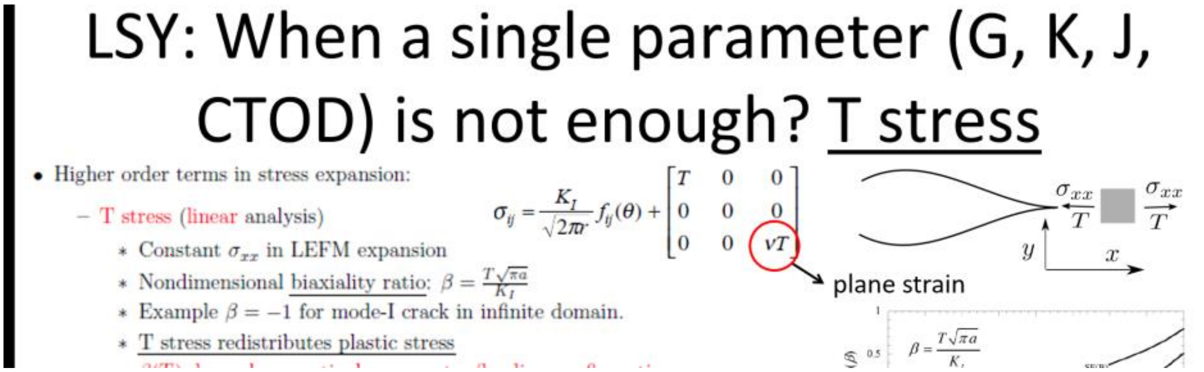
2020/10/14

Wednesday, October 14, 2020 2:50 PM

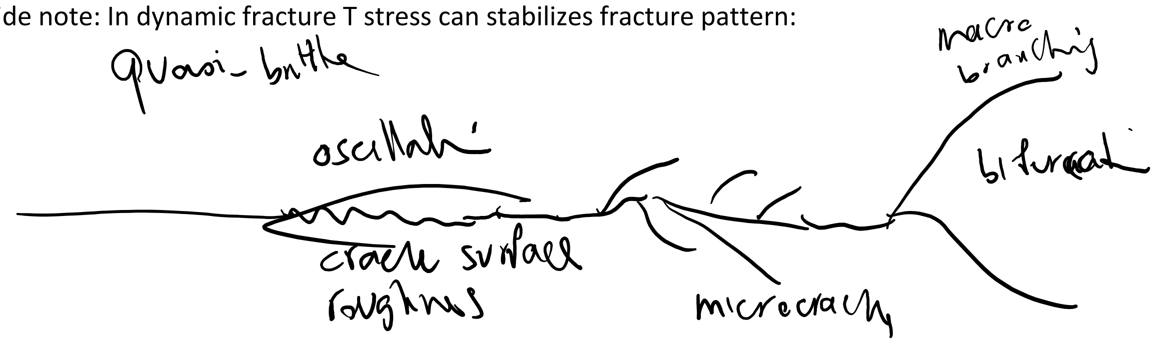
LSY we need something beyond J to characterize the response (global crack propagation, local stress solution, etc.)

If not using advanced computational tools, one approach is to use Q + some measure as mentioned above.



$\sigma_{xx} = f(\theta, \theta = 0) = \frac{K_I}{\sqrt{2\pi r}} + T$

Side note: In dynamic fracture T stress can stabilizes fracture pattern:

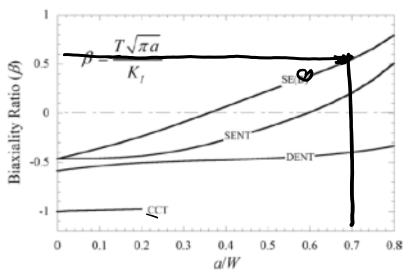


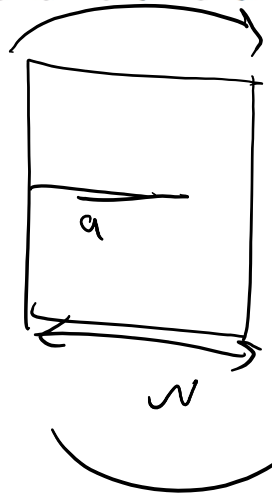
T ↗ more tensile these instabilities are retarded

* Constant σ_{xx} in LEFM expansion $\begin{bmatrix} 0 & 0 & \nu T \end{bmatrix}$
* Nondimensional biaxiality ratio: $\beta = \frac{T\sqrt{\pi a}}{K_I}$
* Example $\beta = -1$ for mode-I crack in infinite domain.
* T stress redistributes plastic stress
* $\beta(T)$ depend on particular geometry/loading configuration
* Effect of $T(\beta)$ on toughness:

High (+) $T \Rightarrow$ Constrained (triaxial) stress $\Rightarrow$ Toughness $\searrow$ Ductility $\searrow$

Low (-) $T \Rightarrow$ Lose constraint $\Rightarrow$ Toughness $\nearrow$ Ductility $\nearrow$

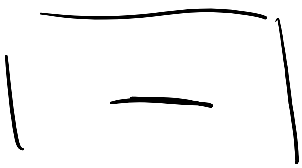




$a/W = .7$

$$\beta = .5 = \frac{T\sqrt{\pi a}}{K_I}$$

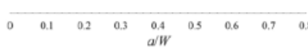
$$T = \frac{.5\,K_I}{\sqrt{\pi a}}$$



$\beta = -1$

$T \nearrow$ more tensile $\Rightarrow R \searrow$

* T stress also influnces crack path stability (particulary in dynamic fracture)

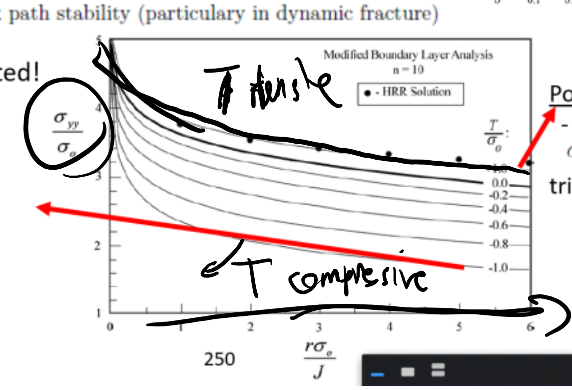


Plastic analysis: σ_{yy} is redistributed!

Kirk, Dodds, Anderson

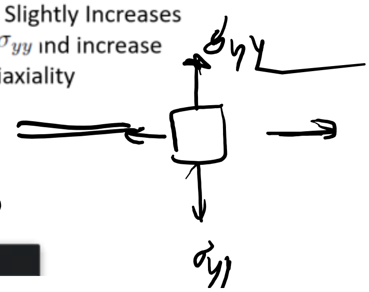
High negative T stress:

- Decreases σ_{yy}
- Decreases triaxiality



Positive T stress:

- Slightly Increases σ_{yy} and increase triaxiality



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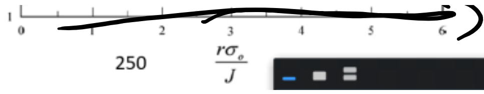
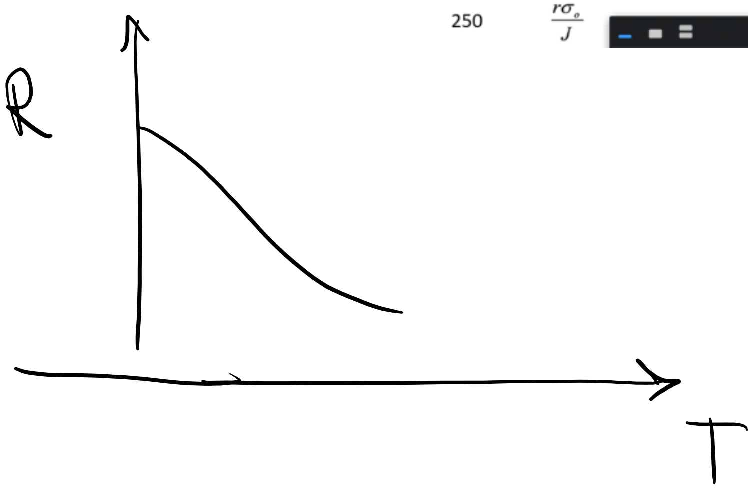


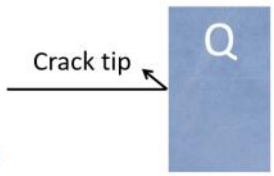
σ_y



LSY: When a single parameter (G, K, J, CTOD) is not enough? J-Q theory

- Q parameter (J-Q theory) Valid for nonlinear analysis
  - Added as a hydrostatic shift in front of crack to (HRR) stress fields

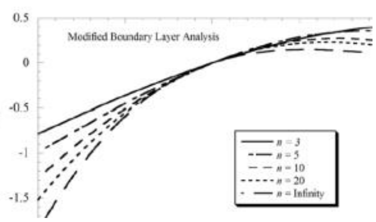
$$\sigma_{ij} \approx (\sigma_{ij})_{T=0} + Q\sigma_0\delta_{ij} \quad \left(|\theta| \leq \frac{\pi}{2}\right)$$



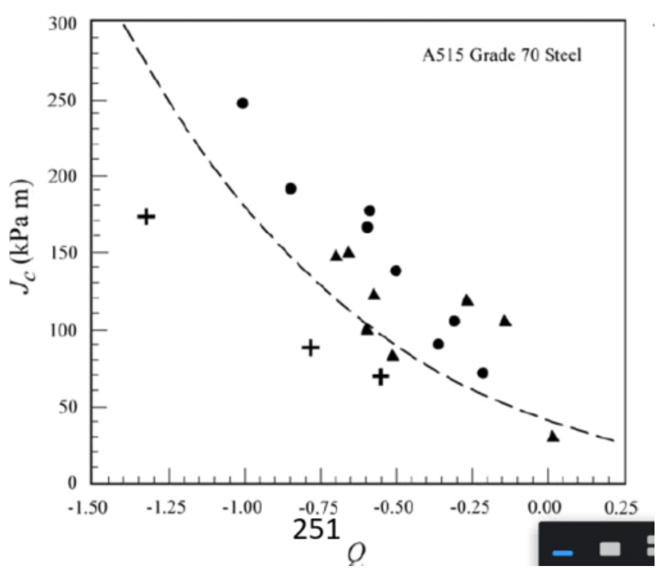
  - Similar to T positive Q increases triaxiality and reduces fracture resistance

$$J_c = J_c(Q)$$

High (+) $Q \Rightarrow$ Constrained (triaxial) stress $\Rightarrow$ Toughness ↘ Ductility ↘
Low (-) $Q \Rightarrow$ Lose constraint $\Rightarrow$ Toughness ↗ Ductility ↗

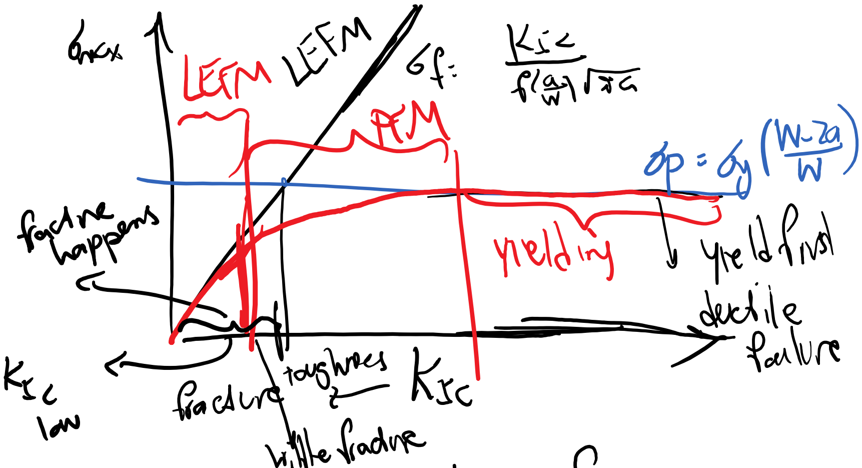


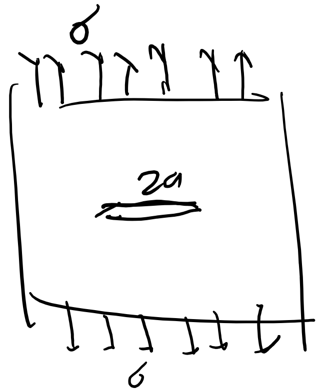
- More number of parameters: With extensive deformation two-parameter models such as K, T or J, Q evnetully break.



5.3. 7. Fracture mechanics versus material (plastic strength

Governing fracture mechanism and fracture toughness





– fracture

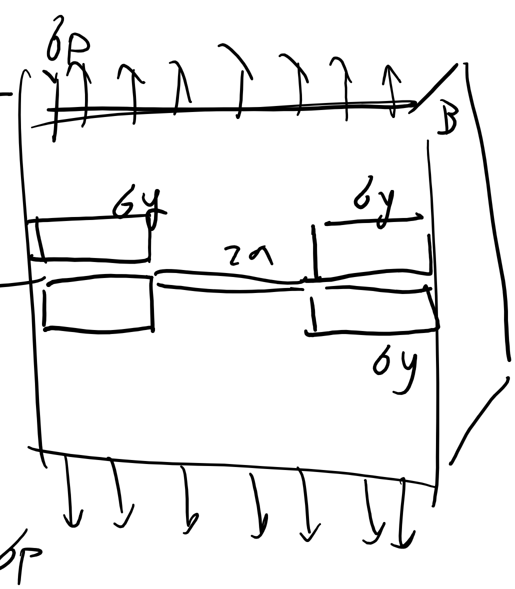
$$K_I = f\left(\frac{a}{W}\right)\sqrt{\pi a}\,\sigma$$

for fracture

$$K_{Ic} = f\left(\frac{a}{W}\right)\sqrt{\pi a}\,\sigma_f \rightarrow \boxed{\sigma_f = \frac{K_{Ic}}{f(\frac{a}{W})\sqrt{\pi a}}}$$

– Yielding

$$\sigma_p \times W \times B$$



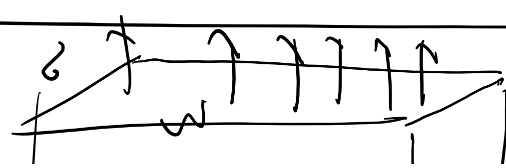
$$\sigma_y \times (W-2a)B$$

$$\rightarrow \boxed{\sigma_p = \sigma_y\left(\frac{W-2a}{W}\right)}$$

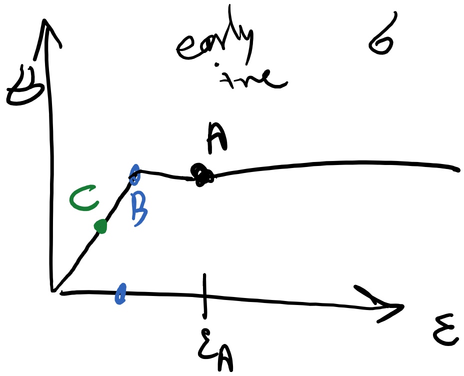
constant

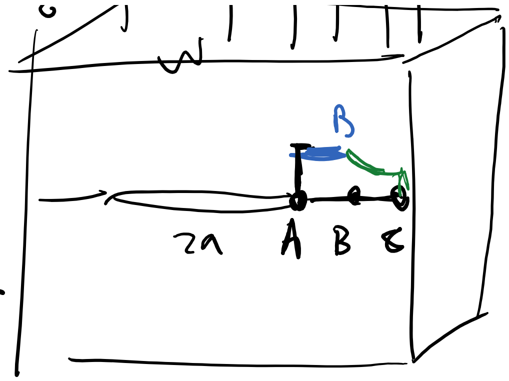
---

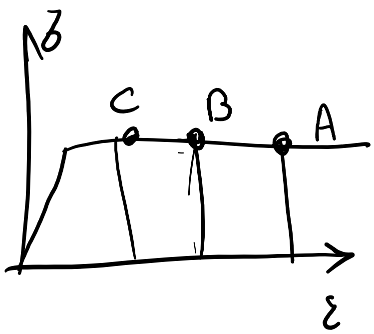
a changes

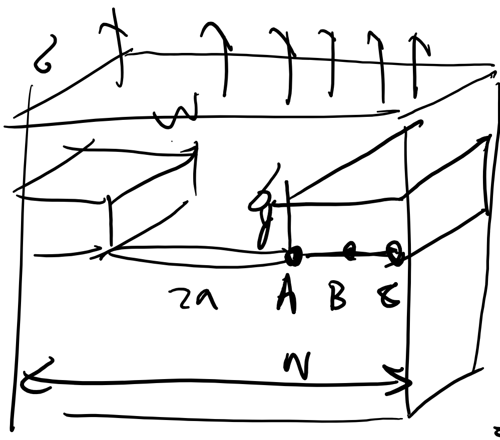


a changes





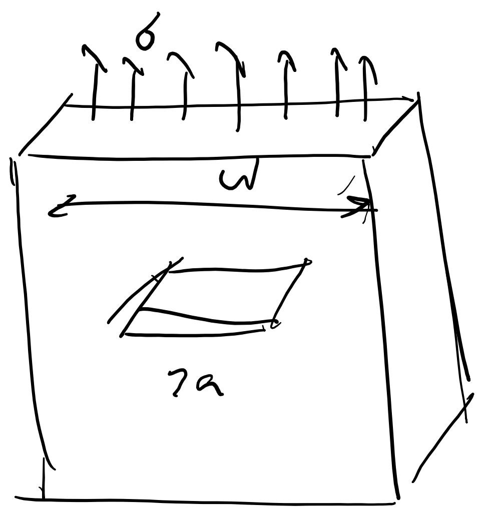




positive force $= \sigma_y (W - 2a) B = \sigma_P W B$

$$\boxed{\sigma_P = \left(\frac{W - 2a}{W}\right) \sigma_y} \quad (1)$$

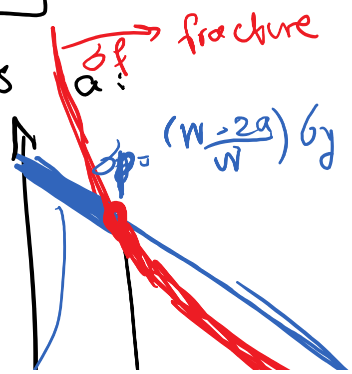
Fracture



$$K_I = f\left(\frac{a}{W}\right) \sqrt{\pi a}\, \sigma$$

$$\rightarrow \boxed{\sigma_f = \frac{K_{Ic}}{f\left(\frac{a}{W}\right) \sqrt{\pi a}}} \quad (2)$$

plot σ_p, σ_f versus a:



$$\sigma_f = \frac{K_{Ic}}{f\left(\frac{a}{W}\right) \sqrt{\pi a}}$$

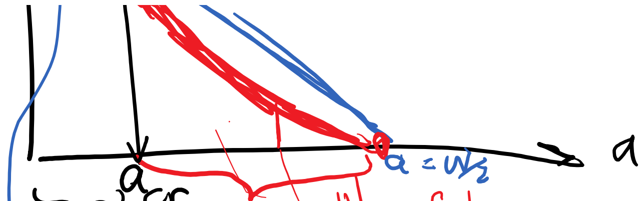
$\frac{a}{W} \rightarrow 0 \quad f\left(\frac{a}{W}\right) \rightarrow 1$

$\frac{a}{W} \to 0 \quad f\left(\frac{a}{W}\right) \to 1$

$\sigma_f = \frac{K_{Ic}}{\sqrt{\pi a}} \quad a \to 0$

$a \to \frac{W}{2}, \longrightarrow f\left(\frac{a}{W}\right) \to \infty$

$\sigma_f = 0$



$a < a_{cr}$

material yields first

ductile failure

$a \geq a_{cr}$

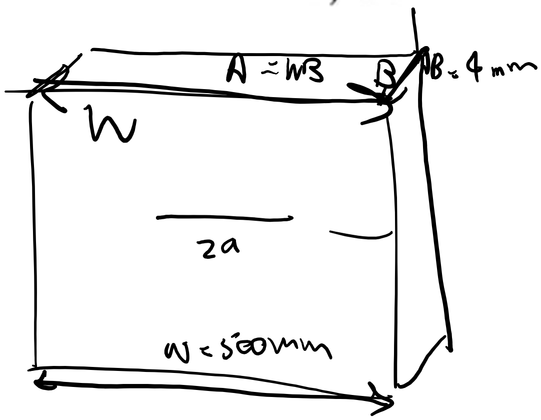
material fractures first

Example

Example 4.11 Estimate the failure load under uniaxial tension for a centre-cracked panel of aluminium alloy of width W=500 mm, and thickness B=4 mm, for the following values of crack length $2a$ = 20 mm and $2a$ = 100 mm. Yield stress σ_y =350 MPa and fracture toughness K_{Ic}=70 MPa $\sqrt{m}$

case 1 $\quad 2a = 20$ mm

case 2 $\quad 2a = 100$ mm



$\sigma_y = 350$ MPa

$K_{Ic} = 70$ MPa$\sqrt{m}$

Does it yield or fractures first?

case 1 $\quad 2a = 20$ mm $\to a = 10$ mm

yielding $\quad \sigma_y (W-2a) B = \sigma_p WB \to$

$\sigma_p = \sigma_y \frac{W-2a}{W} = 350 \times \frac{500-20}{500} \Rightarrow$

$\boxed{F_p = \sigma_p \times B \times W = 672\ \text{kN}}$

for fracture $\quad K_{Ic} = f\left(\frac{a}{W}\right) \sigma_p \sqrt{\pi a}$

$\to \sigma_f = \frac{K_{Ic}}{f\left(\frac{a}{W}\right)\sqrt{\pi a}} = \frac{70\ \text{MPa}\sqrt{m}}{\sqrt{\sec\left(\frac{\pi a}{W}\right)}\sqrt{\pi \times 10e{-3}\,m}}$

$500e\text{-}3$

$\sigma_f = 394.6\,MPa$

$F_f = \sigma_f\,(\overset{WB}{A}) = \boxed{790\,kN}$

The domain with $a = 10\,mm$ fails by yielding

$2a = 100\,mm$

$F_Y = 560\,kN$

$\boxed{F_f = 172.2\,kN}$

For longer crack the domain fails by fracture (brittle failure mode)

6. Computational fracture mechanics

6.1. Fracture mechanics in Finite Element Methods

6.2. Traction Separation Relations (TSRs)

6.1Fracture mechanics in Finite Element Methods (FEM)

6.1.1. Introduction to Finite Element method

6.1.2. Singular stress finite elements

6.1.3. Extraction of K (SIF), G

6.1.4. J integral

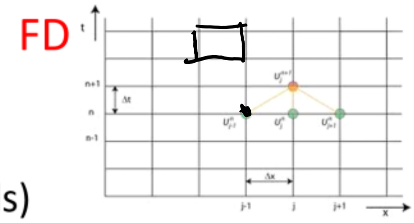
6.1.5. Finite Element mesh design for fracture mechanics

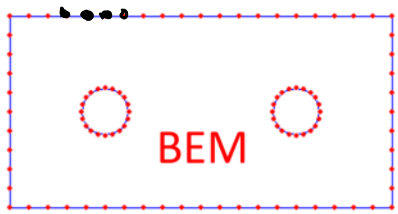
6.1.6. Computational crack growth

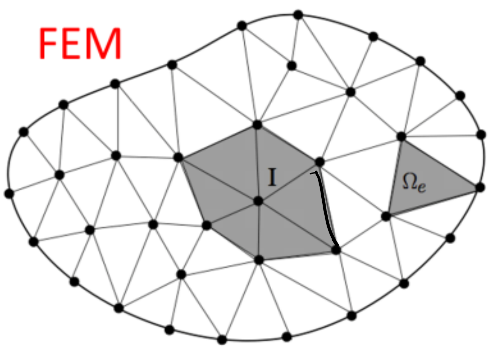
6.1.7. Extended Finite Element Method (XFEM)

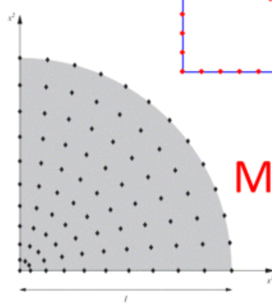
Numerical methods to solve PDEs

- Finite Difference (FD) & Finite Volume (FV) methods
- FEM (Finite Element Method)
- BEM (Boundary Element Method)
- MMs (Meshless/Meshfree methods)







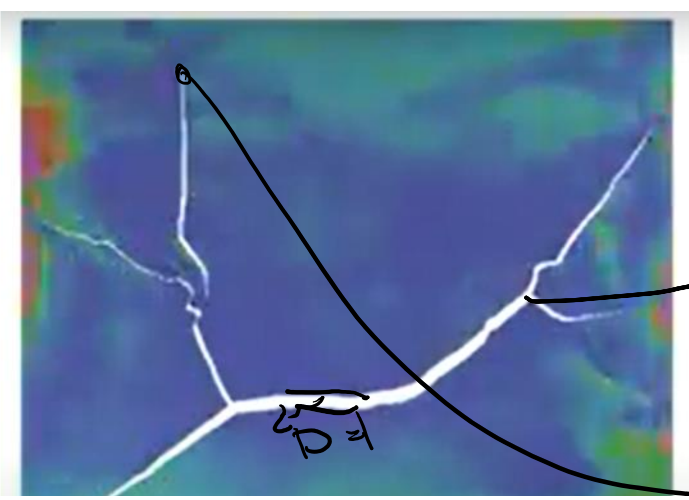


264

How is fracture and failure is modeled computationally?

Fracture models

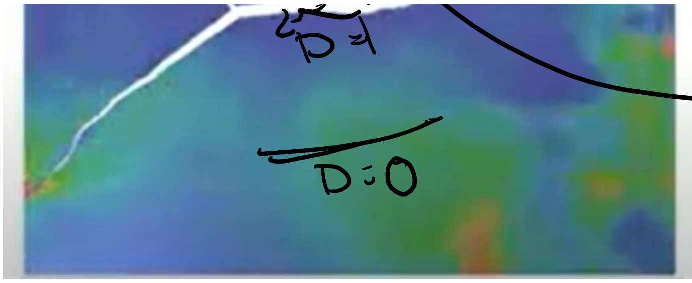
- Discrete crack models (discontinuous models):
  Cracks are explicitly modeled
  - LEFM
  - EPFM
  - Cohesive zone models



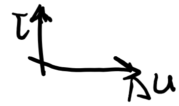
cracks

– cracks are explicitly modeled in the computational domain

LEFM + EPFM

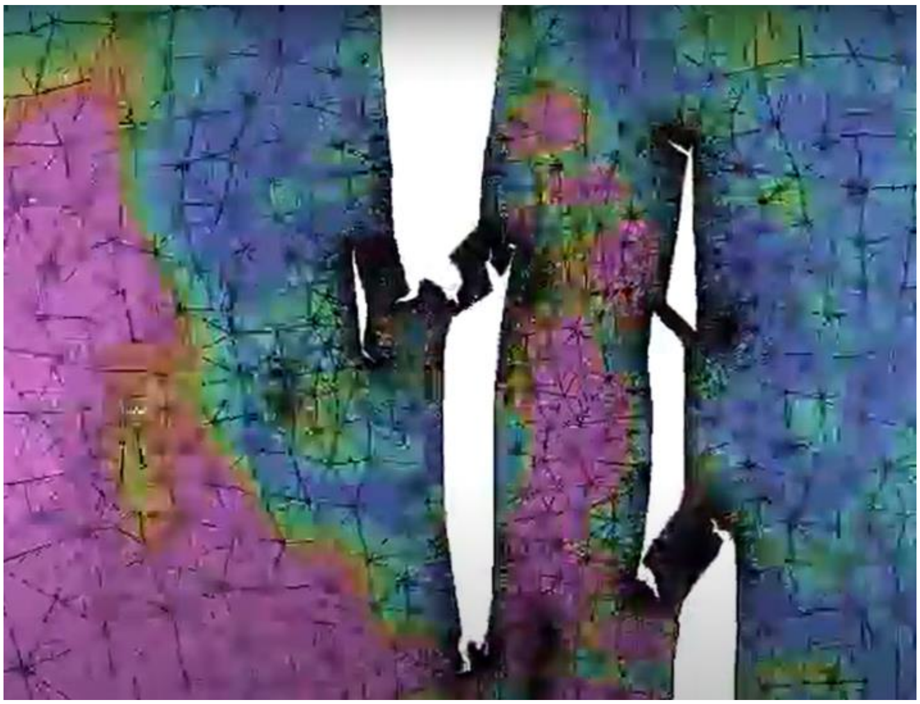


- LEFM + FEM
- cohesive model



- Interfacial damage models

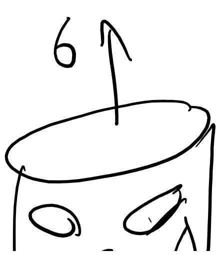
High fidelity models (as we directly model the cracks), but these models are expensive and computationally very challenging to track the cracks:

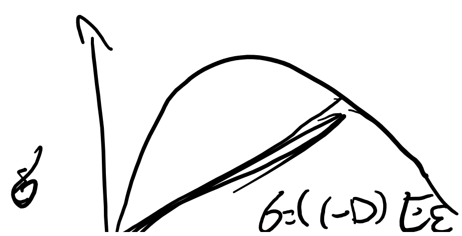


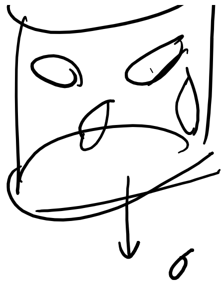
B

- Continuous models: Effect of (micro)cracks and voids are incorporated in bulk damage
  - Continuum damage models
  - Phase field models

regularizes LEFM



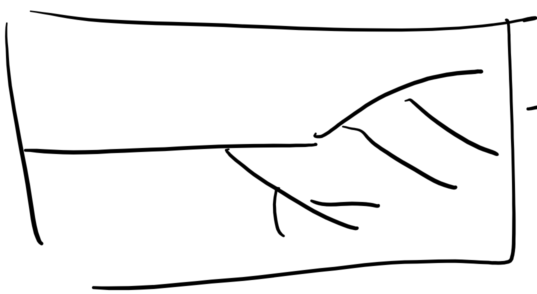


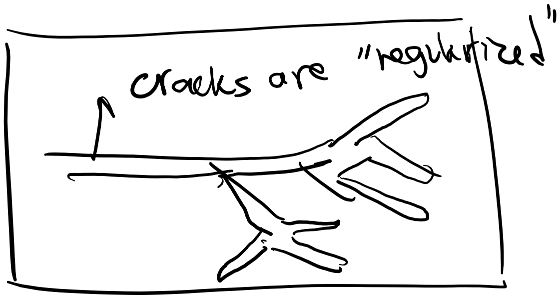


D: bulk damage parameter

D=0 perfect elastic

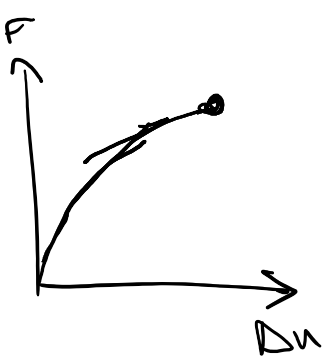
D=1 σ=0 fully failed material

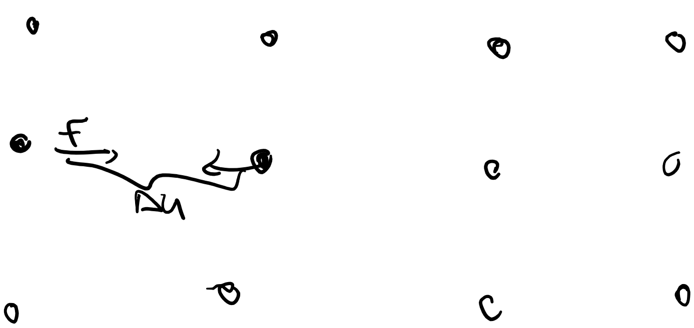




- Peridynamic models: Material is modeled as a set of particles

Discrete models:



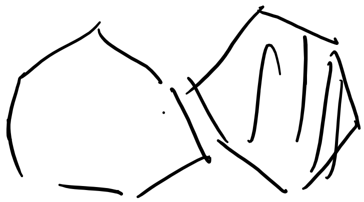


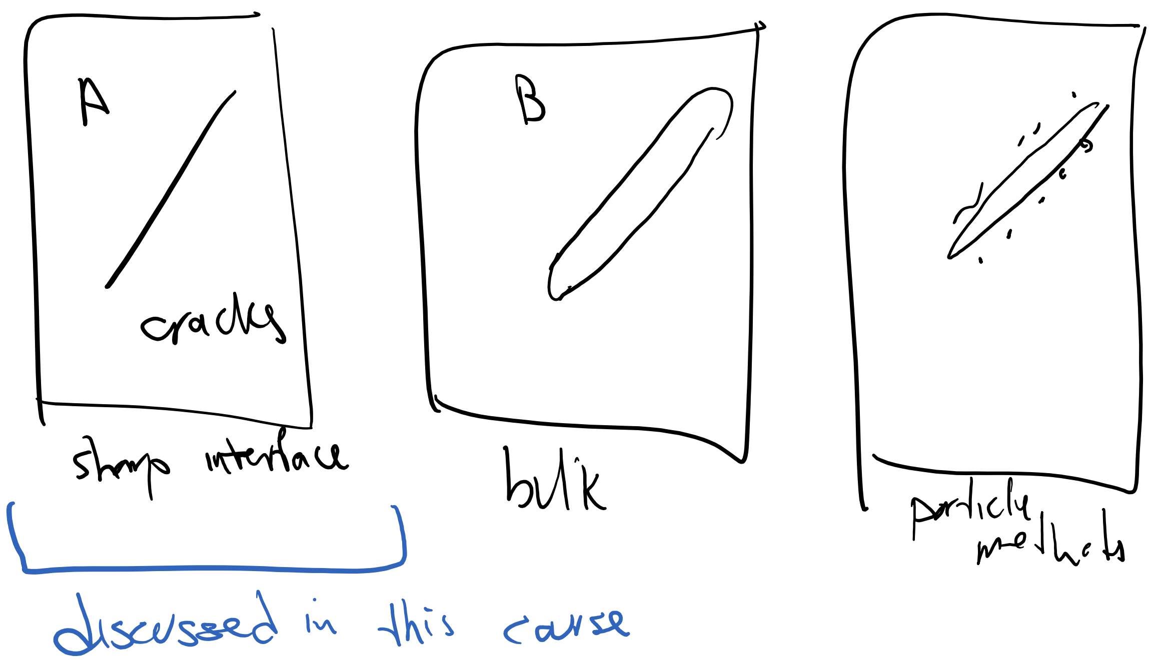
Peridynamic

or the "particles" can be

deformable or undeformable bodies

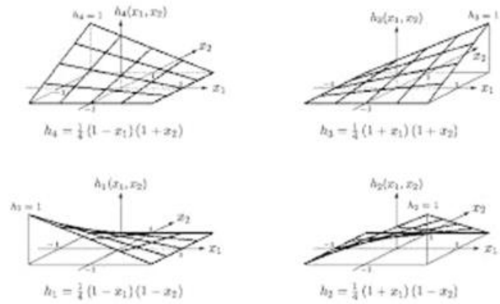
$\Rightarrow$ Discrete Element Method (Geomechanics)

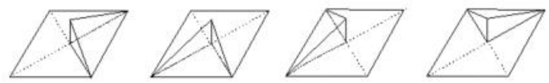




Next, we will discuss how to calculate K, G, J, etc. for LEFM / PFM theory from FEM solutions.

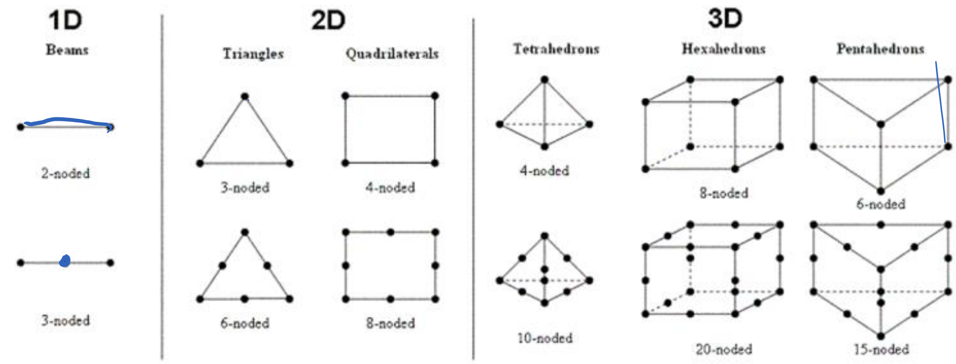
Finite Element Method





Global level: The nodal dof at the center node is shared by all four elements

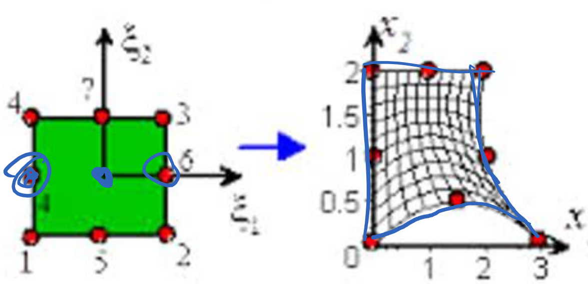
Element level: 4 Shape functions for a linear quad element



266

6.1.2. Singular stress finite elements

Isoparametric Elements



parent element

Actual element

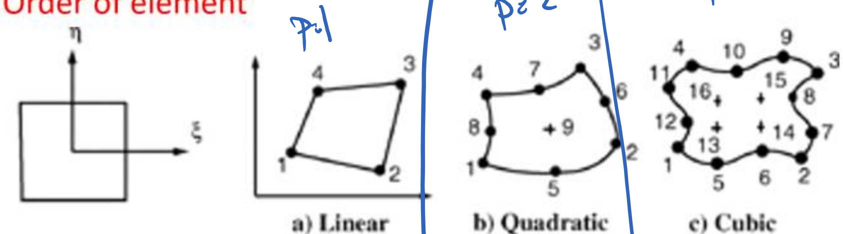
- Geometry is mapped from a parent element to the actual element
- The same interpolation is used for geometry mapping and FEM solution (in the figure 2nd order shape functions are used for solution and geometry)
- Geometry map and solution are expressed in terms of ξ

Order of element

p=1

p=2

p=3



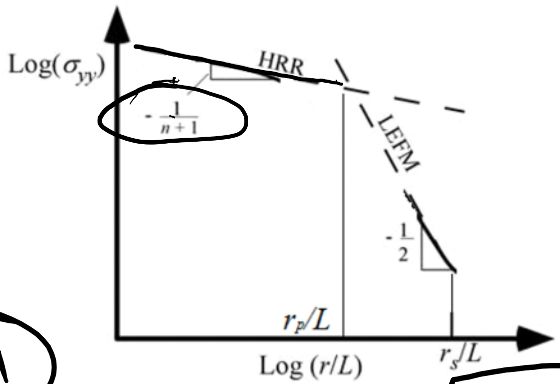
Interpolation space
(Same number of nodes based on the order)

Physical space

268

useful for fracture mechanics

Singular crack tip solutions



$\sigma \propto \left(\frac{1}{r}\right)^{\frac{1}{n+1}}$

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r}\right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n,\theta)$$

$$\epsilon_{ij} = \frac{\alpha\sigma_0}{E} \left(\frac{EJ}{\alpha \sigma_0^2 I_n r}\right)^{\frac{n}{n+1}} \tilde{\epsilon}_{ij}(n,\theta)$$

$\varepsilon \propto \left(\frac{1}{r}\right)^{\frac{n}{n+1}}$

case 1

$n = 1 \longrightarrow$ LEFM $\boxed{\varepsilon, \sigma \propto \frac{1}{\sqrt{r}}}$

case $n \to \infty$ Elastic - perfectly plastic

$\boxed{\varepsilon \propto \frac{1}{r}}$ $\sigma = O(\sigma_y)$

We can we design FEs that reproduce $\varepsilon \propto \frac{1}{\sqrt{r}}$ (LEFM) and $\varepsilon \propto \frac{1}{r}$ for E. Perfectly plastic

---

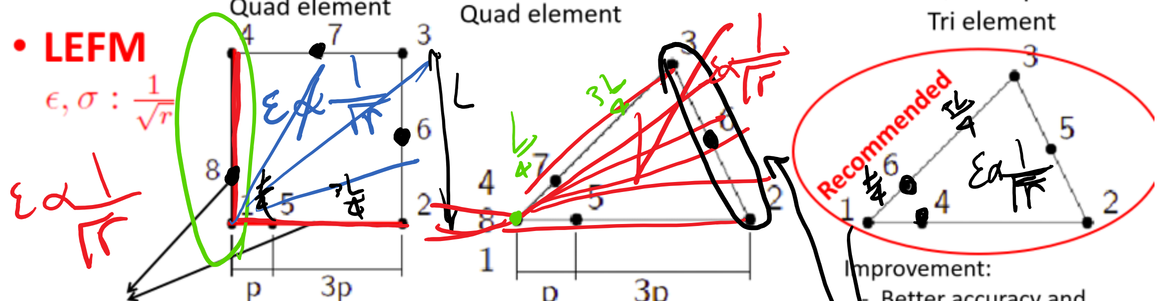
Isoparametric singular elements

Quarter point Quad element | Quarter point collapsed Quad element | $p=2$ Tri-Element, Quarter point Tri element

- **LEFM**

$\epsilon, \sigma : \frac{1}{\sqrt{r}}$

$\varepsilon \propto \frac{1}{\sqrt{r}}$



singular form $\frac{1}{\sqrt{r}}$ nly along these lines
NOT recommended

Improvement:
- $\frac{1}{\sqrt{r}}$ rom inside all element

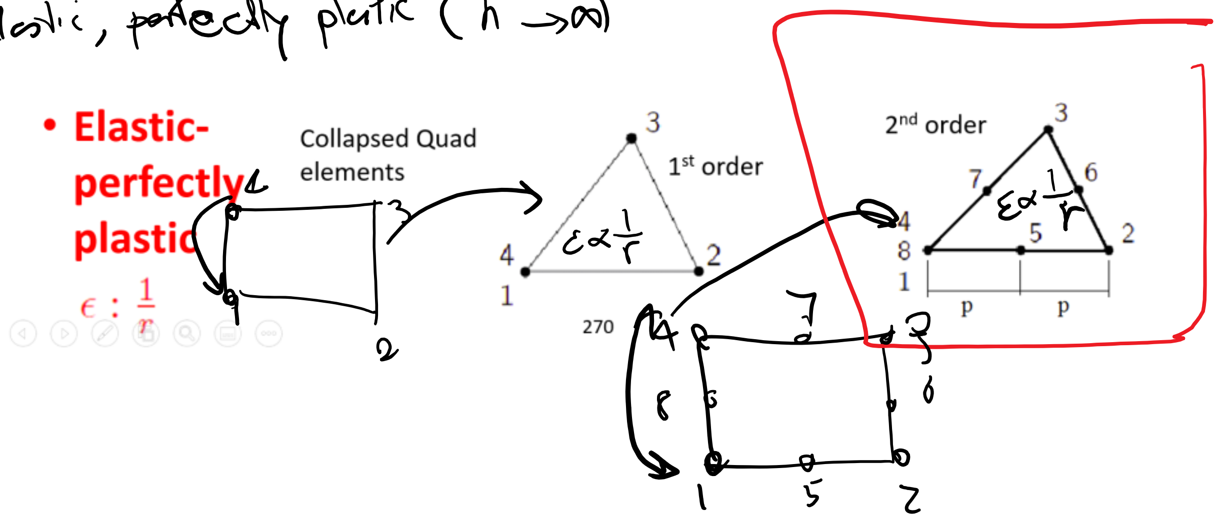
Problem
- Solution inaccuracy and sensitivity when opposite edge 3-6-2 is curved

Improvement:
- Better accuracy and less mesh sensitivity

bad for

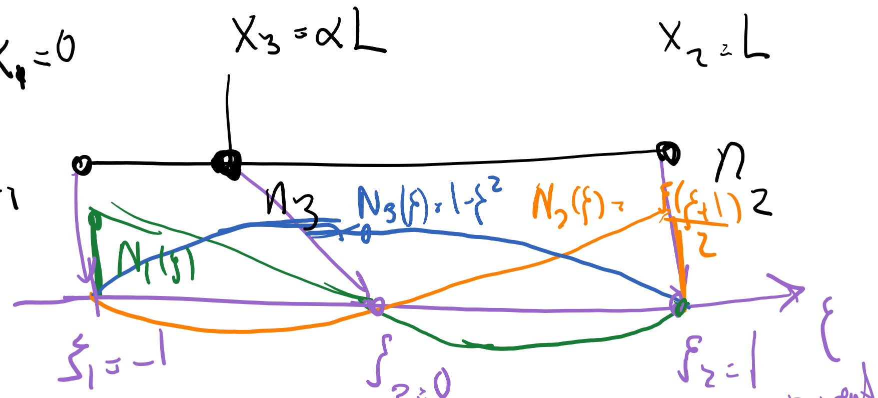
↓ good for LEFM

Elastic, perfectly plastic ($n \to \infty$)



Motivation:
Why 1/4, 3/4 ratio generates the singularity needed for FEM?

Real geometry



$\alpha = \frac{1}{4}$ $\quad \varepsilon \propto \frac{1}{\sqrt{x}}$

$x \sim r$

$$N_1(\xi) = \frac{(\xi-\xi_2)(\xi-\xi_3)}{(\xi_1-\xi_2)(\xi_1-\xi_3)} = \frac{(\xi-1)(\xi-0)}{(-1-1)(-1-0)} = \frac{\xi(\xi-1)}{2}$$

$$\boxed{u(\xi) = u_1 N_1(\xi) + u_2 N_2(\xi) + u_3 N_3(\xi)}$$

displacements @ node 1, 2, 3

u_1 $\quad u_2$ $\quad u_3$

$\varepsilon = \frac{du}{dx} \propto \frac{1}{\sqrt{x}}$ as $x \to 0$ for $\alpha = \frac{1}{4}$

$u(\xi)$ How to calculate $\frac{du}{dx}$