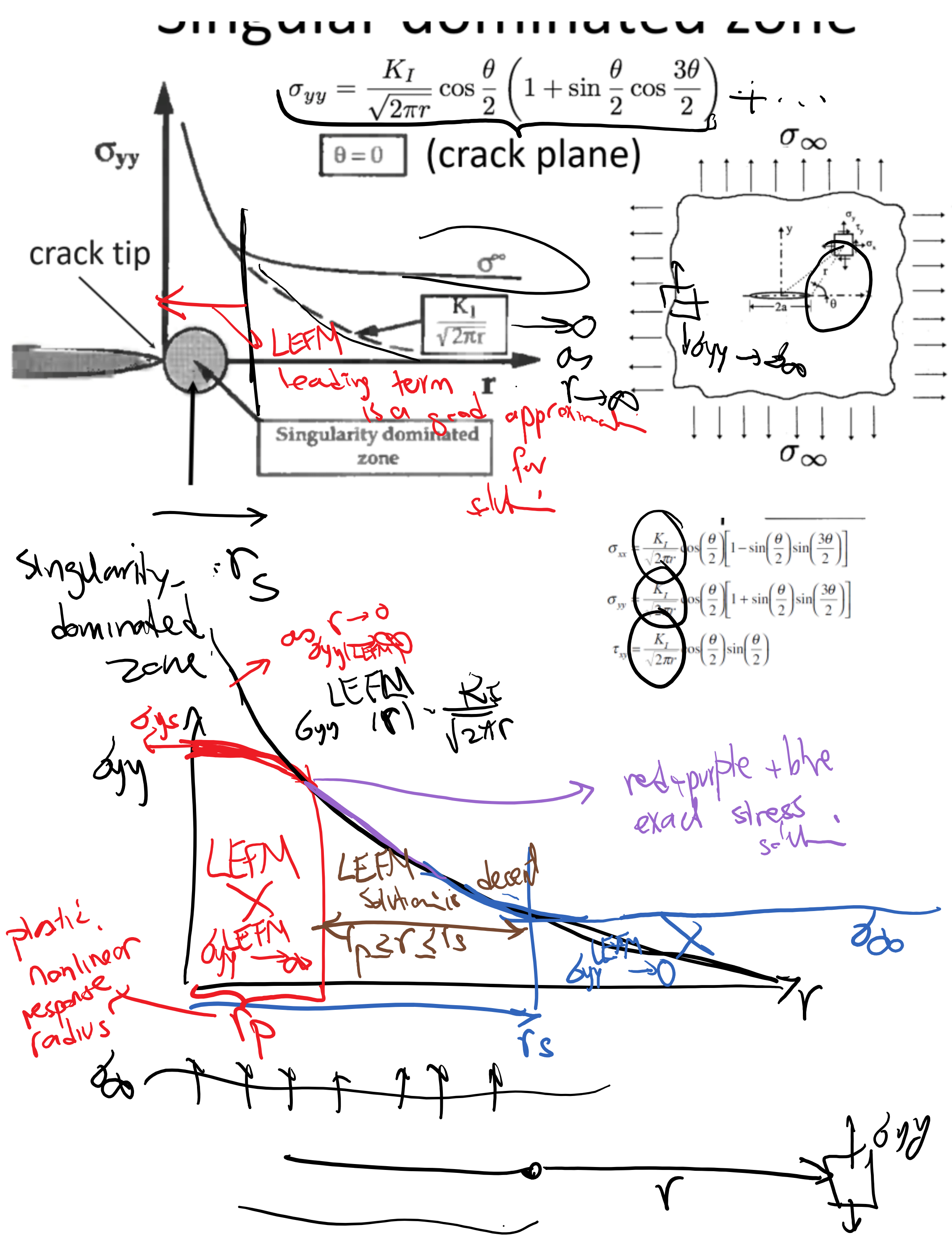
2020/09/28

Monday, September 28, 2020 2:47 PM

When can we use LEFM?

5.2. Plastic zone models

- 1D Models: Irwin, Dugdale, and Barenbolt models



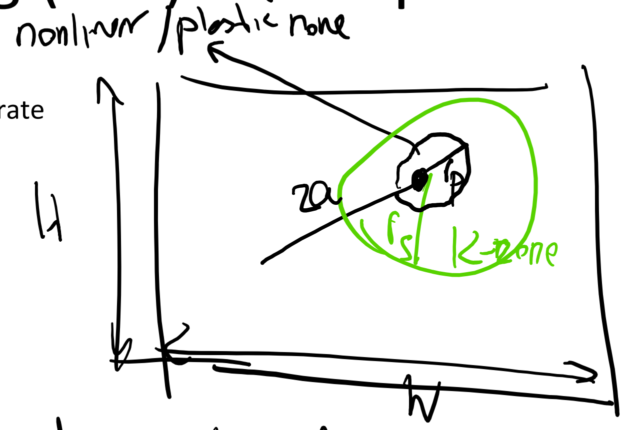
- We want to derive equations for r_p (plastic or nonlinear zone radius) and r_s (singular-dominant zone (K-zone) radius)

- We want to discuss when we can use LEFM theory

Small Scale Yielding (SSY) assumption

SSY states that LEFM solution provides an accurate / acceptable description of the problem if

Nonlinear zone size is significantly smaller than all relevant length scales of the problem.

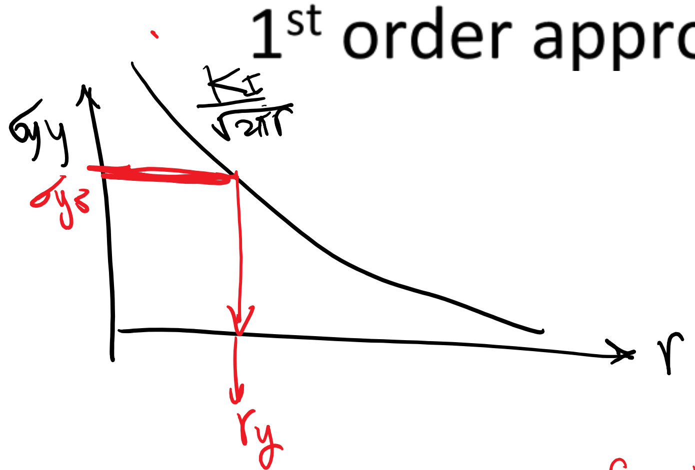


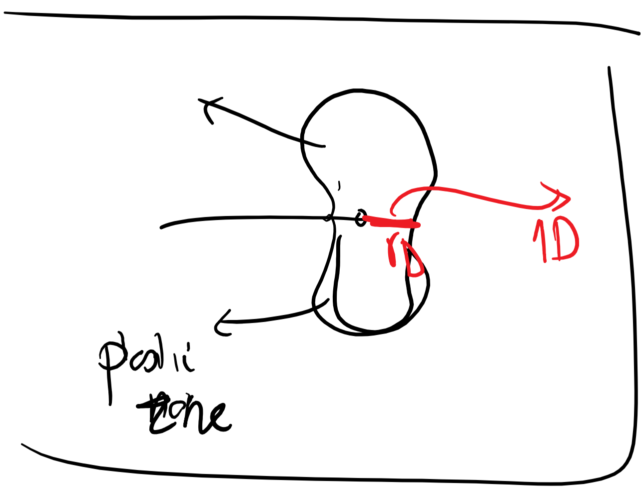
$r_p \ll$ all relevant length scales

e.g. H, W, r_s, ...

So, to check SSY, we first need to have estimates for r_p

Plastic correction: 1[st] order approximation





r_y: 1st order estimate for plastic zone

$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$: $\sigma_{yy} \to \sigma_{ys}$, $r \to r_y$ $\rightarrow$ $\sigma_{ys} = \frac{K_I}{\sqrt{2\pi r_y}}$ $\rightarrow$ $2\pi r_y = \left(\frac{K_I}{\sigma_{ys}}\right)^2$ $\rightarrow$

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

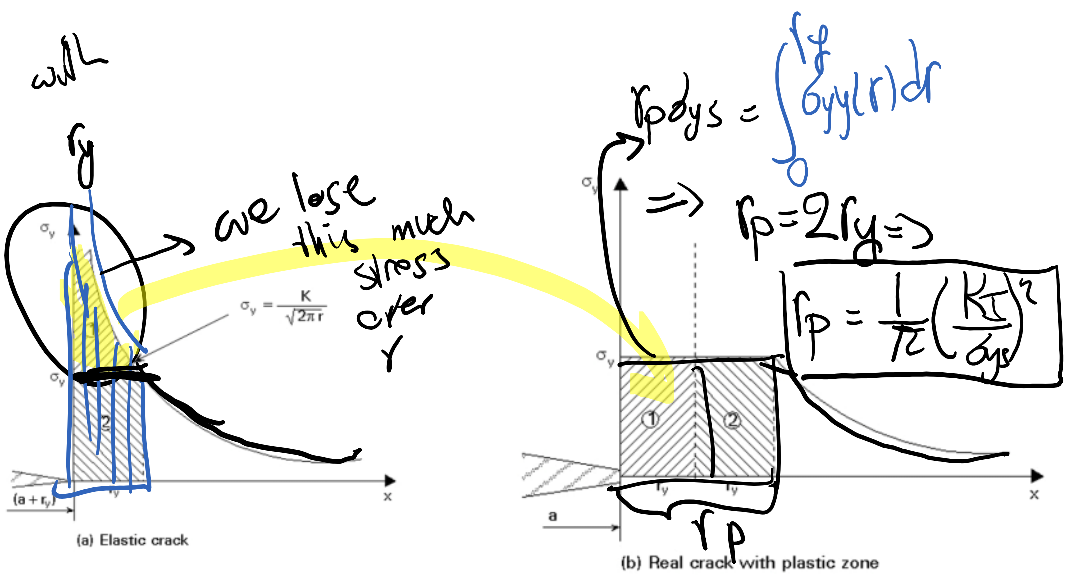
$[K] = [\sigma] L^{1/2}$

$[\sigma_{ys}] = [\sigma]$

If we want to do better than the approximation above, we need to take the lost stress force into account

-> Irvin plastic zone size estimate (r_p)

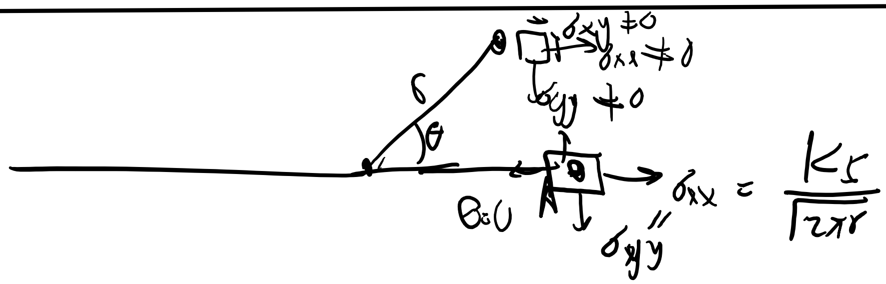
2.Irwin's plastic correction



To get the actual process zone size, we need to solve the problem (BC + PDE + constitutive equations) from the beginning to find the correct solution for that problem. The stress solution for the exact problem is NOT the same as LEFM solution even outside r_p and LEFM stress solution is distributed.

We'll have the Yield Strip model that somehow does this shortly.

Before that, let's compare plane-stress versus plane-strain modes.



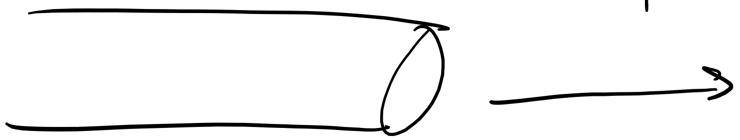
How do we evaluate if point A is yielding or not?

2D & 3D yield criteria

von-Mises criterion

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \leq \sigma_Y$$

principal stresses



$\sigma_1 = \sigma_{xx} = \sigma$

$\sigma_2 = 0$

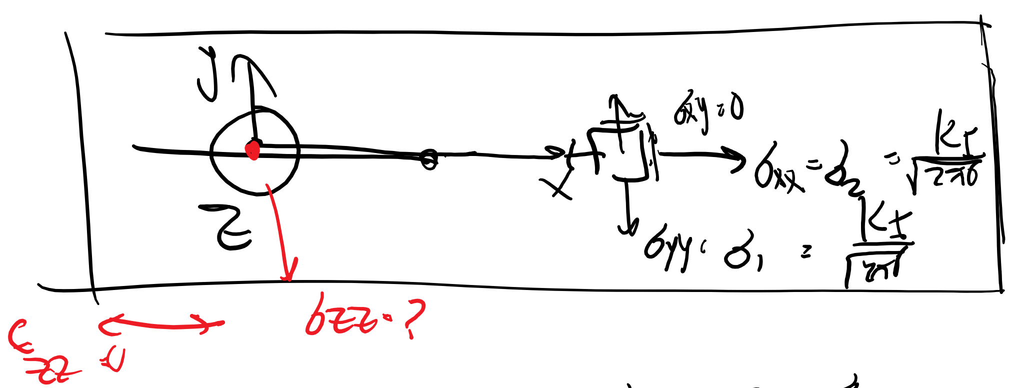
$\sigma_3 = 0$

$\sigma_e = \sigma_1 = \sigma$

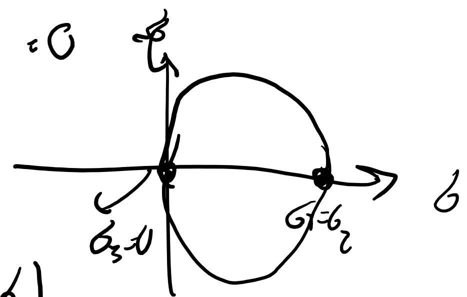
$\sigma = \sigma_e = \sigma_Y$ at yield

$\sigma_{max} = \sigma_Y$ for yielding

2D ahead of the crack



Case 1 : Plane stress $\sigma_3 = \sigma_{zz} = 0$



$\sigma_3 = 0 \quad \sigma_1 = \sigma_2$

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)+(\sigma_3-\sigma_1)^2} = \sigma_1$$

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} = \sigma_1$$

($\sigma_2 \to \sigma$, $\sigma_3 \to 0$, $\sigma_3 \to 0$)

Case 2 : Plane strain

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{xx}+\sigma_{yy})\right) = 0 \rightarrow$$

$$\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$$

ahead of the crack

$$\sigma_{xx} = \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \rightarrow \sigma_{zz} = \sigma_3 = 2\nu\sigma_1$$

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - 2\nu\sigma_1)^2 + (\sigma_1 - 2\nu\sigma_1)^2 + (\sigma_1-\sigma_1)^2}$$

($\sigma_1 - \sigma_3$; σ_2, σ_3; σ_1, σ_2)

$$= (1-2\nu)\,\sigma_1$$

Ahead of crack for which we have $\sigma_1 = \sigma_2 = \frac{K}{\sqrt{2\pi r}}$

$$\sigma_e = \begin{cases} \sigma_1 & \text{p. stress} \\ \sigma_1(1-2\nu) & \text{p. strain} \end{cases}$$

$\sigma_e = \sigma_Y$ is the yield condition $\rightarrow$

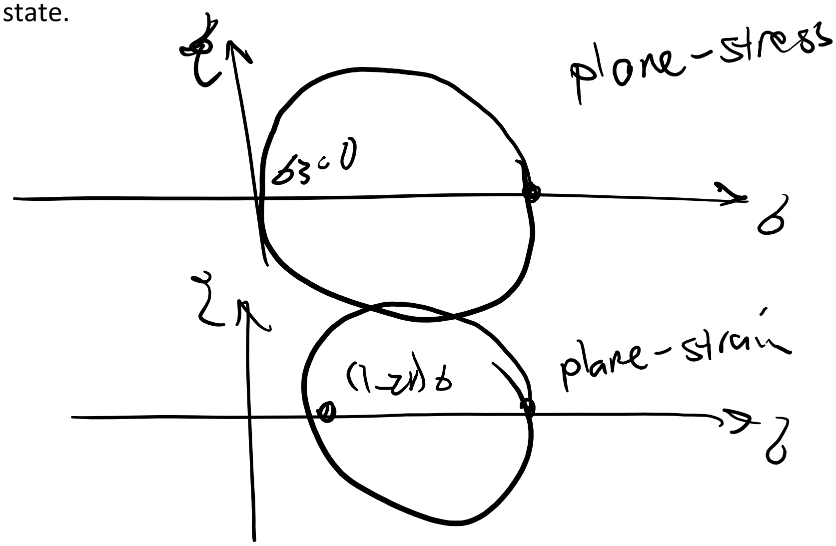
$$\sigma_1 = \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

$$(\sigma_{yy})_{max} = \sigma_{ys} = \begin{cases} \sigma_Y & \text{p. stress} \\ \dfrac{\sigma_Y}{1-2\nu} & \text{p. strain} \end{cases}$$

(ys: maximum)

σ_{yy} we can have

For plane-strain we have a higher yield limit for stress in yy direction (σ_{ys}). This is because of its more triaxial stress state.



Higher effective yield-strength for plane-strain results in a more brittle response and confirms our earlier discussion of having more brittle response for plane strain.

$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

$$\sigma_{ys} = \begin{cases} \sigma_Y & \text{p stress} \\ \dfrac{\sigma_Y}{1-2\nu} & \text{p. strain} \end{cases}$$

$$r_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{p. stress} \\ \dfrac{(1-2\nu)^2}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{p strain} \end{cases}$$

$$\approx \frac{1}{3\pi}\left(\frac{K_I}{\sigma_Y}\right)^2 \quad \text{for } \nu = .2$$

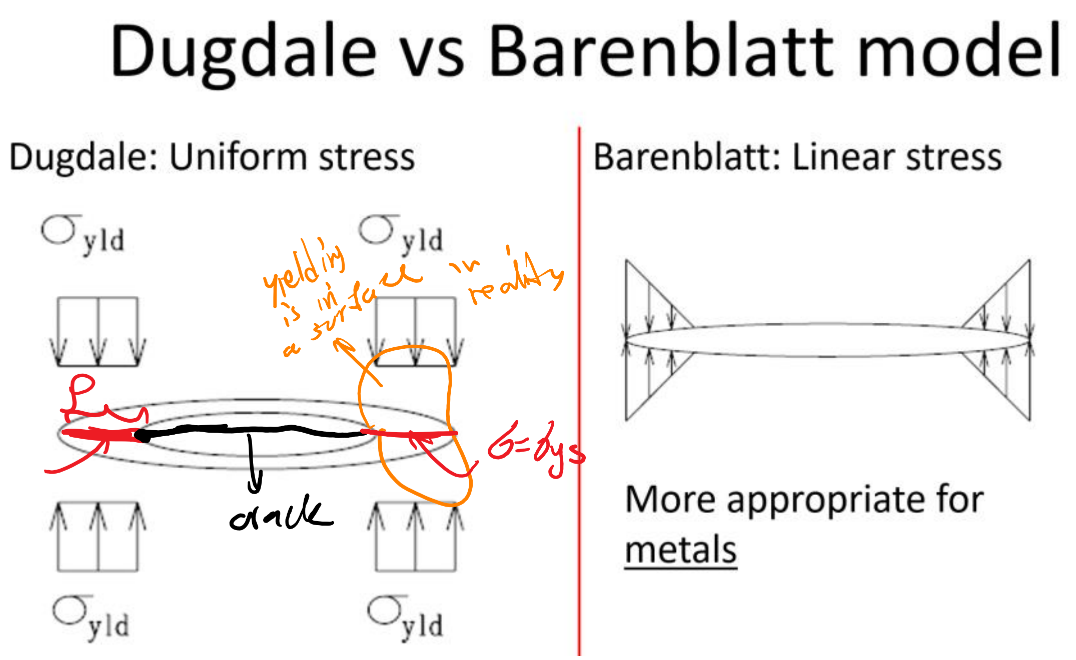
$$\approx \frac{1}{3\pi}\left(\frac{K_I}{\sigma_y}\right)^2 \quad \text{for } \nu = .2$$

r_p for plane-strain

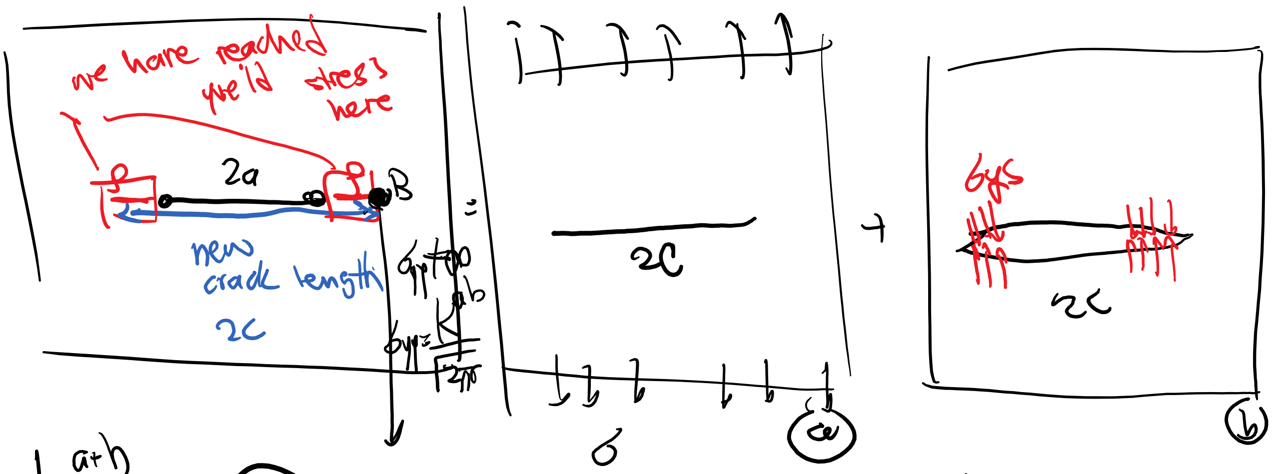
r_p plane-stress

3rd 1D plastic zone size model.

It has 1 dimensional stress redistrubution.



The goal is to find the size rho such as the solution is valid. It means we take stress-redistribution into effect.

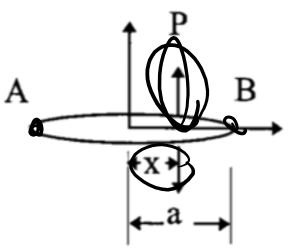


$$K^{a+b} = 0$$

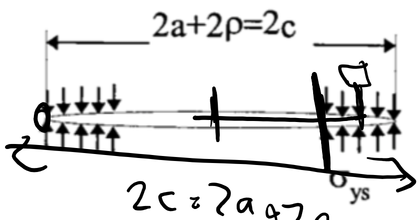
otherwise stress ahead of the yield strip is ∞

$$K^{a} = \sigma\sqrt{\pi c}$$

$$K^{b} = ?$$



$$P = -\sigma_{ys}dx$$



$$2c = 2a + 2\rho$$

$$P = -\sigma_{ys}dx$$

$$K_A = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$K_B = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$

Anderson, p64

$$K^{b} = \frac{-\sigma_{ys}}{\sqrt{\pi c}}\int_a^c\left(\sqrt{\frac{c+x}{c-x}} + \sqrt{\frac{c-x}{c+x}}\right)dx$$

$$\left(K_I^{\sigma_{ys}}\right)^{b} = -2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right)$$

problem b