2020/10/26

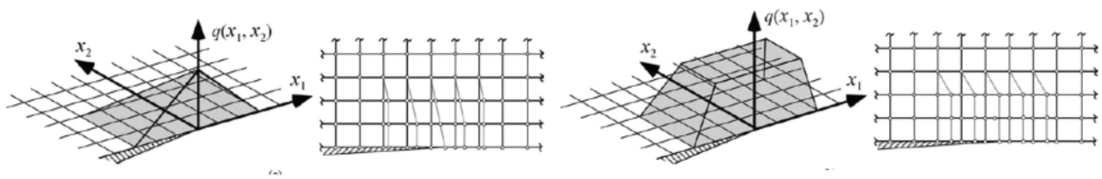
Monday, October 26, 2020 2:48 PM

Ansys, computation of J

```
>> sqrt(0.15890E-01 * 100)

ans =

   1.2606

>> sqrt(pi*0.5)

ans =

   1.2533
```

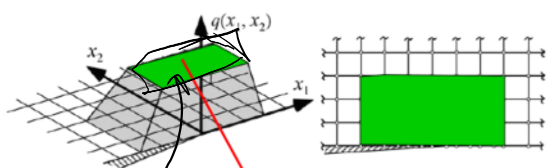
Continue with the theory of J integral computation using equivalent domain integration (EDI)

- Shape of decreasing function *q*:



Pyramid *q* function   Plateau *q* function

- Plateau q function useful when inner elements are not very accurate: e.g. when singular/quarter point elements are not used



$$\frac{\partial q}{\partial x_i} = 0$$ These elements do not contribute to J

$$J = \int_{A^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right]\frac{\partial q}{\partial x_j}dA$$

288

EDI for Jk

$$\bar{J}_k \Delta L = \int_{V^*} \left(\sigma_{ij} \frac{\partial u_i}{\partial x_k} \frac{\partial q}{\partial x_j} - w \frac{\partial q}{\partial x_k} \right) dV$$

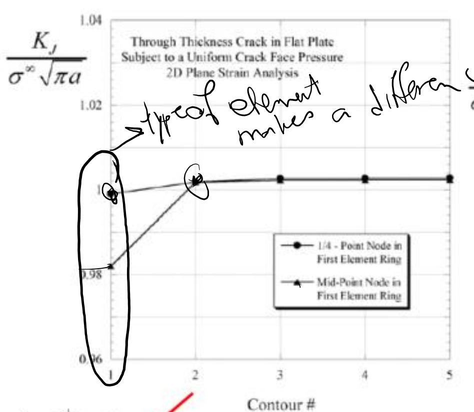
$$G_{III} = \int_{V^*} \left(\sigma_{3j} \frac{\partial u_3}{\partial x_1} \frac{\partial q}{\partial x_j} - w^{III} \frac{\partial q}{\partial x_1} \right) dV$$

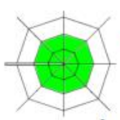
$$\begin{aligned} K_I &= \tfrac{1}{2}\sqrt{E^*}\left(\sqrt{(J_1 - J_2 - G_3)} + \sqrt{(J_1 + J_2 - G_3)}\right) \\ K_{II} &= \tfrac{1}{2}\sqrt{E^*}\left(\sqrt{(J_1 - J_2 - G_3)} - \sqrt{(J_1 + J_2 - G_3)}\right) \\ K_{III} &= \sqrt{2\mu G_3} \end{aligned}$$

Nikishkov and Vainshtok 1980

$$E^* = E\left[\frac{1}{1-\nu^2} + \left(\frac{\nu}{1+\nu}\right)\frac{\varepsilon_{33}}{\varepsilon_{11}+\varepsilon_{22}}\right] \qquad E^* = \frac{E}{1-\nu^2} \text{ for plane strain and } E^* = E \text{ for plane stress}$$

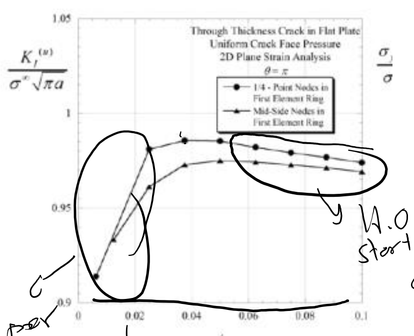
Which of these methods is the best in terms of accuracy of J integral?





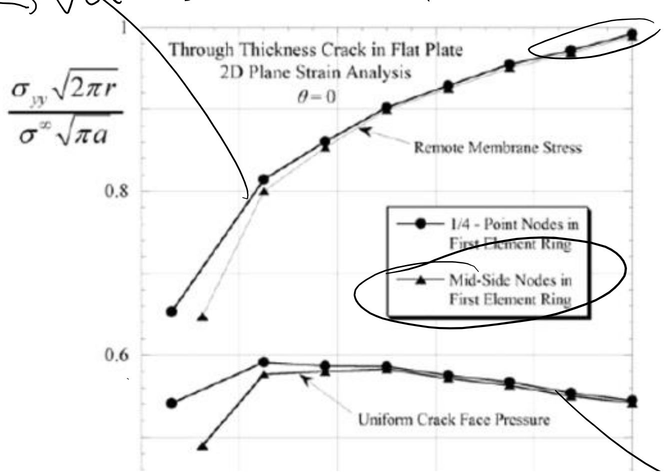
J integral EDI

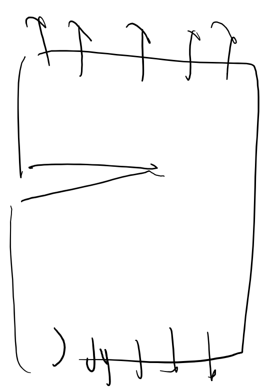
$$J = \int_{A^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right]\frac{\partial q}{\partial x_i}\, dA$$

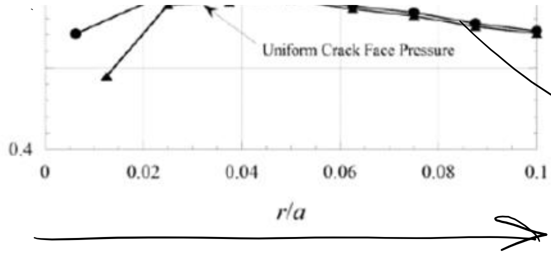


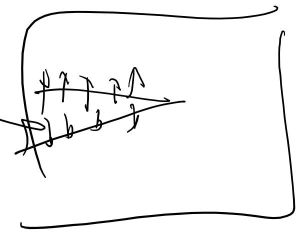
K from displacement *u*

$$K_I = \lim_{r\to 0}\left[\frac{E' u_y}{4}\sqrt{\frac{2\pi}{r}}\right] \quad (\theta = \pi)$$





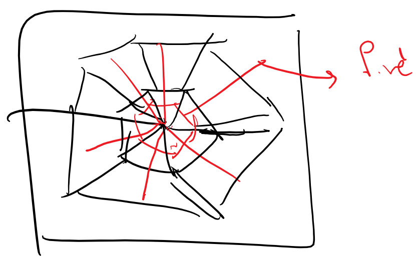


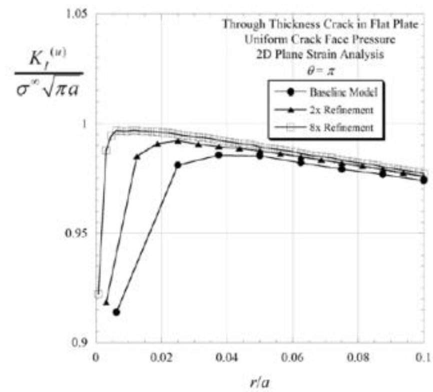


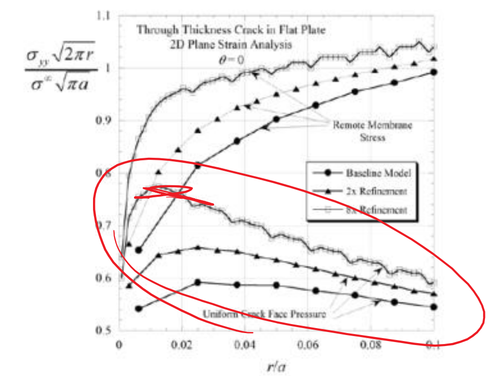
K from stress σ

$$K_I = \lim_{r \longrightarrow 0} \left(\sqrt{2\pi r}\ \sigma_{22}|_{\theta=0} \right)$$

What if we use finer meshes? Is that going to help?







K from displacement ***u***

$$K_I = \lim_{r \to 0} \left[\frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$

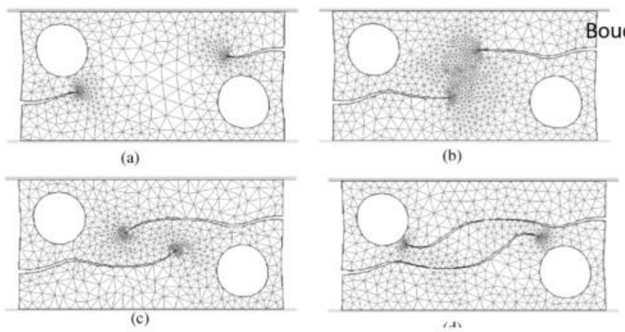
K from stress σ

$$K_I = \lim_{r \longrightarrow 0} \left(\sqrt{2\pi r}\ \sigma_{22}|_{\theta=0} \right)$$

Even element h-refinement cannot improve K values by much particularly for stress based method

6.1.6. Computational crack growth

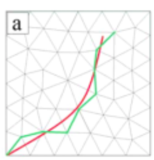
- Element edges must conform to the crack geometry: make such a mesh is time-consuming, especially for 3D problems.
- Remeshing as crack advances: difficult. Example:



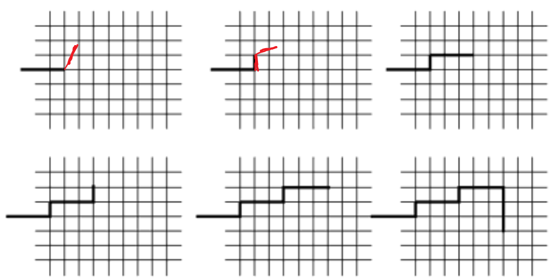
Bouchard et al. CMAME 2003

- Nodal release method (typically done on fixed meshes)
  - Crack advances one element edge at a time by releasing FEM nodes
  - Crack path is restricted by discrete geometry

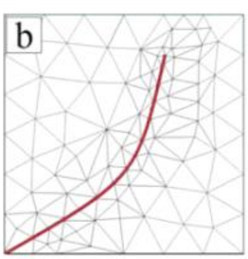
older approach



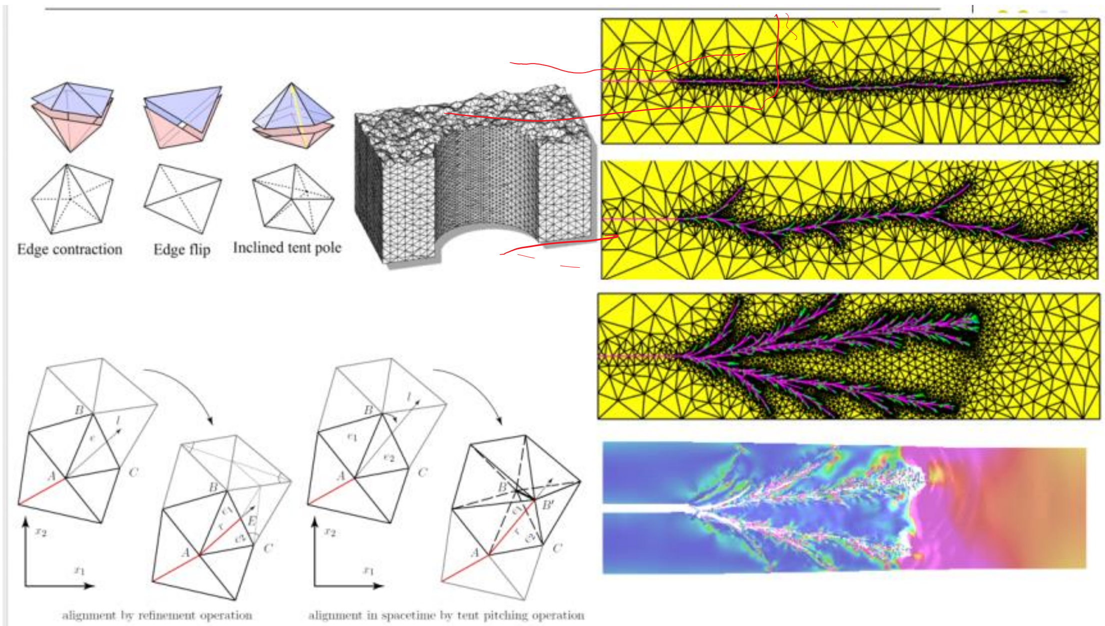
Fixed mesh



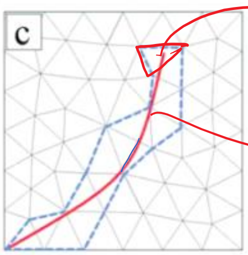
- Also for cohesive elements they can be used for both extrinsic and intrinsic schemes. For intrinsic ones, cohesive surfaces between all elements induces an artificial compliance (will be explained later)



Crack tracking



Spacetime mesh adaptive operations align element boundaries with crack path.

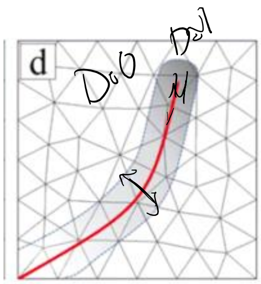


crack tip can also be within an element

we allow the cracks to go through elements

XFEM enriched elements

Brief overview in the next section



bulk / continuum damage $\sigma = (1-D)(C\varepsilon)$ → intact stress

$D=0$: $\sigma = C\varepsilon$ intact

$D=1$ $\sigma = 0$ fully damaged

- phase field method regularizes LEFM

Crack/void capturing by bulk damage models

Brief overview in continuum damage models

6.1.7. Extended Finite Element Method (XFEM)

asymptotic expression

6.1.7. Extended Finite Element Method (XFEM)

asymptotic expression

- Direct incorporation of singular terms: e.g. enriched elements by Benzley (1974), shape functions are enriched by K_I, K_{II} singular terms
  - XFEM method falls into this group (discussed later) ← More accurate

$$u_i = \sum_{k=1}^{4} f_k \bar{u}_{ik} + K_{\mathrm{I}}\left(Q_{1i}\sum_{k=1}^{4} f_k \bar{Q}_{1ik}\right) + K_{\mathrm{II}}\left(Q_{2i}\sum_{k=1}^{4} f_k \bar{Q}_{2ik}\right)$$

$$Q_{ij} = \frac{u_{ij}}{k_i}$$

$$u' = \left(1 + \frac{2x}{L} - 3\sqrt{\frac{x}{L}}\right)u_1' + \left(4\sqrt{\frac{x}{L}} - 4\frac{x}{L}\right)u_2' + \left(\frac{2x}{L} - \sqrt{\frac{x}{L}}\right)u_3'$$

Displacement field

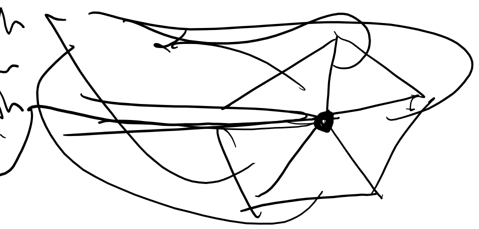
$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$

$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

129

we know what local displacement field should be

we can enrich them with

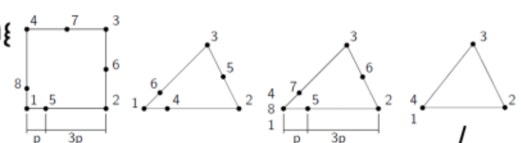


rms: $\sum_{k=1}^{4} f_k \bar{u}_{ik} + K_{\mathrm{I}}\left(Q_{1i}\sum_{k=1}^{4} f_k \bar{Q}_{1ik}\right) + K_{\mathrm{II}}\left(Q_{2i}\sum_{k=1}^{4} f_k \bar{Q}_{2ik}\right)$

ed $Q_{ij} = \frac{u_{ij}}{k_i}$

$u' = \left(1 + \frac{2x}{L} - 3\sqrt{\frac{x}{L}}\right)u_1' + \left(4\sqrt{\frac{x}{L}} - 4\frac{x}{L}\right)u_2' + \left(\frac{2x}{L} - \sqrt{\frac{x}{L}}\right)u_3'$

- Quarter point (LEFM) and Collapsed half point (Elastic-perfectly plastic) elements: By appropriate positioning of isoparametric element nodes create strain singularities ← Can be easily used in FEM software

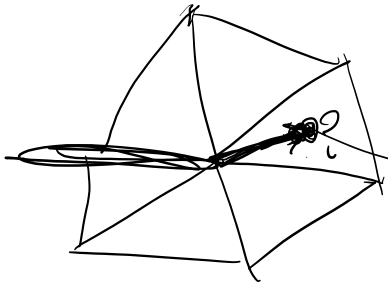


LEFM: $\epsilon, \sigma : \frac{1}{\sqrt{r}}$ singular

Elastic-perfectly plastic: $\epsilon : \frac{1}{r}$ singular

303

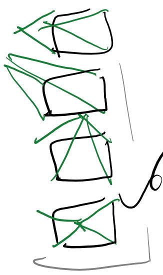
Problem: what if the crack extends?



innovation of XFEM / GFEM is that they allow the crack tip to be inside the element

Belytschko et al 1999

set of enriched nodes

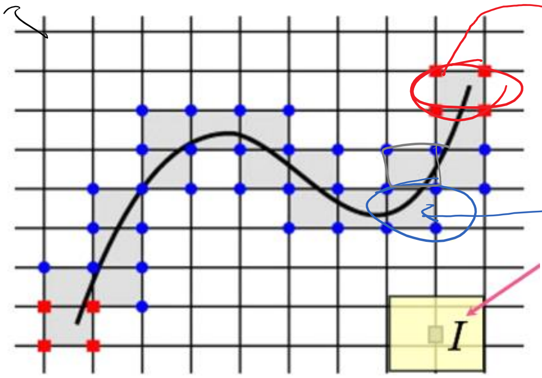


$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in \mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J \in \mathcal{S}^c} N_J(\mathbf{x})\Phi(\mathbf{x})\mathbf{a}_J$$

standard part enrichment part

enrichment for – crack tip

– crack faces

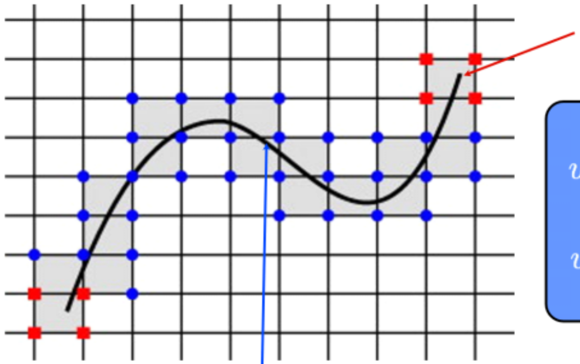


$$\sum_J N_J(\mathbf{x})\Phi(\mathbf{x}) = \Phi(\mathbf{x})$$

$\sum N_J = 1$

if a_J's in one element are 1

we recover enrichment ϕ



$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$
$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \Phi_1 = f(\sqrt{r}, \theta)$$

$\{\sqrt{r}\ \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ \sin\frac{\theta}{2}$

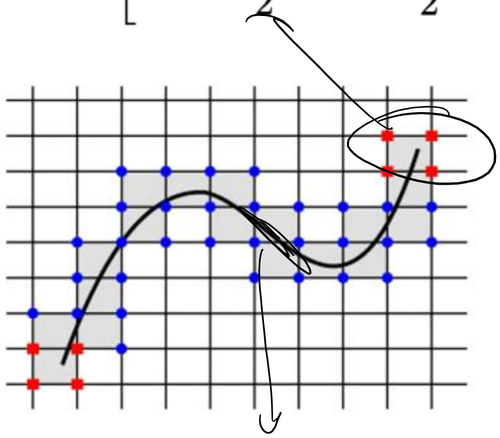
$\sin\frac{\theta}{2}$

$\{\sqrt{r}\ \sin\frac{\theta}{2}\ \sin\theta,\ \sqrt{r}\cos\frac{\theta}{2},\ \sqrt{r}\ \sin\frac{\theta}{2},\ \sqrt{r}\ \sin\theta\cos\frac{\theta}{2}\}$

Crack tip enrichment functions:

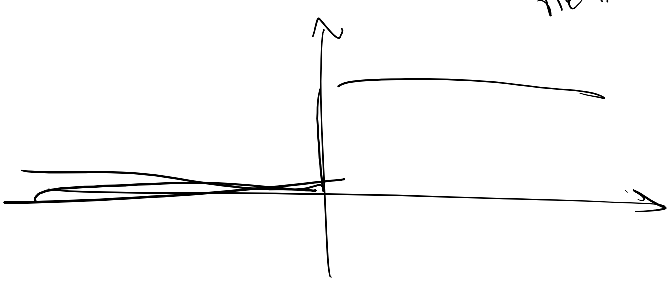
$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$
$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

$$[B_\alpha] = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$



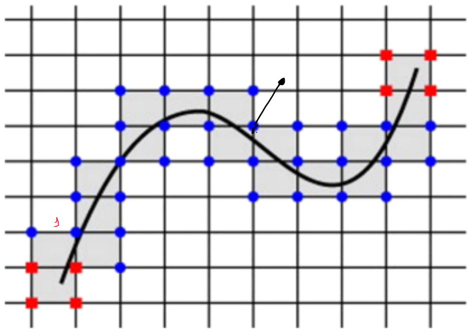
there is jump in displacement field

$H(x)$



$$H(\mathbf{x}) = \begin{cases} +1 & \text{if } (\mathbf{x}-\mathbf{x}^*)\cdot\mathbf{n} \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

S^c	blue nodes
S^t	red nodes

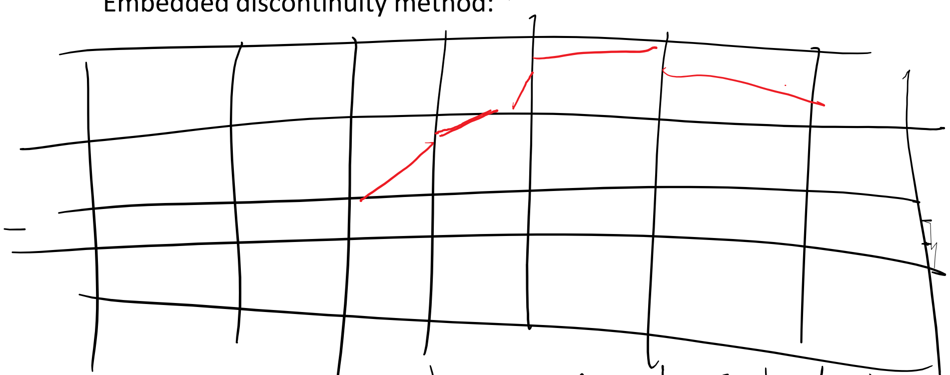


$$\mathbf{u}^h(\mathbf{x}) = \sum_{I\in\mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J\in\mathcal{S}^c} N_J(\mathbf{x})H(\mathbf{x})\mathbf{a}_J + \sum_{K\in\mathcal{S}^t} N_K(\mathbf{x})\left(\sum_{\alpha=1}^{4} B_\alpha \mathbf{b}_K^\alpha\right)$$

307

Another approach:

Embedded discontinuity method: → element centered

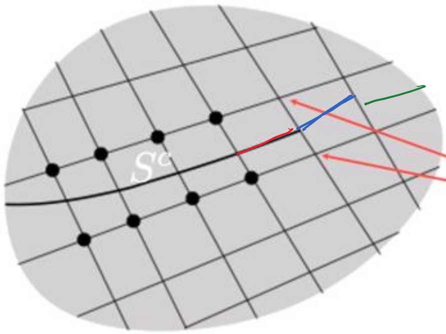


since this is element centered, the crack does not necessarily follow a continuous path.

Some of difficulties with XFEM / GFEM?

1. For the crack tip to be inside an element, we need to have the particular enrichment functions. For example, what are the enrichments when we want to use PFM, or cohesive models?

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in \mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J \in \mathcal{S}^c} N_J(\mathbf{x})H(\mathbf{x})\mathbf{a}_J$$

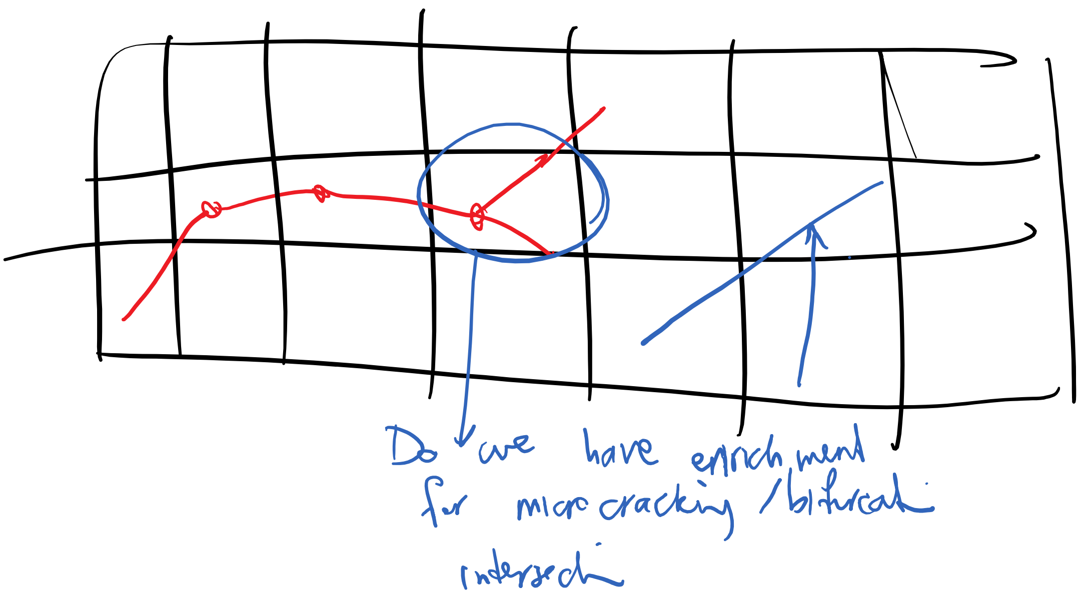


No crack tip solution is known, no tip enrichment!!!

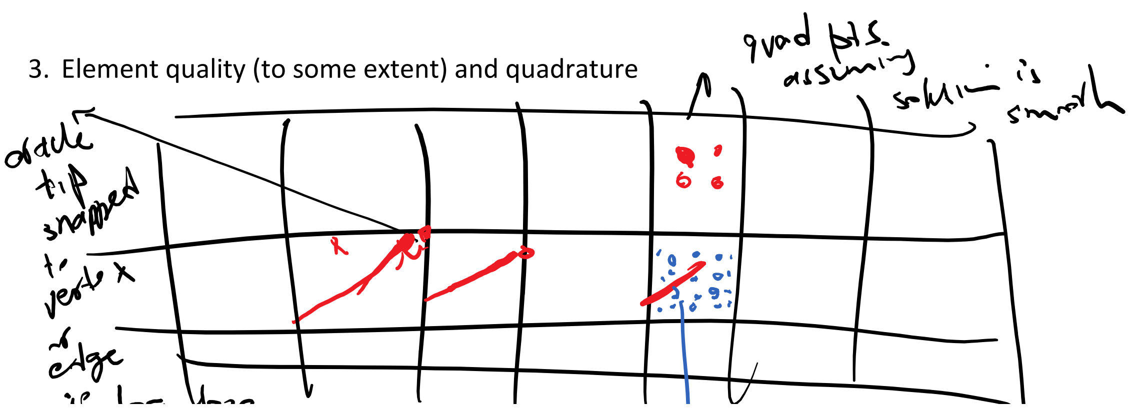
not enriched to ensure zero crack tip opening!!!

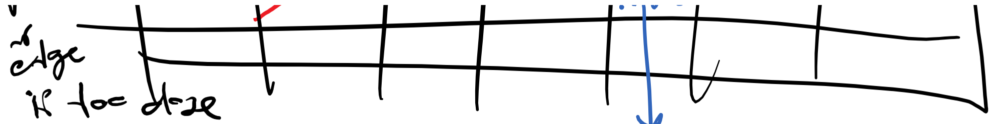
$$H(\mathbf{x}) = \begin{cases} +1 & \text{if } (\mathbf{x} - \mathbf{x}^*) \cdot \mathbf{n} \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

2. Complex fracture patterns:



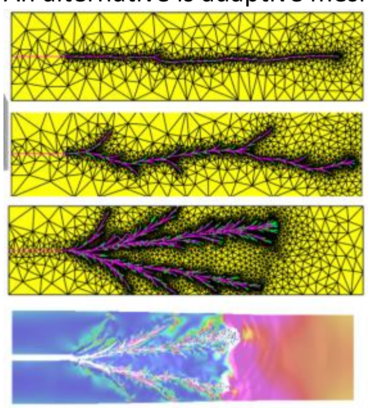
3. Element quality (to some extent) and quadrature



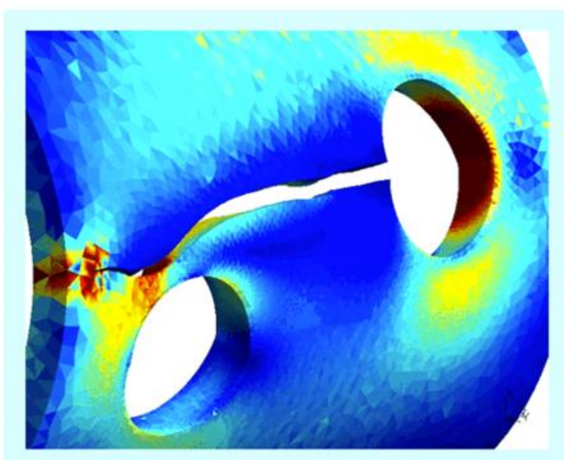


Since we no longer integrate FEM polynomials (mapped) we often need to increase # quadrature, ...

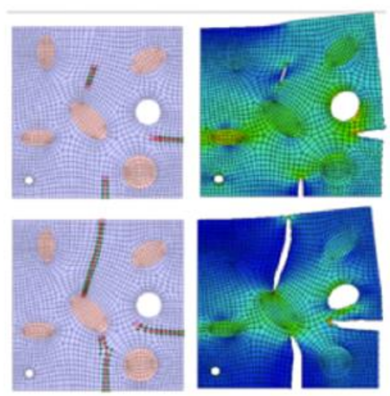
An alternative is adaptive meshing



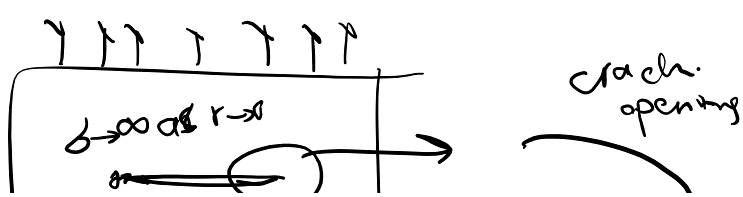
But these methods are very complicated especially for 3D

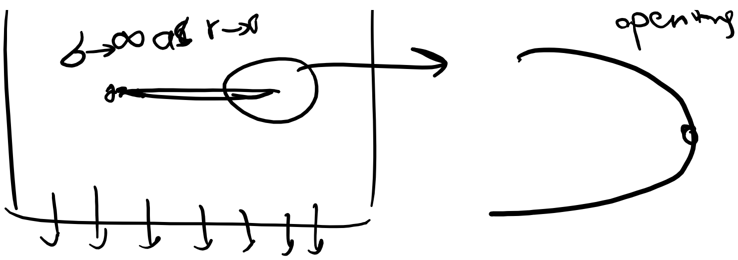


CENAERO, M. Duflot

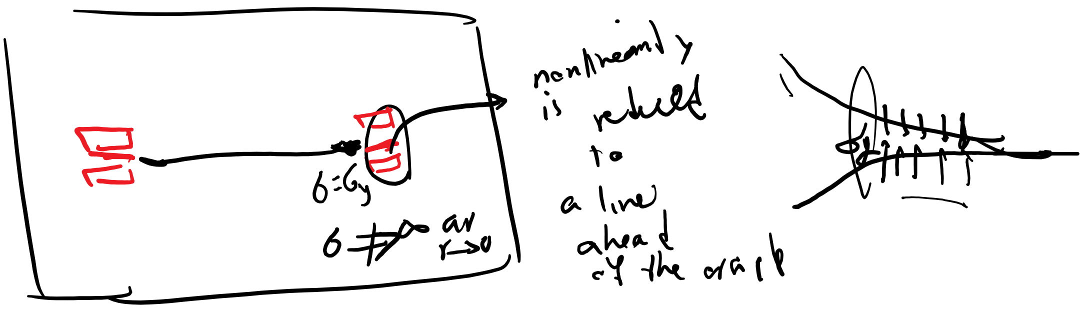


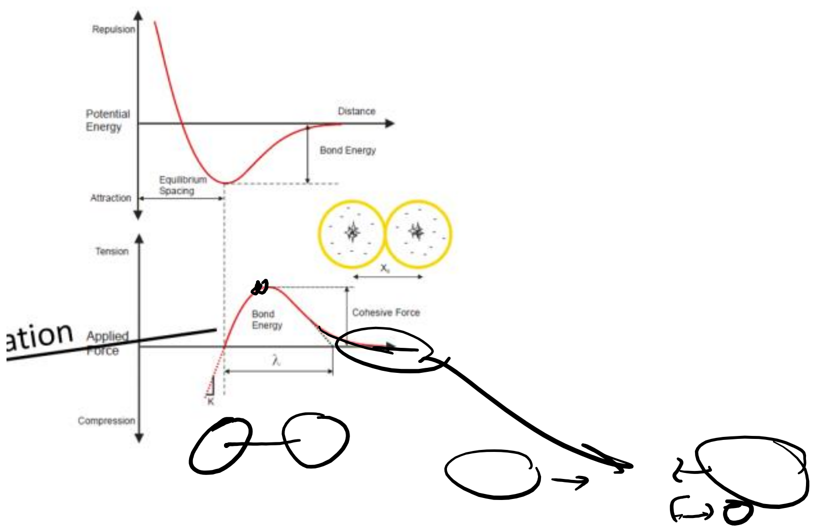
6.2. Traction Separation Relations (TSRs)

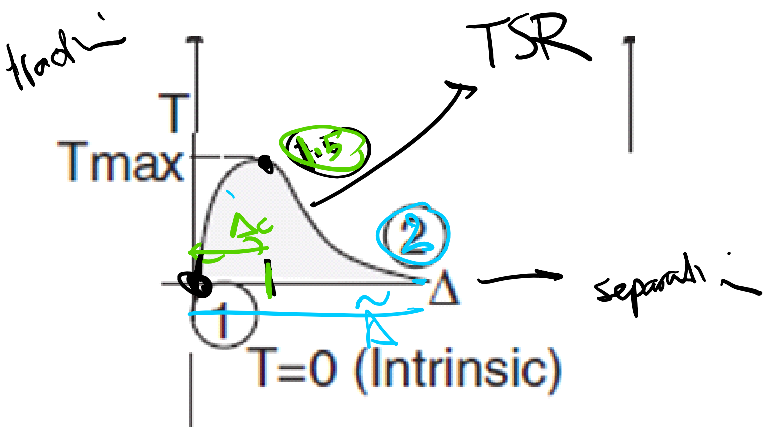


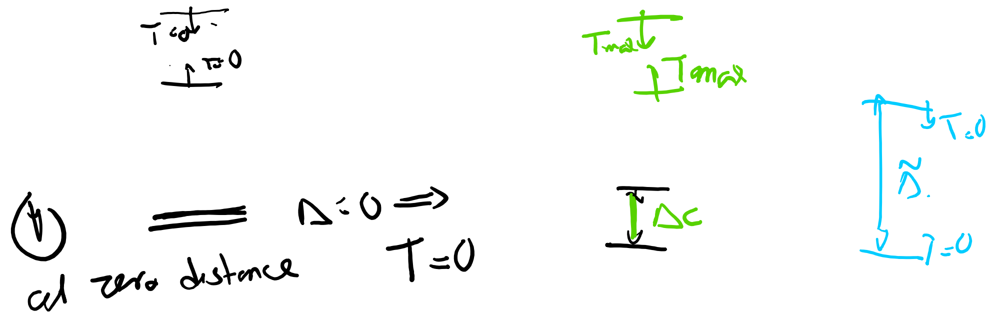


Strip yield model (Dugdale, Barenblatt)

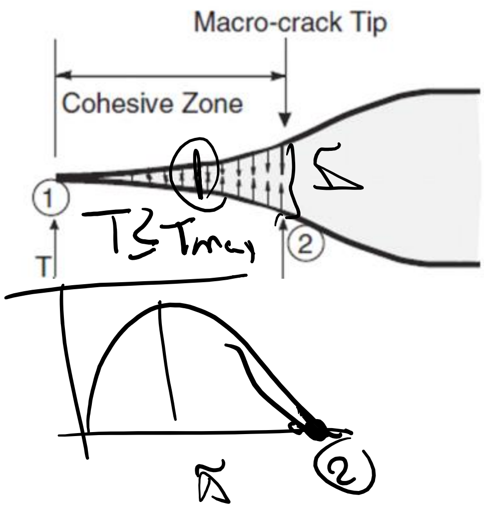






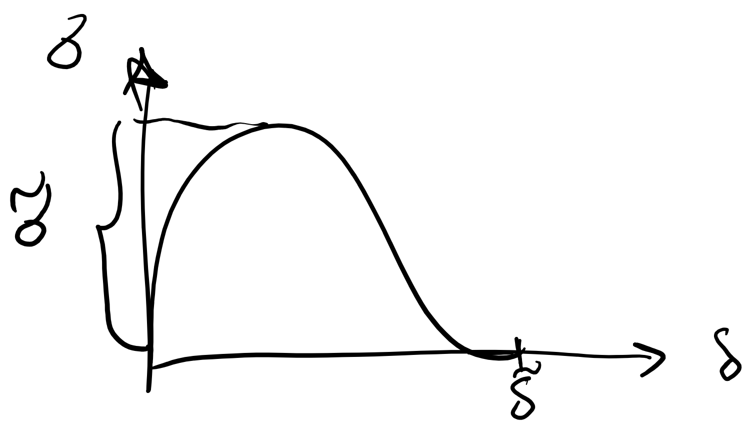


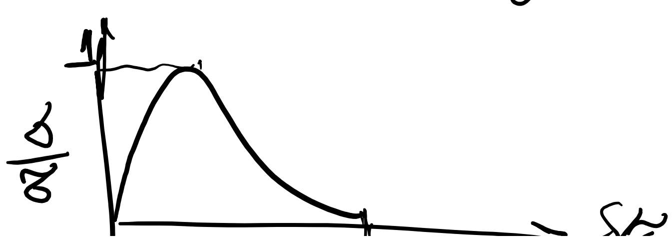
TSRs remove crack tip stress singularity:

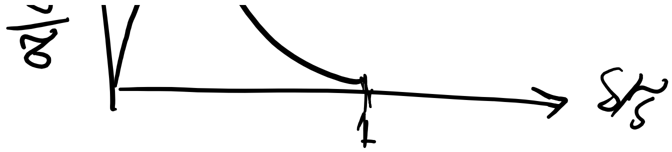


TSRs have traction (stress) scale $\tilde{\sigma}$

& displacement " $\tilde{\delta}$

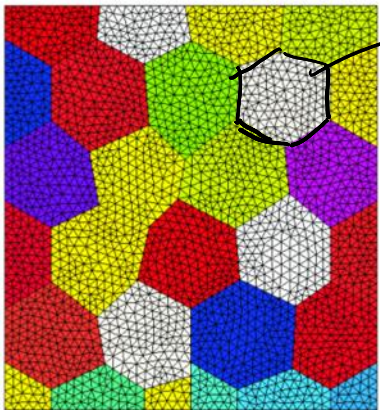






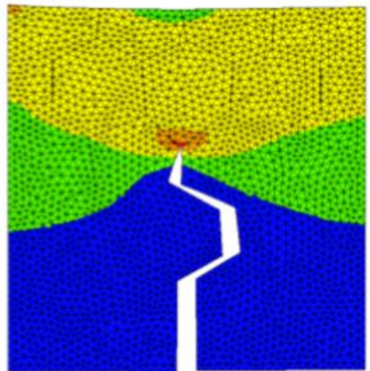
$\sigma/\tilde{\sigma} = f\left(\frac{\delta}{\tilde{\delta}}\right)$

fracture of polycrystalline material

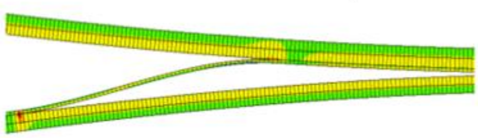


we can put TSR between grains → TG

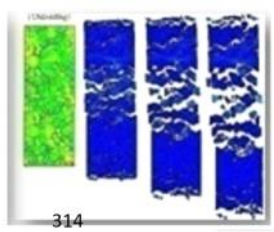
and even between the grains → IG



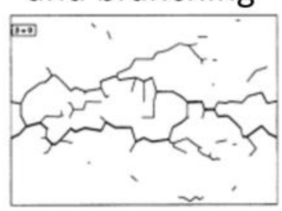
delamination of composites



fragmentation

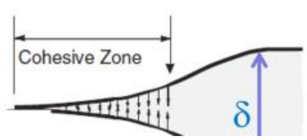


Microcracking and branching



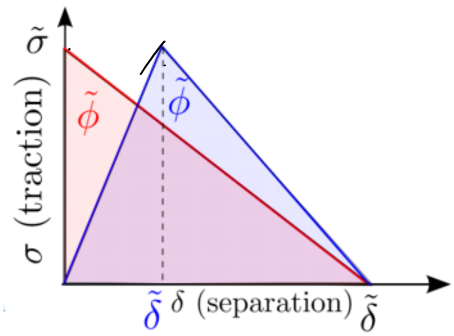
- Traction Separation Relation (TSR): Relation between traction (stress) and displacement jump

$$\sigma = \tilde{\sigma} f(\delta/\tilde{\delta})$$

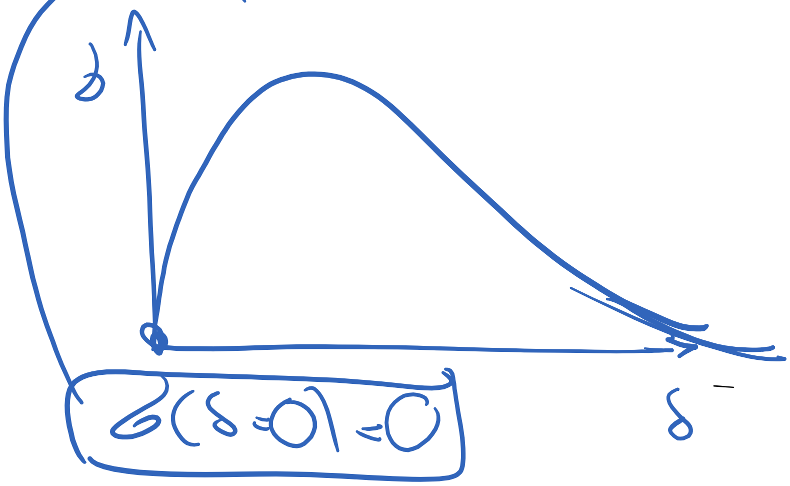


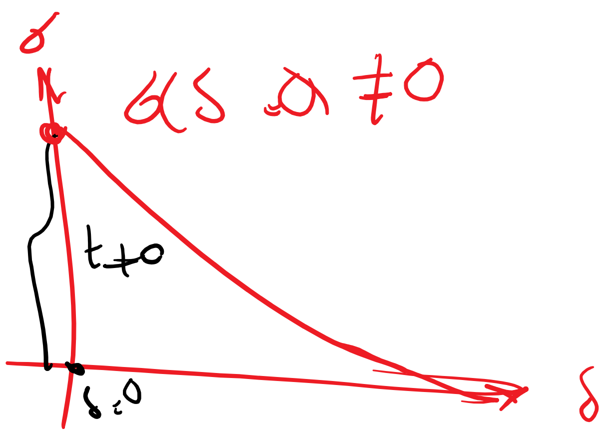
- Parameters of a cohesive model(Only 2 out of 3 are needed)
  - Stress (traction) scale $\tilde{\sigma}$: Maximum traction in TSR
  - Displacement scale $\tilde{\delta}$: Separation corresponding to maximum traction (extrinsic models) or maximum nonzero traction
  - Work of Separation $\tilde{\phi}$: Area under $\sigma - \delta$ curve is the work needed to complete debond a unit surface area. This can be associated with G_c in LEFM theory.

resistance



intrinsic cohesive model





zero stress / no loading