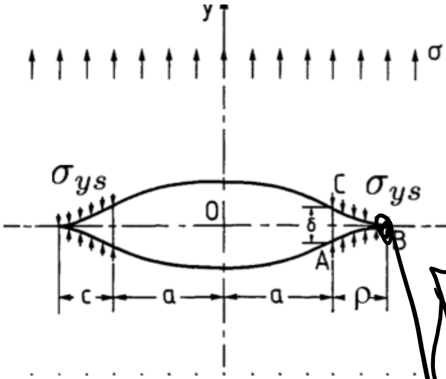
2020/09/30

Wednesday, September 30, 2020   2:52 PM

Continued from the last time:



Superposition principle

$$K_I = K_I^{\sigma} + K_I^{\sigma_{ys}}$$

$$K_I^{\sigma} = \sigma\sqrt{\pi(a+\rho)}$$

$$K_I^{\sigma_{ys}} = +2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right)$$

after requiring K @ a+ρ location = 0

$$K = K_I^{\sigma} + K_I^{\sigma_{ys}} = 0 \rightarrow$$

$$2\sigma_{ys}\sqrt{\frac{\cancel{a+\rho}}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right) = \sigma\sqrt{\pi(\cancel{a+\rho})}$$

$$\cos\left(\frac{\pi\sigma}{2\sigma_{ys}}\right) = \frac{a}{a+\rho} \qquad \rho = ?$$

$$\frac{\sigma}{\sigma_{ys}} \ll 1 \quad \underline{SSY}$$

$\cos x = 1 - \frac{x^2}{2}$ for small $x = \frac{\pi}{2}\frac{\sigma}{\sigma_{ys}}$

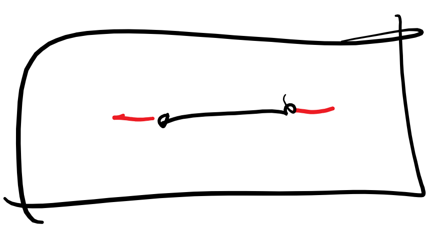
We obtain: Strip yield model

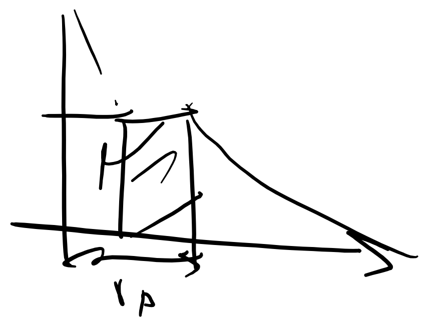
$$\boxed{\rho = \frac{\pi^2\sigma^2 a}{8\sigma_{ys}^2} = \underbrace{\frac{\pi}{8}}_{.392}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

Irwin

$$\boxed{r_p = \underbrace{\frac{1}{\pi}}_{.318}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

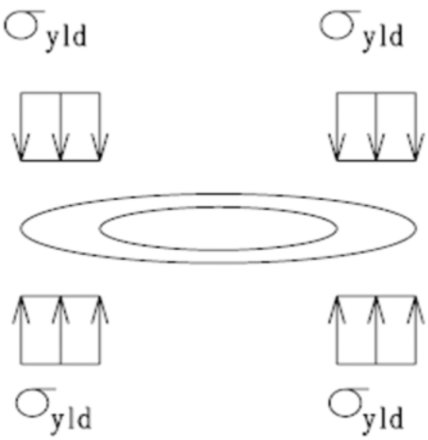
with 1D stress redistribution





3. Strip Yield Model: Dugdale vs Barenblatt model

Dugdale: Uniform stress



More appropriate for <u>polymers</u>

Barenblatt: Linear stress

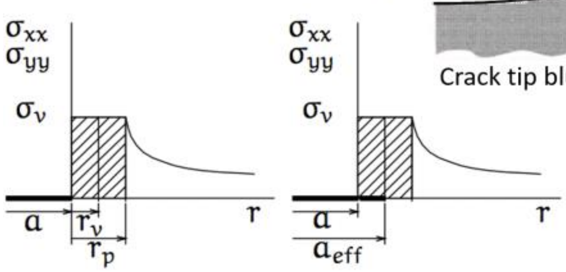
More appropriate for <u>metals</u>

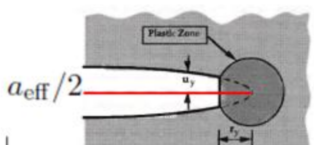
182

These are predecessors to Traction-Separation-Relations (TSRs) or cohesive models that we will later discuss.

Effective crack length

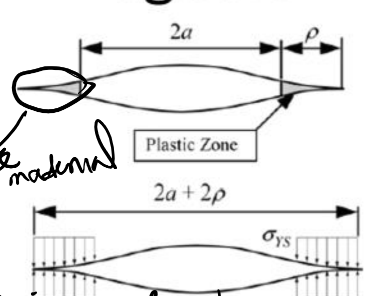
Irwin





Crack tip blunting

Dugdale

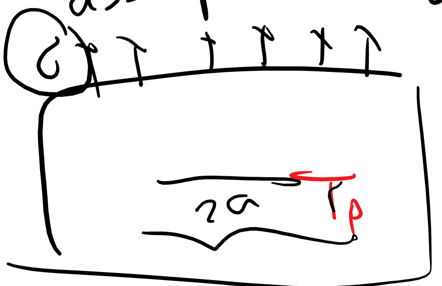


since the material is yielding here, it is as if the effective crack length is longer than 2a

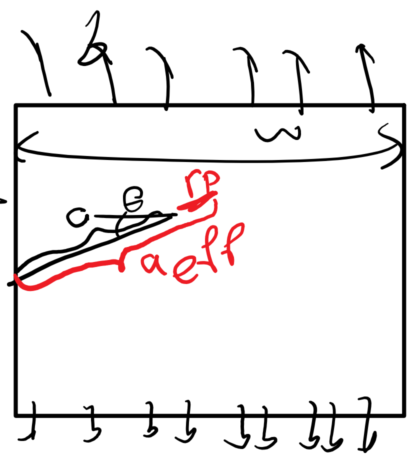
$a \longrightarrow a_{eff} = a + \rho \longrightarrow$ Fracture Process Zone (FPZ) size

- the applicability of LEFM theory is increased!

This means we can apply the theory for slightly higher loads that we could have had with SSY assumption & still get acceptable results



$\sigma\nearrow \rightarrow r_p\nearrow$



$$\boxed{\begin{aligned} &(1)\quad K_{eff} = f(a_{eff}, W, \theta, \ldots)\sqrt{\pi a_{eff}}\,\sigma \\ &(2)\quad a_{eff} = a + r_p = a + \frac{1}{\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2 \end{aligned}}$$

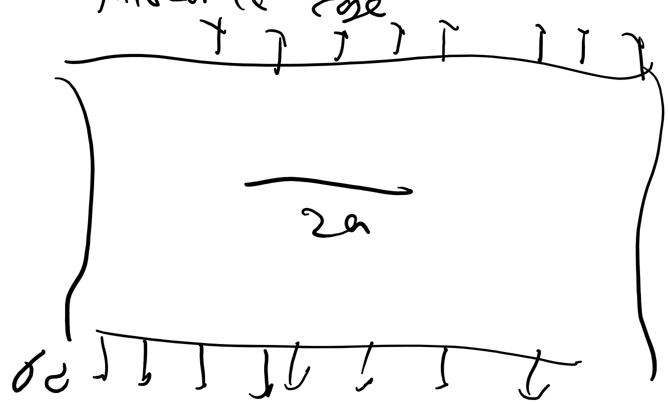
2 nonlinear equation, 2 unknowns

$\longrightarrow$ Iterations needed in general.

Idea (1) $f(a, W, \theta)\sqrt{\pi a}$ $\quad a_{eff} = a \rightarrow K_{eff}$ use a_{eff} 2

(2) $\rightarrow K_{eff} \rightarrow a_{eff}$

Infinite domain   Mid-crack case



$$\begin{cases} K_{eff} = f\sqrt{\pi a_{eff}}\,\sigma \\ a_{eff} = a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2 \end{cases}$$

1st order approximation

$$\hookrightarrow K_{eff} = \sqrt{\pi\left(a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2\right)}\,\sigma$$

$$K_{eff} = \sqrt{\pi\left(a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2\right)}\,\sigma$$

$$\to K_{eff}^2 = \pi a \sigma^2 + \frac{1}{2} K_{eff}^2 \left(\frac{\sigma}{\sigma_{ys}}\right)^2$$

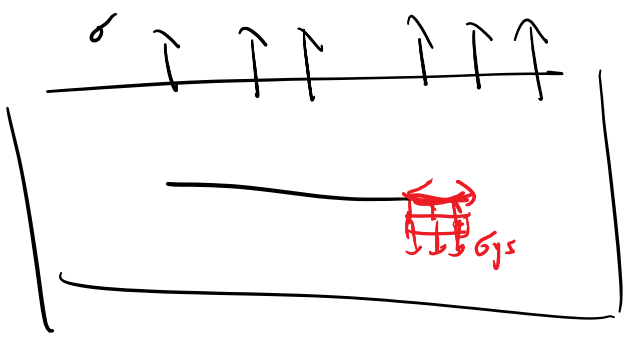
$$\to \boxed{K_{eff} = \frac{\sqrt{\pi a}\,\sigma}{\sqrt{1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}}$$

$\sqrt{\pi a}\,\sigma \to K$ without a_{eff} correction

$\frac{\sigma}{\sigma_{ys}} \ll 1$

we can ignore * term

$K_{eff} = \sqrt{\pi a}\,\sigma$



$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \ll 1$

LEFM :)

$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \lesssim 1$

σ getting close to σ_{ys}

a_{eff} idea extends the applicability of the theory

$\frac{\sigma}{\sigma_{ys}} \gg 1$

PFM

↓

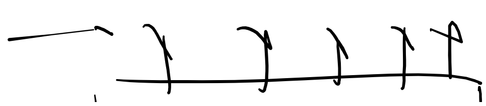
other theories

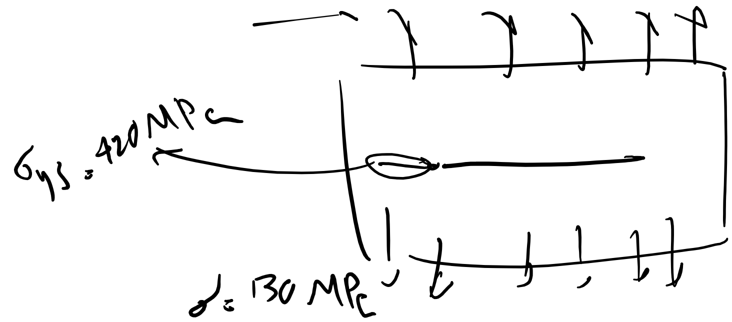
Example:

Consider a large central cracked plate subjected to a uniform stress of 130 MPa. The fracture toughness K_c=50MPa√m, the yield strength σ_{ys}=420MPa.

(a) What is the maximum allowable crack length? without correction

(b) What is the maximum crack length if plastic correction is taken into account. Plane stress and Irwin's correction.





$\frac{\sigma}{\sigma_{ys}} = \frac{130}{420} \approx .2 \rightarrow .3$ rel. large

use the correction

(a) part

$K_{Ic} = \sigma\sqrt{\pi a} \rightarrow$

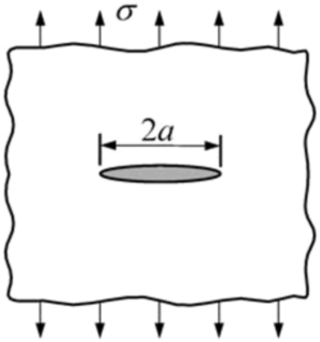
$K_{Ic} = 420$, $\sigma = 130$

$\boxed{a = 94.2\text{mm}}$

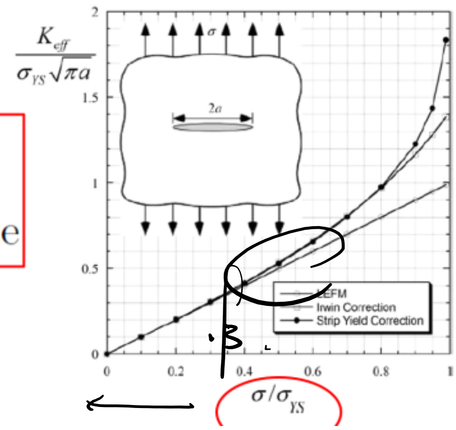
(part b)

$\text{MPa}\sqrt{\text{m}}\ 420 = K_{Ic} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}} = \frac{130\sqrt{\pi a}}{\sqrt{1 - \left(\frac{130}{420}\right)^2 \times \frac{1}{2}}} \rightarrow$

$a = \ldots m \rightarrow \boxed{a_{eff} = 89.7\text{mm}}$



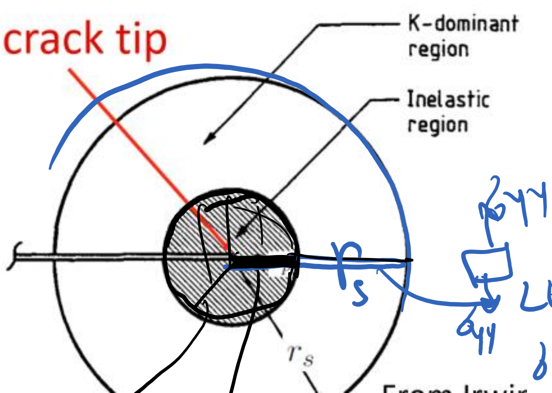
$$K_{eff} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{YS}}\right)^2}}$$



$\boxed{\text{As } \frac{\sigma}{\sigma_{ys}} \text{ increases} \Rightarrow \text{LEFM becomes less accurate}}$

$\left(\frac{\sigma}{\sigma_{ys}}\right) \approx .1$

Relating this to SSY:

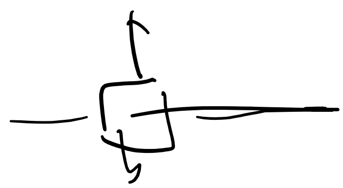


LEFM leading term is dominant

$\sigma = \frac{K}{\sqrt{2\pi r}}$

$r \to \infty$

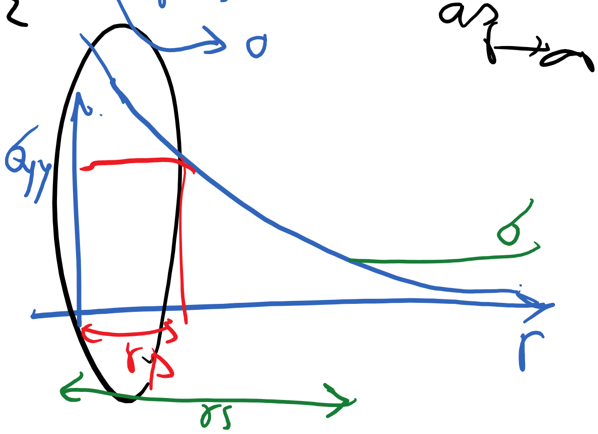
$\to \sigma$



$\sigma_{yy} \to \sigma$

$$r_p = \frac{1}{\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

$r_s = ?$



$$\sigma(r_s, \theta = 0) = \frac{K}{\sqrt{2\pi r_s}} = \sigma$$

$$\boxed{r_s = \frac{1}{2\pi}\left(\frac{K}{\sigma}\right)^2}$$

$$r_s = \frac{1}{2\pi}\left(\frac{K}{\sigma}\right)^2$$

$$\boxed{r_p = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2} \rightarrow \boxed{\frac{r_p}{r_s} = \left(\frac{\sigma}{\sigma_{ys}}\right)^2}$$

$$\frac{r_p}{r_s} \ll 1$$

SSY $\frac{r_p}{r_s}, \frac{r_p}{a}, \frac{r_p}{W}, \frac{r_p}{H} \ll 1$

all relevant lengths

LEFM provides accurate global response
& local = outside FPZ

$$\frac{r_p}{r_s} \lesssim .1 \qquad \frac{\sigma}{\sigma_{ys}} \lesssim .3$$

Plastic yield criteria

von-Mises criterion

$$\sigma_v = \sqrt{3J_2}$$
$$= \sqrt{\frac{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)}{2}}$$
$$= \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}}$$
$$= \sqrt{\tfrac{3}{2}\, s_{ij}s_{ij}}$$

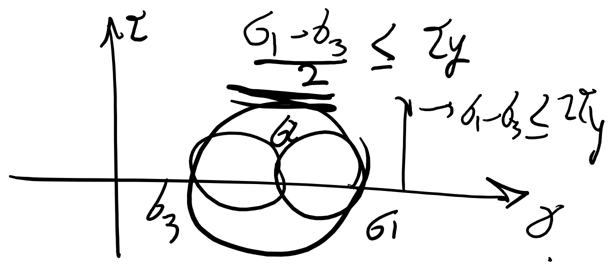
s is stress deviator tensor

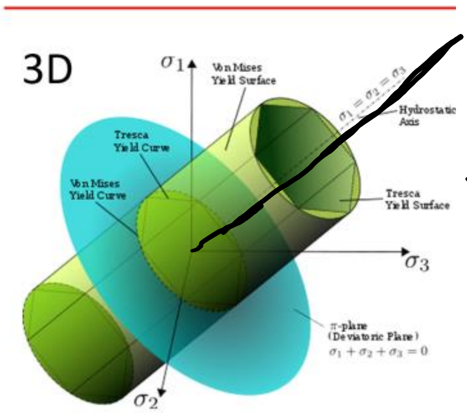
$$\boldsymbol{\sigma}^{dev} = \boldsymbol{\sigma} - \frac{1}{3}(\mathrm{tr}\,\boldsymbol{\sigma})\mathbf{I}$$

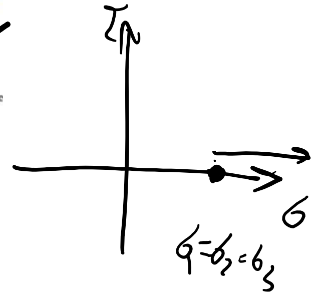
Tresca criterion
Maximum shear stress

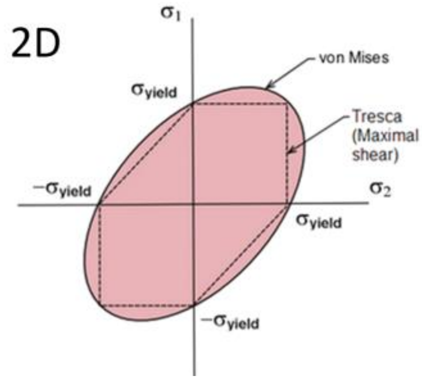
$$\sigma_{tresca} = \sigma_1 - \sigma_3 > \sigma_{max}$$

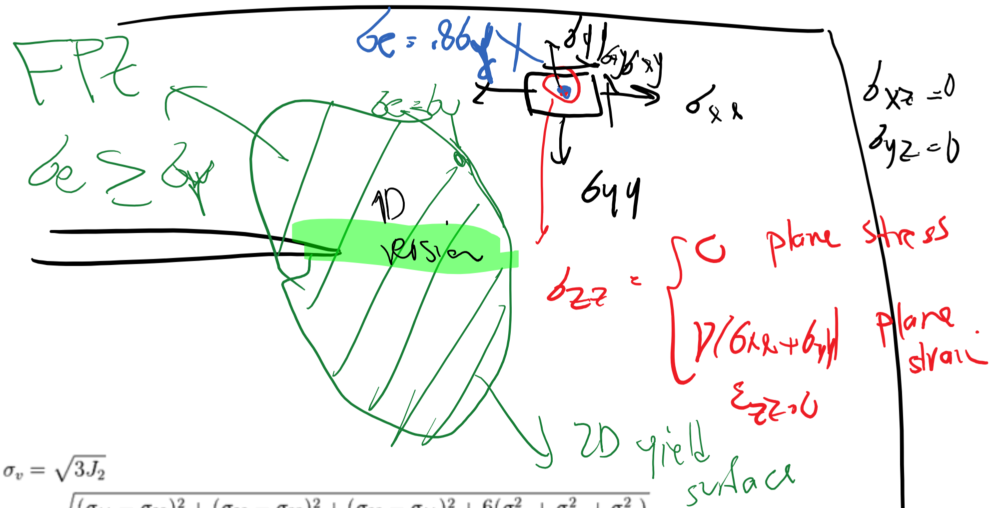
$$\frac{\sigma_1 - \sigma_3}{2} \le \tau_y$$











$$\sigma_v = \sqrt{3J_2}$$
$$= \sqrt{\frac{(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)}{2}}$$

von-Mises criterion

$$\sigma_e = \sigma_{ys}$$

Principal stresses:

Mode I, principal stresses

$$\begin{cases} \sigma_1 = \dfrac{K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1 + \sin\dfrac{\theta}{2}\right) \\ \sigma_2 = \dfrac{K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\right) \end{cases}$$

$$\sigma_3 = \begin{cases} 0 & \text{plane stress} \\ \dfrac{2\nu K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2} & \text{plane strain} \quad \nu(\sigma_1+\sigma_2) \end{cases}$$

$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right]}$$ plane stress

$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[(1-2\mu)^2(1+\cos\theta)+\frac{3}{2}\sin^2\theta\right]$$ plane strain

191

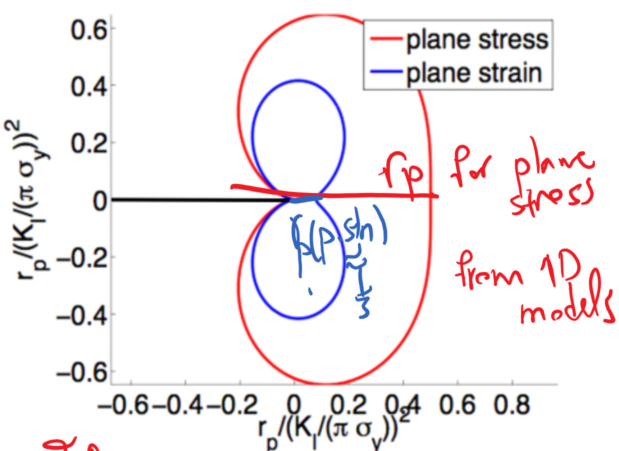
we had this from 1D models

angle-dependent terms

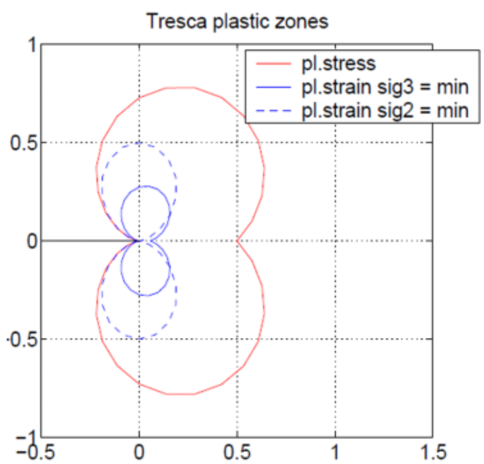
von-Mises criterion

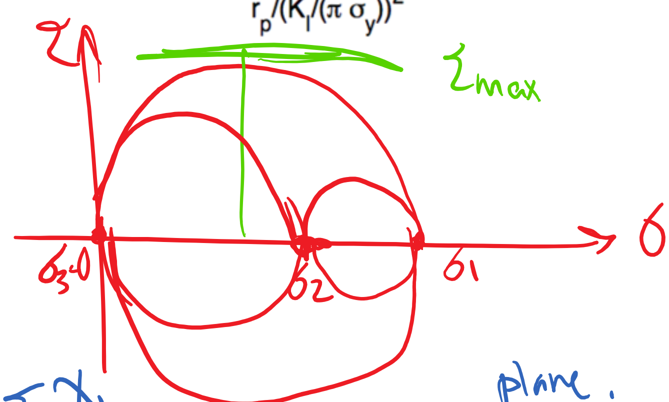
$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right]}$$

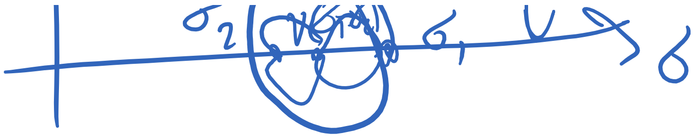
plane stress



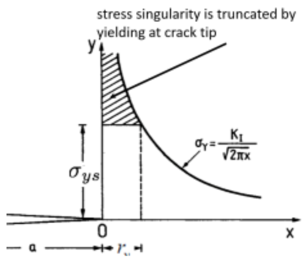
Tresca criterion

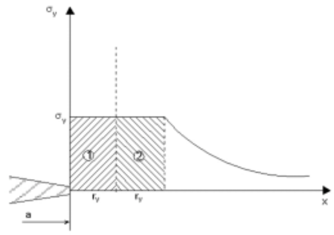




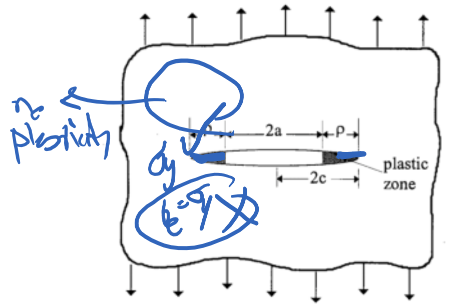


Remember 1D analyses





(b) Real crack with plastic zone



Strip yield model

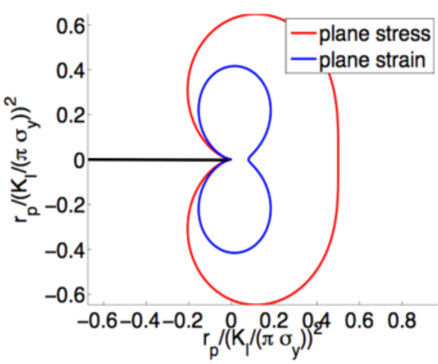
2D

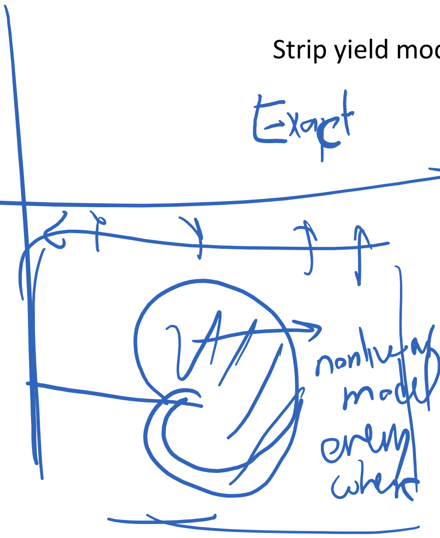
Exact

von-Mises criterion

$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right]}$$

plane stress



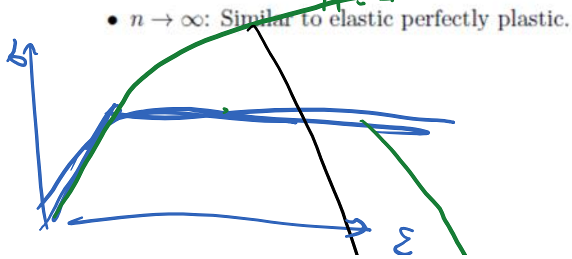


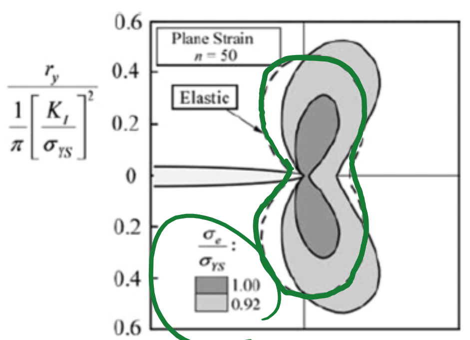
Dodds, 1991, FEM solutions
Ramberg-Osgood material model

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n$$

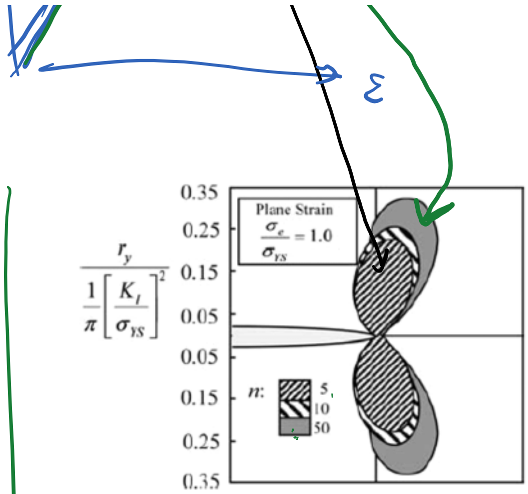
$n \to \infty$

- Low n: High strain-hardening.
- $n \to \infty$: Similar to elastic perfectly plastic.



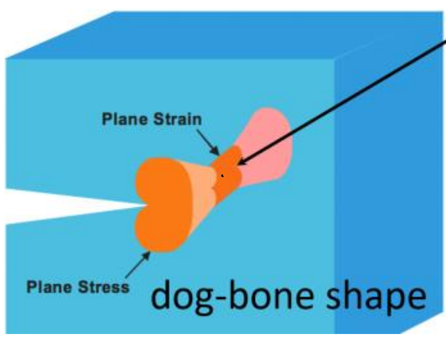


Effect of definition of yield (some level of ambiguity)

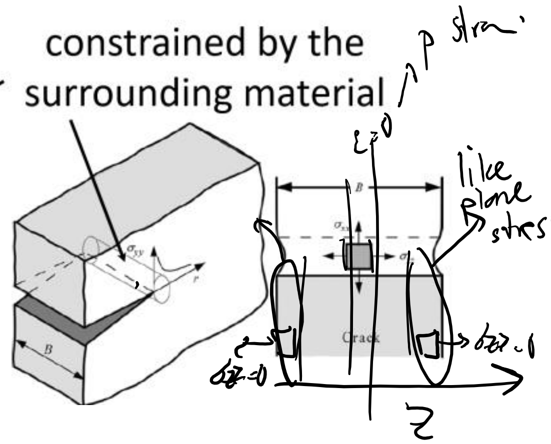


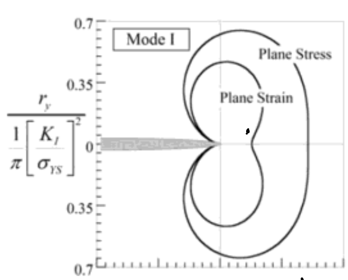
Effect of strain-hardening:
Higher hardening (lower n) => smaller zone

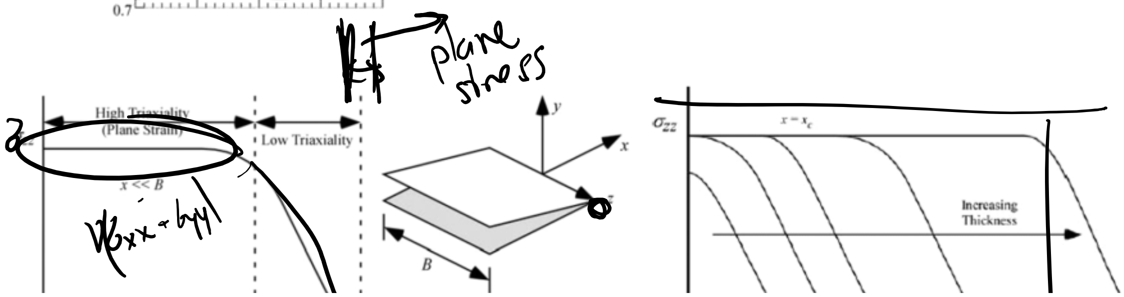
Plane stress/plane strain

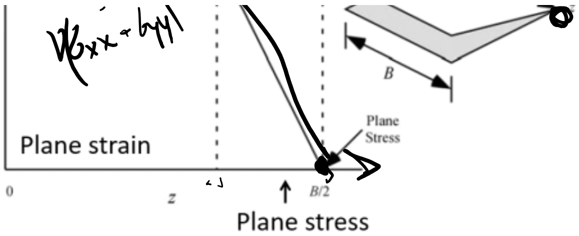


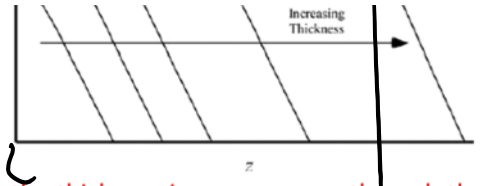
constrained by the surrounding material





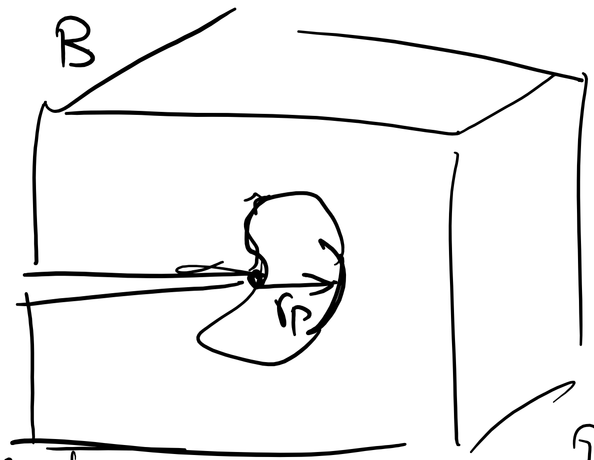






As the thickness increases more through the thickness bahaves as plane strain

Fracture Mechanics $r_p \propto \left(\frac{K_I}{\sigma_y}\right)^2$



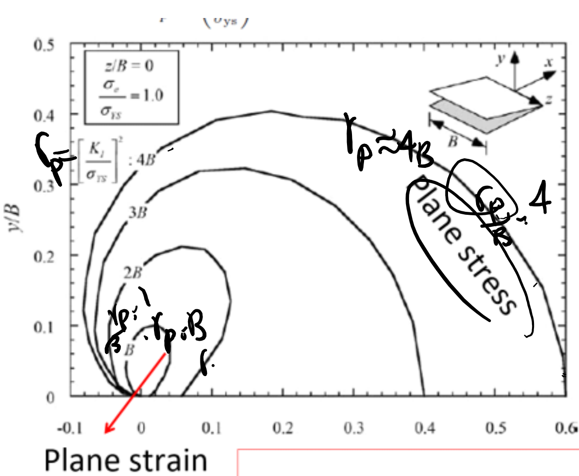
$\frac{r_p}{B}$	very small	p. strain
$\frac{r_p}{B}$	large	p stress

$B \uparrow$

r_p fixed

B changed and look @

FP2 size & shape change



$$\frac{\left(\frac{K}{\sigma_{ys}}\right)^2}{B} \propto \frac{r_p}{B} : \begin{cases} \text{low (high } B) & \text{plane strain} \\ \text{high (low } B) & \text{plane stress} \end{cases}$$

Change of plastic loci to plane stress mode as "relative B decreases". Nakamura & Park, ASME 1988

For plane strain condition we must have $B > \left(\frac{K}{\sigma_{ys}}\right)^2 \propto r_p$