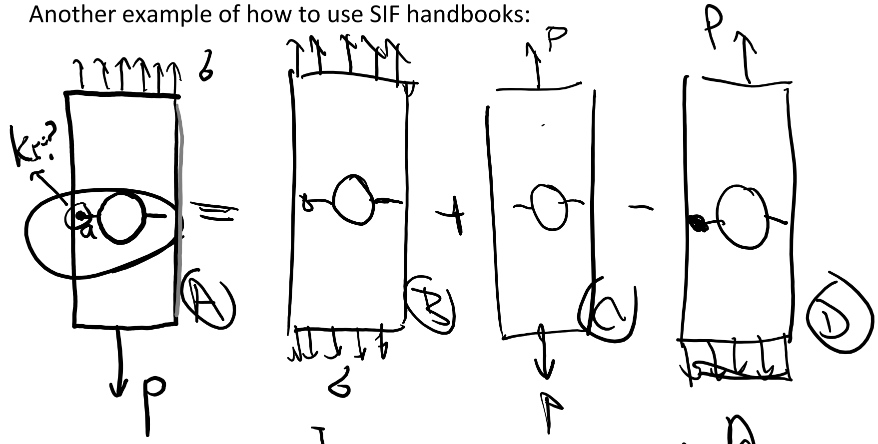
2020/09/23

Wednesday, September 23, 2020 2:42 PM

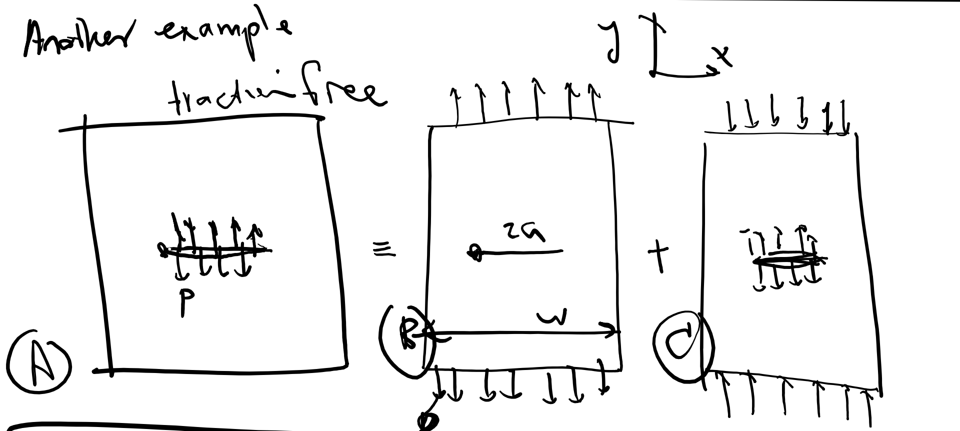
Another example of how to use SIF handbooks:



$$K^A = K^I_\sigma + K^I_p - K^A$$

$$\rightarrow \boxed{K^A = \frac{K^I_\sigma + K^I_p}{2}}$$

Another example

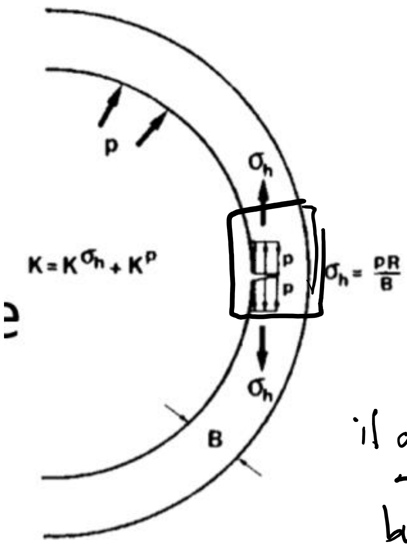


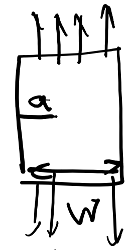
$$\boxed{K^I = K^I = \frac{1}{\cos\left(\frac{a\pi}{w}\right)} \sigma\sqrt{\pi a}}$$

uniform solution $\boxed{\sigma_{yy} = -p}$

$$K^I_A = K^I_B + K^I_C \rightarrow 0$$

Internal pressure is generally exerted from a fluid. Some examples are hydraulic fracturing, pressure vessel fracture, porous media, …





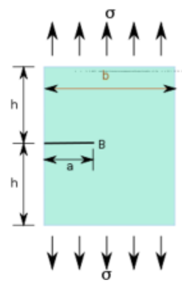
if $a \ll W$

σ

$$K_I = 1.12\sigma\sqrt{\pi a}$$

but $\sigma = \sigma_{\theta\theta} = p\frac{R}{t}$

$$K_I \approx 1.12\frac{pR}{t}\sqrt{\pi a}$$



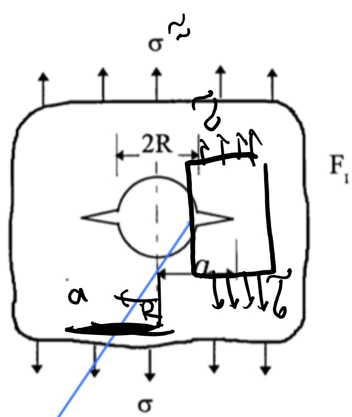
0.4	1.257
0.5	1.500
0.6	1.915

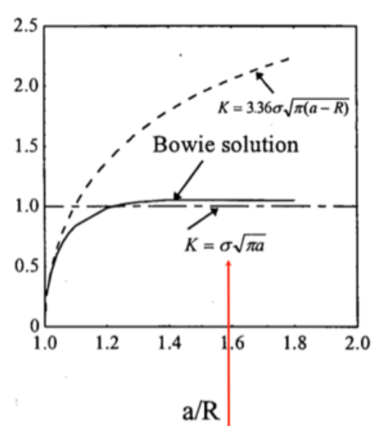
$h/b \geq 1$ and $a/b \leq 0.6$

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4\right] \approx 1.12\sigma\sqrt{\pi a}$$

144

Two small cracks at a hole





3σ edge crack hole as a part of the crack

$$K = 1.12\overset{3\sigma}{\tilde{\sigma}}\sqrt{\pi a_{crack}} = 3.36\sigma\sqrt{\pi(a-R)}$$

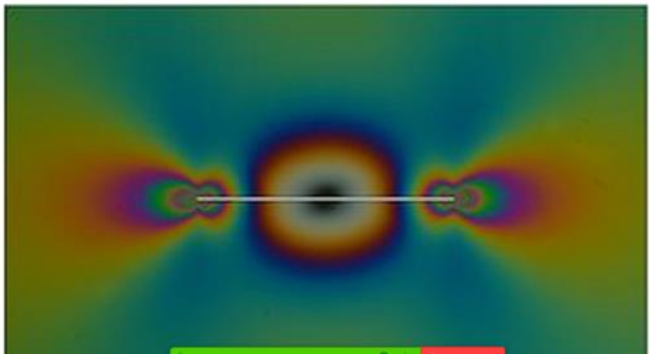
SEN crack approach

$a_{crack} = a - R$

Photoelasticity

Wikipedia

Photoelasticity is an experimental method to determine the stress distribution in a material. The method is mostly used in cases where mathematical methods become quite cumbersome. Unlike the analytical methods of stress determination, photoelasticity gives a fairly accurate picture of stress distribution, even around abrupt discontinuities in a material. The method is an important tool for determining critical stress points in a material, and is used for determining stress concentration in irregular geometries.

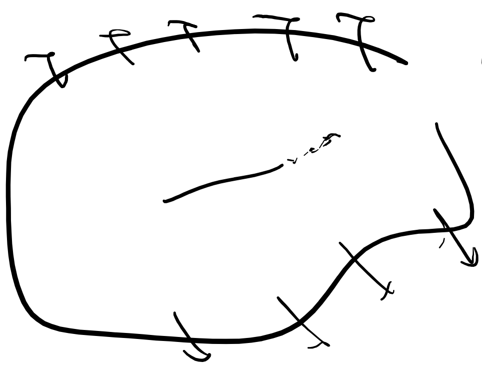


Relation between G and K

G = energy release rate

= how much energy release per unit area of the crack

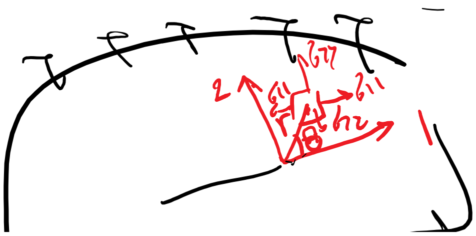
K = stress intensity factor



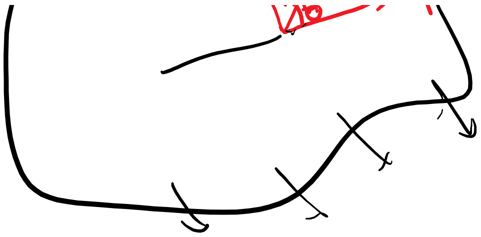
G = Global

: energy

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f^{I}_{ij}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f^{II}_{ij}(\theta) + \frac{K_{III}}{\sqrt{2\pi r}} f^{III}_{ij}(\theta)$$



K: local stress related



K: local stress related

Although G is energy-based and a global measure while K is stress-based and local, they can be related!

Let's look at their dimensions

$$[G] = \frac{\overset{\text{energy}}{[\mathcal{E}]}}{\underset{\text{area}}{[A]}} = \frac{\overset{[\sigma][L]^2}{[F]}[L]}{[L]^2} \Rightarrow \boxed{[G] = [\sigma][L]}$$

N/m , MPa-m

$$\sigma = \frac{K_I}{2\pi r} \underbrace{f_{ij}^{\theta}}_{\text{dimensionless}} \rightarrow [\sigma] = \frac{[K]}{[L]^{1/2}} \rightarrow \boxed{[K] = [\sigma][L]^{1/2}}$$

MPa$\sqrt{m}$

$$\left.\begin{array}{l} [K]^2 = [\sigma]^2[L] \\ [G] = [\sigma]\,[L] \end{array}\right\} \longrightarrow \frac{[G]}{[K^2]} = \frac{1}{[\sigma]} \rightarrow \text{strength stiffness measure}$$

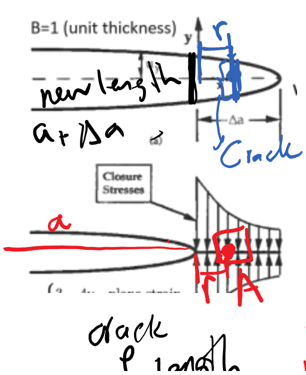
we'll find out that

$$\boxed{G \propto \frac{K^2}{\underset{\text{elastic modulus}}{E}}}$$

we want to prove this

K-G relationship

Irwin



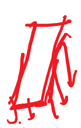
stage ii)

Crack opening = $\delta = 2v$

force = 0

$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$

stage i



σ_{yy}

δ opening = 0

$\Delta F = (\Delta x B)\sigma_{yy}$

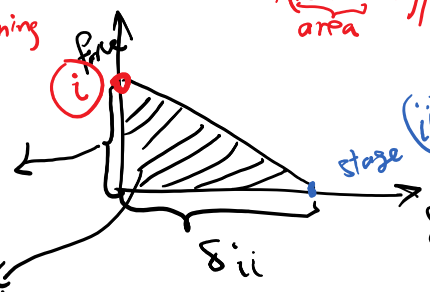
crack of length a

stage i before crack opening

$\Delta x \quad \sigma_{yy} \quad \delta_{opening} = 0$

$\Delta F_i = \underbrace{(\Delta x\, B)}_{area}\, \sigma_{yy}$

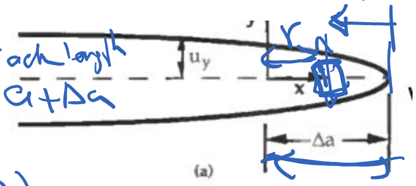
ΔF_i



work done = the energy released

$$= \frac{1}{2}\, \Delta F_i\, (\delta_{ii})$$

$\delta = 2V \quad r^{ii} = \Delta a - r$

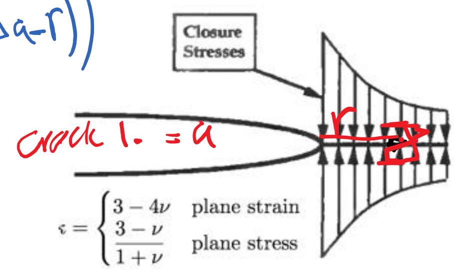


SIF for crack length a

$$\frac{1}{2}\left(\frac{K_I(a)}{\sqrt{2\pi r}}\Delta x B\right)\left(2V\left(K(a+\Delta a), \Delta a - r\right)\right)$$

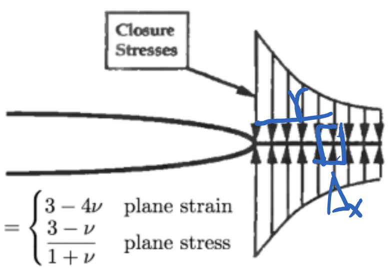
dist. behind the crack

$$V = \frac{K}{2\mu}\sqrt{\frac{r^{ii}}{2\pi}}\,(\kappa+1)$$



$$\kappa = \begin{cases} 3-4\nu & \text{plane strain} \\ \dfrac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$$

$$dW = \frac{1}{2}\left(\frac{K_I(a)\,\Delta x B}{\sqrt{2\pi r}}\right) 2\,\frac{K(a+\Delta a)}{2\mu}\sqrt{\frac{\Delta a - r}{2\pi}}\,(\kappa+1)$$



$$\kappa = \begin{cases} 3-4\nu & \text{plane strain} \\ \dfrac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$$

$$G = \frac{W}{\underbrace{B\Delta a}_{\text{increment of crack surface}}}$$

W → work released

$$W = \int_{r=0}^{r=\Delta a} dW$$

$$G = \int_{r=0}^{\Delta a} \underbrace{\frac{K(a)\,K(a+\Delta a)}{4\pi\mu\,\Delta a}}_{\text{constant}} \sqrt{\frac{\Delta a - r}{r}}\, dr$$

constant go out of the integral

$$G = \frac{(\kappa+1)K^2}{8\mu}$$

μ → shear modulus $= \frac{E}{2(1+\nu)}$

$$G = \frac{K_I^2}{E'} \qquad E' = \begin{cases} E & \text{p. stress} \\ \frac{E}{(1-\nu^2)} & \text{p. strain} \end{cases}$$

Mode I only

Mode I

$$G_I = \begin{cases} \frac{K_I^2}{E} & \text{plane stress} \\ (1-v^2)\frac{K_I^2}{E} & \text{plane strain} \end{cases}$$

mode II

$$G = \frac{K_{II}^2}{E'}$$

Mixed in-plane mode

$$G = \frac{K_I^2 + K_{II}^2}{E'}$$

Mixed mode

$$G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu} \qquad E' = \begin{cases} \frac{E}{1-\nu^2} & \text{for plane strain} \\ E & \text{for plane stress} \end{cases}$$

$\frac{E}{2(1+\nu)}$

- Equivalence of the strain energy release rate and SIF approach
- Mixed mode: G is scalar => mode contributions are additive

For crack propagation

$$G = R \xrightarrow{} (\Gamma) \text{ resistance}$$

$G = R \quad (\Gamma)$

but $G = \frac{K_I^2 + K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}$

Mode I Fracture

$G = \frac{K_I^2}{E'} \rightarrow$

$K_I = \sqrt{GE'}$ when the crack can grow $G = R$

$K_I \rightarrow K_{Ic}$, $G \rightarrow R$

$\underbrace{K_{Ic}}$ critical mode I SIF

$$\boxed{K_{Ic} = \sqrt{RE'}}$$

mode I fracture toughness — resistance

MPa-$\sqrt{m}$ — MPa-m

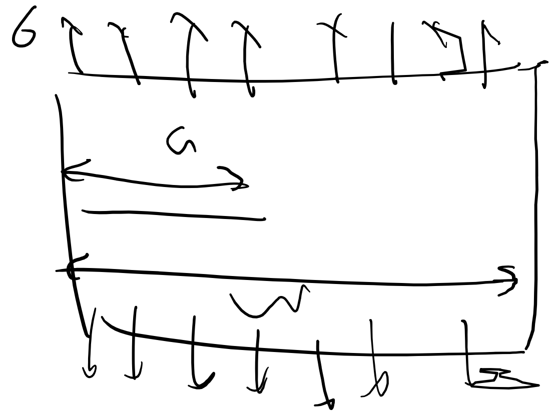
Fracture toughness can also be used for R. So you need to look at the units to see what is being referred to.

$$\boxed{\begin{array}{ll} G = R & \text{general relation (mixed mode) crack can grow} \\ K_I = K_{Ic} & \text{only for mode I " " "} \end{array}}$$

$K_{II} = K_{IIc}$ mode II

from energy eqn above $K_{IIc} = K_{Ic} = \sqrt{RE'}$

Typical fracture mechanics problems:



$$K_I = f\left(\frac{a}{W}\right) \sigma\sqrt{\pi a} = \underbrace{K_{Ic}}_{\text{material property}}$$

- a → crack length
- σ → loading
- K_{Ic} material

2 given → 3rd one obtained

i. a given, material (K_{Ic}) given → obtain σ_{max}

$$\sigma_{max} = \frac{K_{Ic}}{f(a/W)\sqrt{\pi a}}$$

ii. σ given, material (K_{Ic}) given → a = ?

$$f\left(\frac{a}{W}\right)\sqrt{\pi a} = \frac{K_{Ic}}{\sigma}$$

generally a non linear problem

iii. design a ✓, σ ✓ → K_{Ic}

$$K_{Ic} = f\left(\frac{a}{W}\right)\sqrt{\pi a}\,\sigma$$

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