delta a}$$

We first show this for two simple loading cases:

A) Fixed grip:







$$U_1 = \frac{1}{2} P_1 u$$

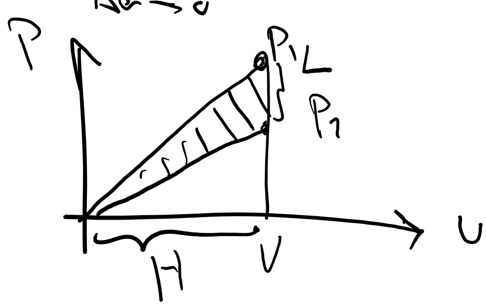
$$U_2 = \frac{1}{2} P_2 u$$

ΔU_e change of internal energy $= \frac{1}{2} P_2 u - \frac{1}{2} P_1 u = \frac{1}{2}(P_2 - P_1) u$

ΔW change of external work $= 0$

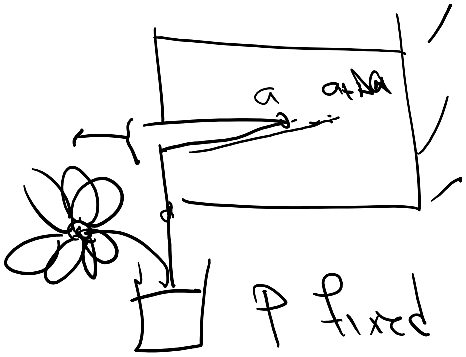
$$G = -\frac{d\Pi}{dA} = \lim_{\Delta A \to 0} -\frac{(\Delta U_e - \Delta W)}{\Delta A} =$$

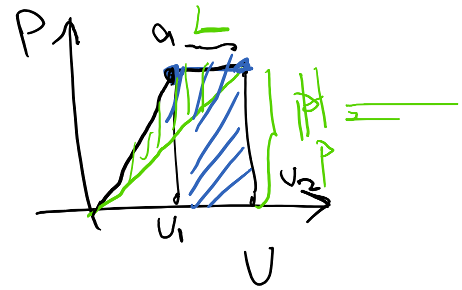
$$\lim_{\Delta a \to 0} \frac{-\frac{1}{2}(P_2 - P_1) u}{B \Delta a} = \lim_{\Delta a \to 0} \frac{\frac{1}{2} \overbrace{(P_1 - P_2)}^{L} \overset{\sim H}{u}}{B \Delta a}$$



Other simple case:

B) Dead load



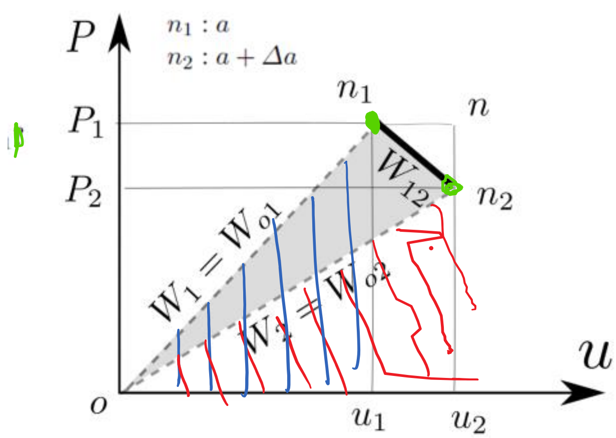


like before $U_1 = \frac{1}{2} P u_1$, $U_2 = \frac{1}{2} P u_2 \rightarrow \Delta U = \frac{1}{2} P(u_2 - u_1)$

$$\Delta W = \int_1^2 P\,du = P(u_2 - u_1)$$

$$G = -\frac{\Delta \Pi}{\Delta A}\bigg|_{\Delta A \to 0} = -\frac{\Delta U + \Delta W}{\Delta A} = \frac{-\frac{1}{2} P(u_2-u_1) + P(u_2-u_1)}{B\,\Delta a} = \frac{\frac{1}{2} P(u_2-u_1)}{B\,\Delta a}$$

Finally, we do this for the most general case:

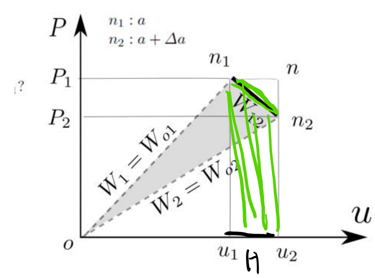


$$\Delta \Pi = \Delta U_e - \Delta W$$

$$\Delta U_e = U_2 - U_1 = \frac{1}{2} P_2 u_2 - \frac{1}{2} P_1 u_1$$

$$\Delta W = \int_{u_1}^{u_2} \underbrace{P(u)}_{\text{force}}\, \underbrace{du}_{\text{displacement increment}} =$$

$$\approx (u_2 - u_1)\left\{\frac{1}{2}(P_1 + P_2)\right\}$$



$$\Delta \Pi = \Delta U_e - \Delta W =$$

$$\frac{1}{2} P_2 u_2 - \frac{1}{2} P_1 u_1 - (u_2 - u_1)\left\{\frac{1}{2}(P_1 + P_2)\right\}$$

$$= \cancel{\frac{1}{2} P_2 u_2} - \cancel{\frac{1}{2} P_1 u_1} - \frac{1}{2} P_1 u_2 - \cancel{\frac{1}{2} P_2 u_2} + \cancel{\frac{1}{2} P_1 u_1} + \frac{1}{2} P_2 u_1$$

$$\Delta \Pi = -\frac{1}{2}(P_1 u_2 - P_2 u_1)$$

Algebraic

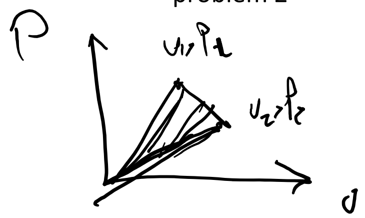
$$\boxed{G = \frac{\frac{1}{2}(P_1 u_2 - P_2 u_1)}{B\,\Delta a}} \quad (*)$$

$$G = \Delta \Pi = \Delta \Pi$$

$$G = -\frac{\Delta\Pi}{\Delta A} = -\frac{\Delta\Pi}{B\Delta a} \longrightarrow \boxed{G = \frac{\frac{1}{2}(P_1u_2 - P_2u_1)}{B\Delta a}}$$

Simple relation to calculate G from a pair of u and P measurements; useful for HW1, problem 2

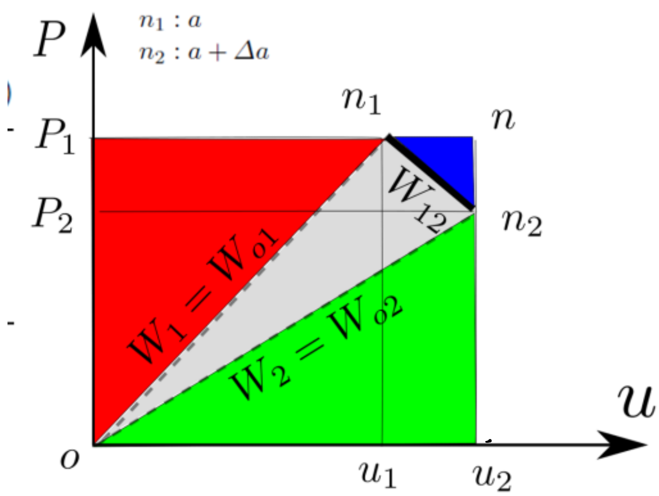
But why $\frac{1}{2}(P_1u_2 - P_2u_1)$ is equal to the shaded area?



From the above derivation, we had:

$$\Delta\Pi = \Delta U_e - \Delta W = \frac{1}{2}P_2u_2 - \frac{1}{2}P_1u_1 - (u_2 - u_1)\left\{\frac{1}{2}(P_1 + P_2)\right\}$$

We continue from here:



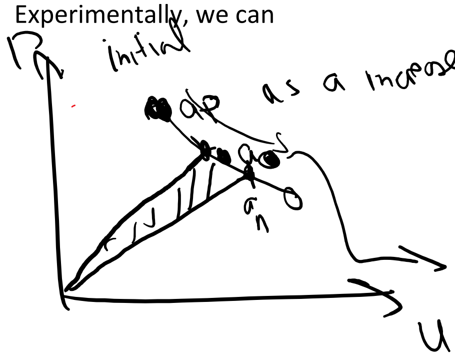
so infact $-\Delta\Pi$ is the shaded area

"usual" interpretation

$$\boxed{\begin{aligned} G &= \frac{1}{B\Delta a}\left(P_1u_2 - \frac{P_1u_1}{2} - \frac{P_2u_2}{2} - \frac{(P_1 - P_2)(u_2 - u_1)}{2}\right) \\ &= \frac{\text{Grey area}}{B\Delta a} \end{aligned}}$$

What is the use of this P - u plot?

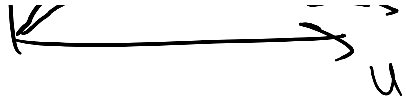
Experimentally, we can



$-\Delta\Pi$

$$G(a_o) = \frac{\text{shaded area}}{B(a_n - a_p)}$$

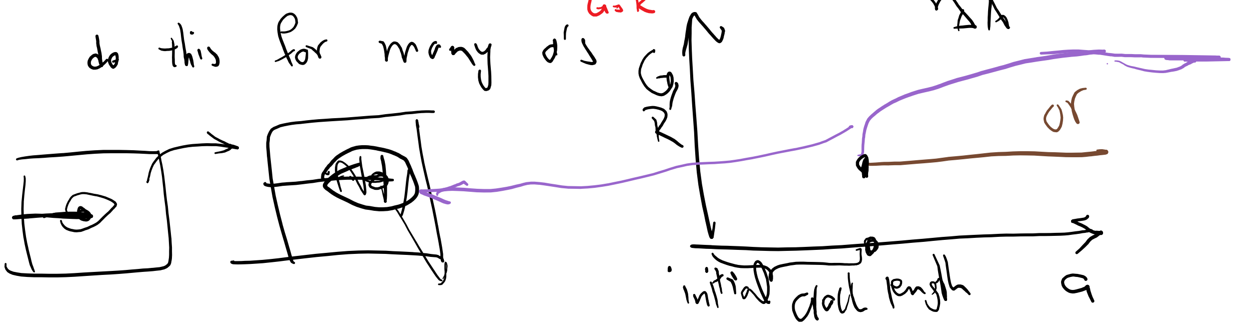
in controlled exp, stable crack growth



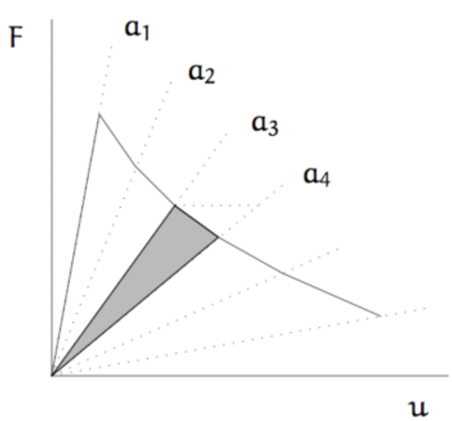
$G(a_0) =$ ... $\underbrace{B(a_n - a_p)}_{\Delta A}$

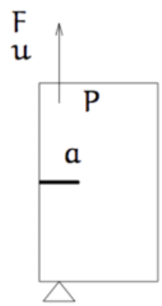
in controlled exp. stable crack growth $G = R$

do this for many a's



This plastic region around it
can get to an almost the same size after some crack
propagation.





$$\boxed{G(a_3) = \frac{1}{B}\frac{\text{shaded area}}{a_4 - a_3}}$$

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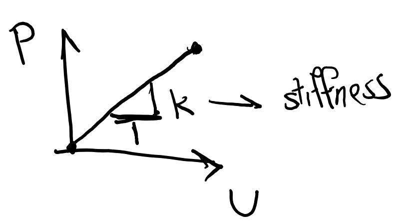
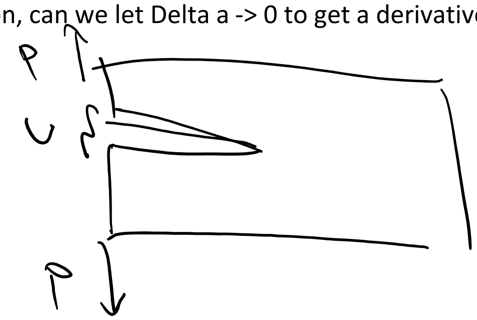
So, instead of this Finite Difference approximation, can we let Delta a -> 0 to get a derivative expression for G?

fixed a





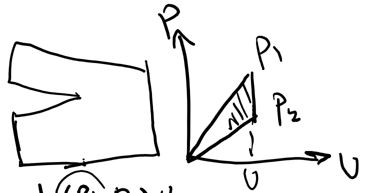
$$P = k\,u \longrightarrow u = \underbrace{\frac{1}{k}}_{\text{compliance}} P$$

$$\boxed{u = CP}$$

We want to get the derivative expression for G as Delta a -> 0

Case A) Fixed grip

u is fixed



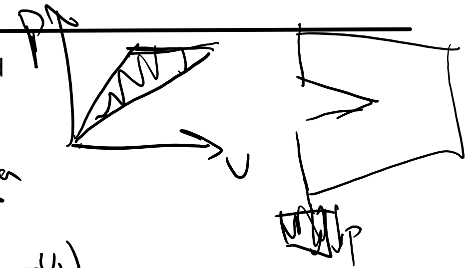
$$G = \frac{\text{shaded area}}{B\Delta a} = \frac{\frac{1}{2}(P_1 - P_2)u}{B\Delta a} \qquad \boxed{u = CP} \rightarrow \frac{u}{C} = P$$

$$= \frac{-\Delta\left(\frac{u}{C}\right)u}{2B\Delta a} = \frac{-d\frac{u^2}{C}}{2B\,da} = -\frac{u^2}{2B}\frac{d\frac{1}{C}}{da}$$

u is constant $\rightarrow$ goes out of derivative

$$= -\frac{u^2}{2B}\left(\frac{-1}{C^2}\frac{dC}{da}\right) = \frac{1}{2B}\left(\frac{u}{C}\right)^2\frac{dC}{da} = \frac{P^2}{2B}\frac{dC}{da}$$

Case B) Dead load



$$G = \frac{\text{shaded area}}{B\Delta a} = \frac{\frac{1}{2}P(u_2 - u_1)}{B\Delta a} = \frac{\frac{1}{2}\Delta P(u_2 - u_1)}{B\Delta a} = \frac{P}{2B}\lim \frac{u_2 - u_1}{\Delta a}$$

P is fixed

$\Delta a \to 0$

$$= \frac{P}{2B}\frac{du}{da} = \frac{P}{2B}\frac{dPC}{da} \qquad u = PC$$

but P is fixed

$$\boxed{= \frac{P^2}{2B}\frac{dC}{da}}$$

So, whether we have fixed grip (u fixed) or dead load (P fixed), the differential expression for G is

$$\boxed{G = \frac{P^2}{2B}\frac{dC}{da} \qquad C \text{ is compliance}}$$

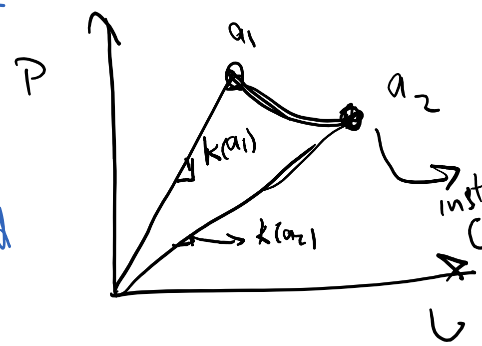
In fact, this is true even for more general loading

HW

Hint use

$$G \approx \frac{\frac{1}{2}(P_1 u_2 - P_2 u_1)}{\Delta a}$$

or other hint in the assignment

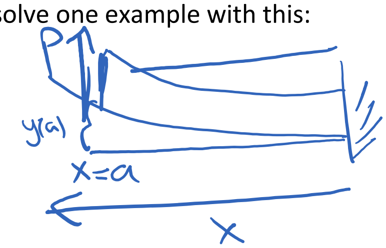


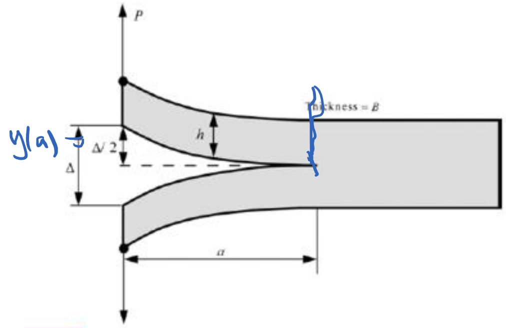
instantaneous $G = \frac{P^2}{2B}\frac{dC}{da}$

How do we use this equation?

We are going to solve one example with this:

$$y(a) = \frac{Pa^3}{3EI}$$





$$\frac{\Delta}{2} = \frac{Pa^3}{3EI} \rightarrow \boxed{\Delta = \frac{2}{3}\frac{Pa^3}{EI}} \quad (1)$$

H is u

From here in fact we can calculate the compliance and from that compute G:

does not depend on P

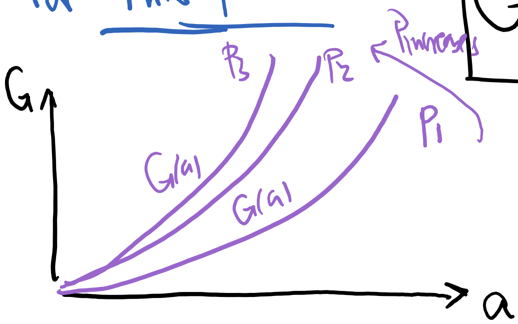
$$\Delta = CP \rightarrow C = \frac{\Delta}{P} = \frac{\frac{2}{3}Pa^3/EI}{P} \rightarrow \boxed{C = \frac{2a^3}{3EI}} \quad (2)$$

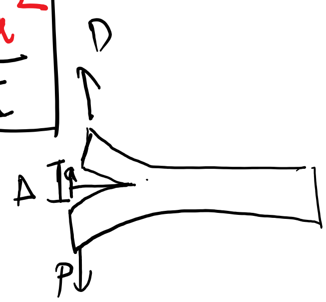
from (1)

$$G = \frac{P^2}{2B}\frac{dC}{da} = \frac{P^2}{2B}\frac{d\left(\frac{2}{3}a^3/EI\right)}{da} = \frac{P^2}{2B}\,\frac{2}{3}\times\frac{3a^2}{EI}$$

for this problem

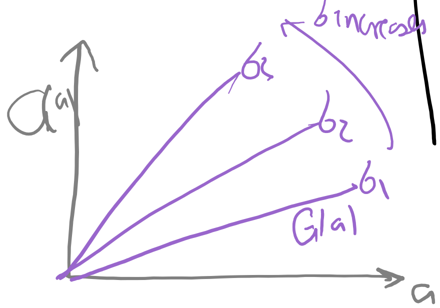
$$\boxed{G = \frac{P^2 a^2}{EI}}$$

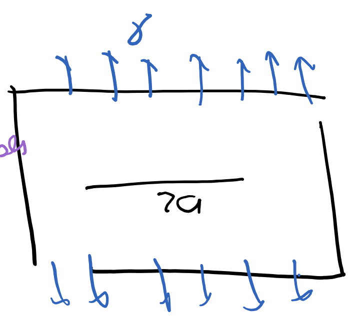




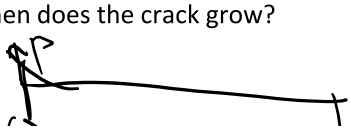
G increases as a^2 versus a (relatively fast pace of increasing G)

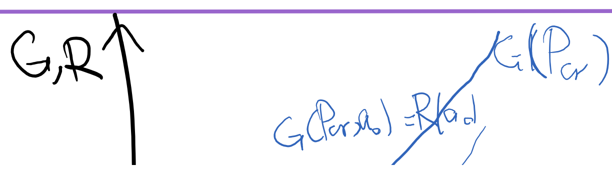
$$G = \frac{\pi\sigma^2 a}{E}$$



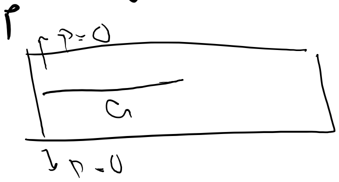


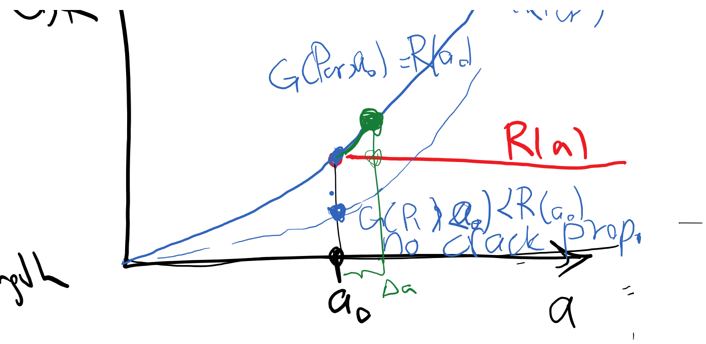
When does the crack grow?





a_0 initial crack length





$R < P_{cr}$

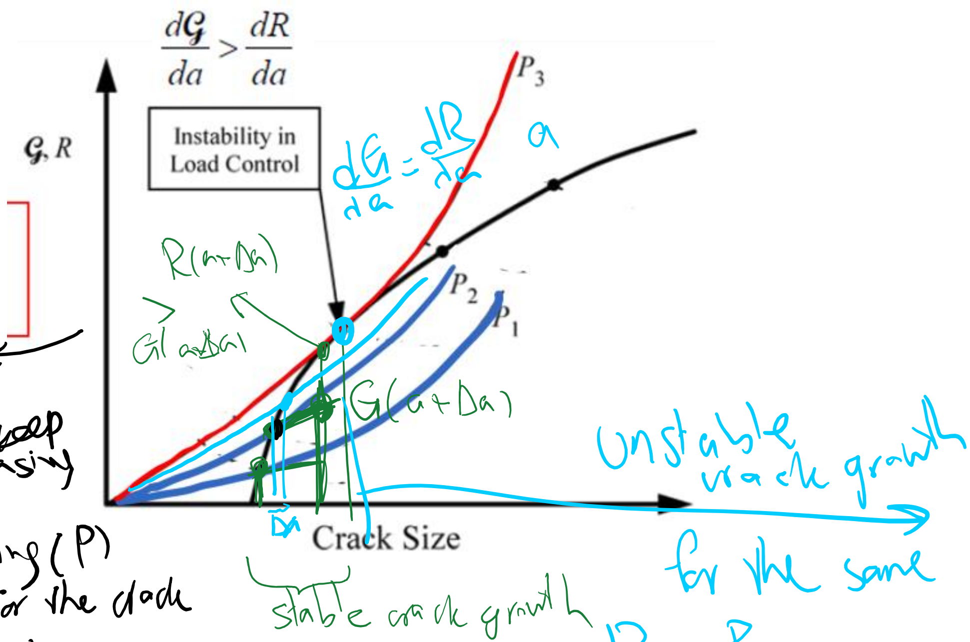
$$G(a_0) = R(a_0)$$

$$\left.\begin{aligned} G(a_0+\Delta a) &\geq R(a_c+\Delta a) \end{aligned}\right\} \rightarrow$$

$$\lim_{\Delta a \to 0} \quad \frac{G(a_0+\Delta a) - G(a_0)}{\Delta a} \geq \frac{R(a_c+\Delta a) - R(a_0)}{\Delta a}$$

necessary for crack propagation ← $G = R$

crack can unstably propagate ← $\dfrac{dG}{da} > \dfrac{dR}{da}$



we need to keep increasing the loading (P) for the crack to grow

$$\frac{dG}{da} < \frac{dR}{da}$$