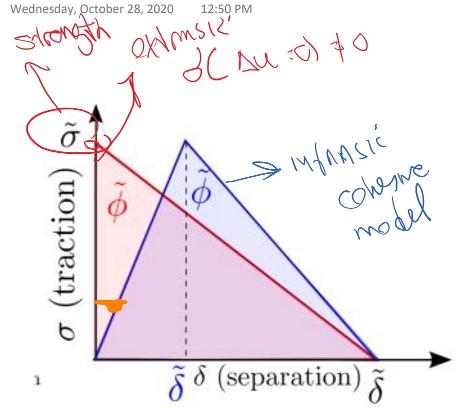
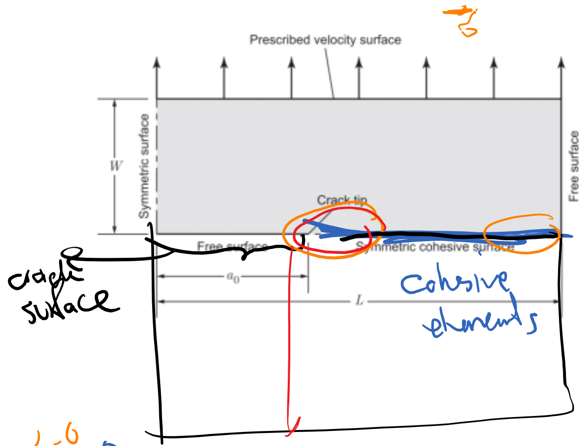
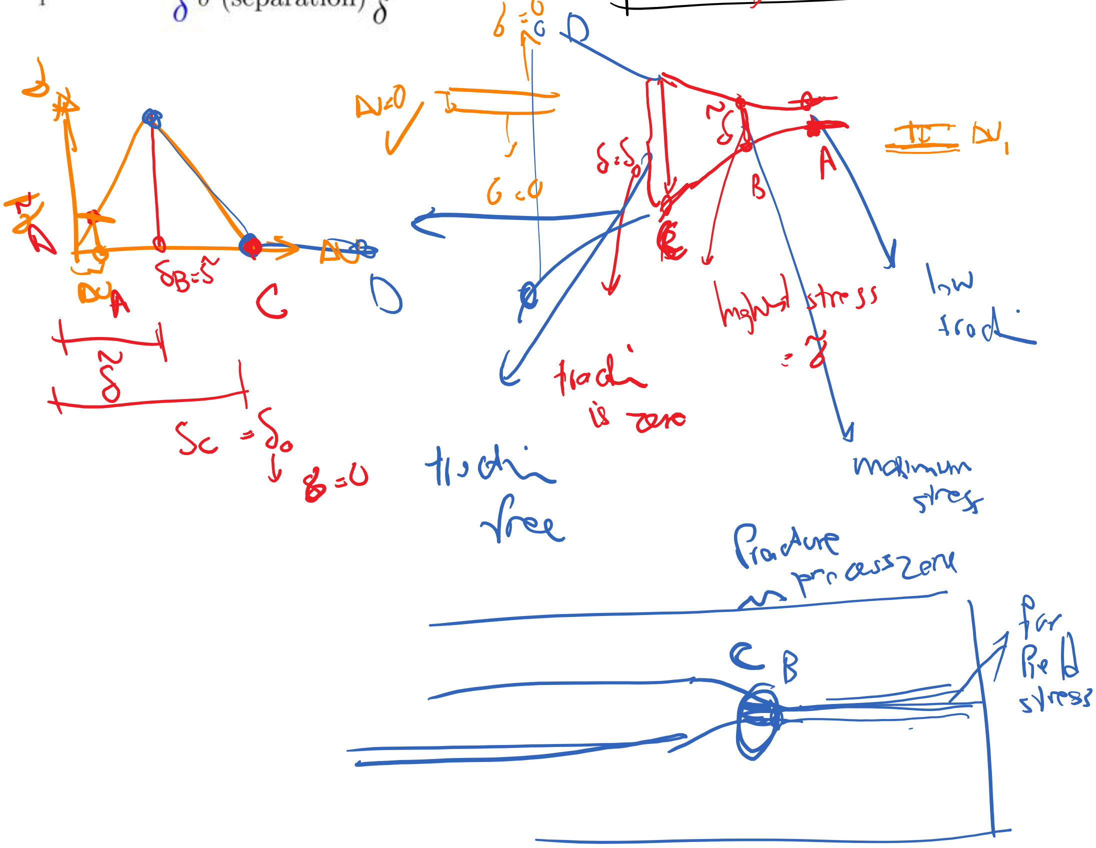
2020/10/28

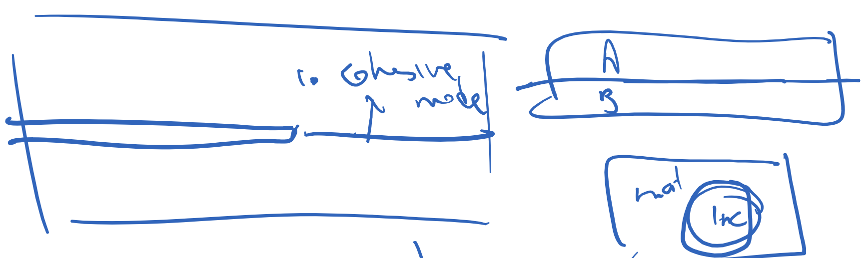
Wednesday, October 28, 2020 12:50 PM





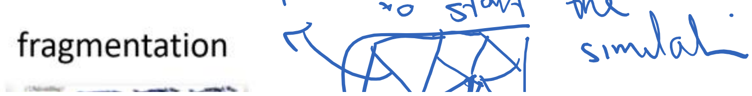


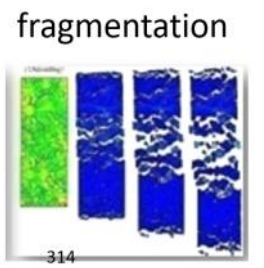
- We call these "intrinsic" cohesive models because they can be inserted between any two material (at any interfaces) from the beginning of simulation.
- Best response is when there are predefined potential crack paths.

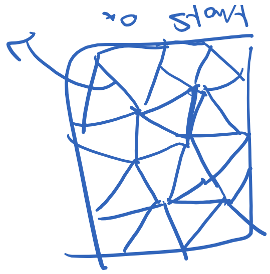


i. coh model to start the simulation

fragmentation

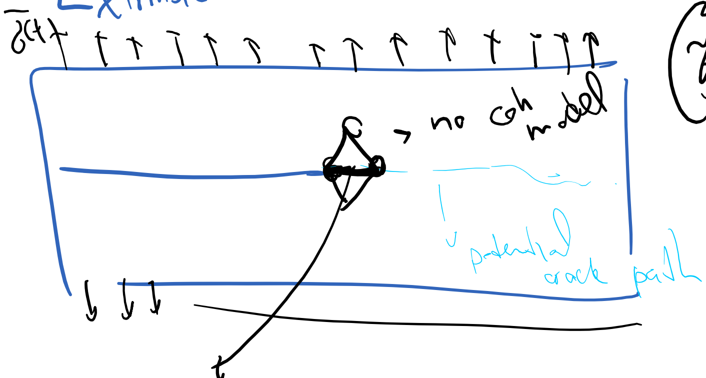


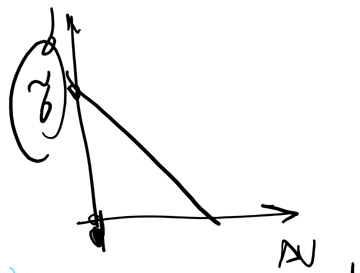




to start the simulation

Extrinsic model



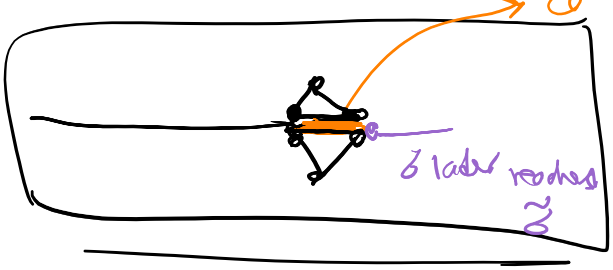


$\Delta u = 0 \rightarrow \sigma = \tilde{\sigma}$ X

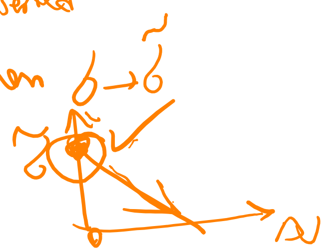
σ must be zero initially

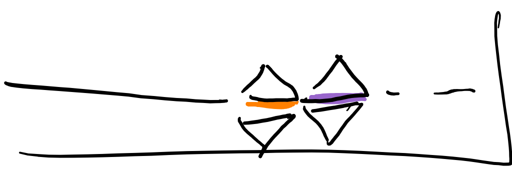
stress increases as $\bar{\sigma} \nearrow$

eventually it reaches $\tilde{\sigma}$



cohesive element inserted when $\sigma \rightarrow \tilde{\sigma}$



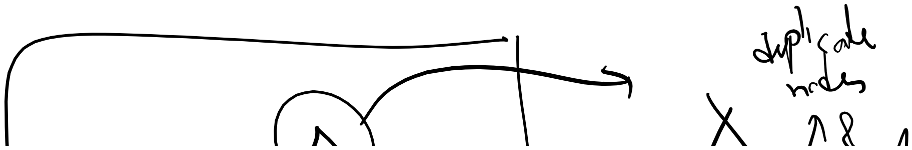


$\rightarrow$ crack gradually growth

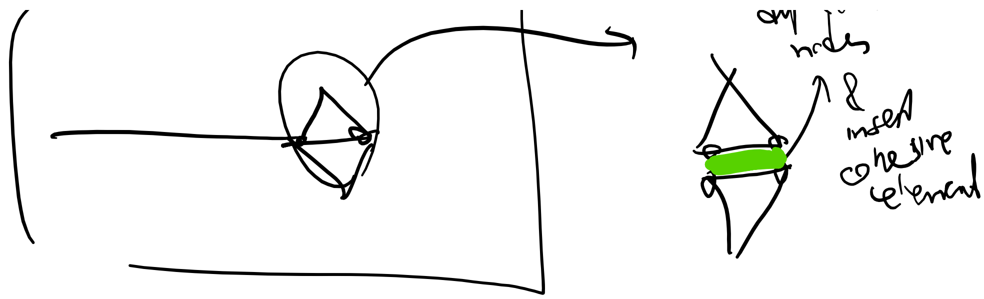
extrinsic

$\longrightarrow$ interactively inserted whenever failure criterion is satisfied ($\sigma \rightarrow \tilde{\sigma}$)

$\longrightarrow$ Difficulty: Changing mesh topology



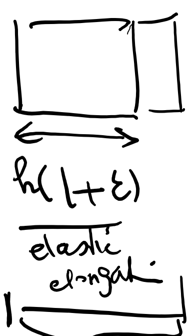
duplicate nodes



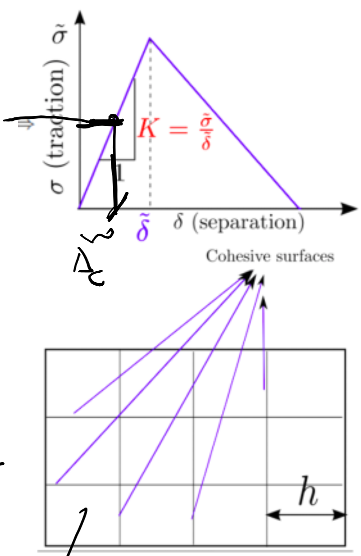
- So, intrinsic models are easier to use as the mesh topology does not change.
- Intrinsic models can be used for fragmentation problems too, but they suffer from **artificial compliance**.

$$\Delta_c = \frac{\bar{\sigma}}{K}$$

$K \rightarrow$ stiffness of cohesive model



$\bar{\sigma}$



$\bar{\sigma} \leftarrow$

new length $= h(1+\varepsilon) + \Delta_c = h + h\varepsilon + \Delta c$

Δ = new length $-\ h = h\varepsilon + \Delta c$

$$= \frac{h\bar{\sigma}}{E} + \frac{\bar{\sigma}}{K}$$

no cohesive surfaces

$$\Delta = \bar{\sigma}\left(\frac{h}{E} + \frac{1}{K}\right) = \frac{\bar{\sigma} h}{E_{eq}}$$

$E_{eq.}$ $\quad C = \frac{1}{E}$

$$\tilde{\sigma}\left(hC + \frac{1}{K}\right) = \bar{\sigma} h C_{eq}$$

$$\boxed{C_{eq} = C + \frac{1}{hK}}$$

material compliance

artificial compliance

$\frac{1}{Kh}$ $\qquad h \rightarrow 0$ better resolution of cracks

$\frac{1}{Kh} \rightarrow \infty$ material becomes too compliant.

what if $K \nearrow \infty$ (let $\frac{1}{Kh} = \alpha C$)

what if $K \nearrow \infty$ (let $\frac{1}{Kh} = \alpha^{c}$)

↓ high stiffness → cause convergence issues

- Artificial compliance becomes important if cohesive surfaces are added between all elements for intrinsic models to find crack propagation path.
- The artifical compliance is computed as,

$$\begin{aligned} \Delta &= \Delta_e + \Delta_c, \quad \Delta_e = \text{ elastic displacement, } \Delta_c = \text{ cohesive separation } \Rightarrow \\ \frac{\sigma}{E_{\text{eff}}} h &= \frac{\sigma}{E} h + \frac{\sigma}{K} \quad \Rightarrow \\ \frac{1}{E_{\text{eff}}} &= \frac{1}{E} + \frac{1}{Kh} \quad \Rightarrow \end{aligned}$$

Artificial compliance is,

$$C_c = \frac{1}{Kh} = \frac{\bar{\delta}}{\bar{\sigma} h} = \frac{1}{E_c}, \text{ where}$$

$$E_c = Kh = \frac{\bar{\sigma} h}{\bar{\delta}}, \text{ and effective elastic modulus is}$$

$$\frac{1}{E_{\text{eff}}} = \frac{1}{E} + \frac{1}{E_c}, \quad \Rightarrow \boxed{E_{\text{eff}} = \frac{E E_c}{E + E_c}}$$

$E_c = Kh$

- That is the smaller element spacing h or softer the initial slope K of TSR the higher artificial compliance (higher errors)
- While extrinsic cohesive models do not have the same problem, adaptive insertion of cohesive surfaces is more challenging for them.

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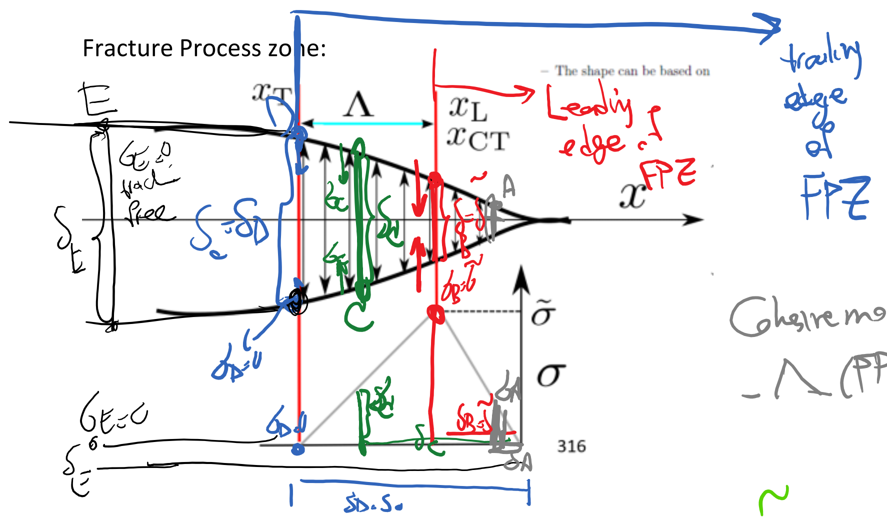
Ideas for better solution for fragmentation problems:

1. Use extrinsic models (difficult to implement)
2. Interfacial damage model ...

Fracture Process zone:

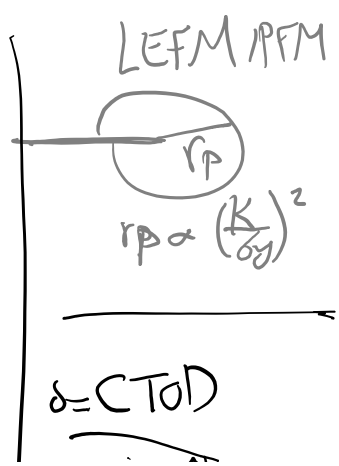


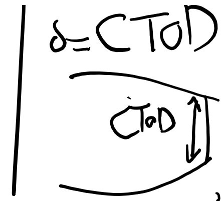
Cohesive model

– Λ (FPZ size)

$\tilde{\delta}$ Crack opening

critical opening corresponding





$\delta = CTOD$

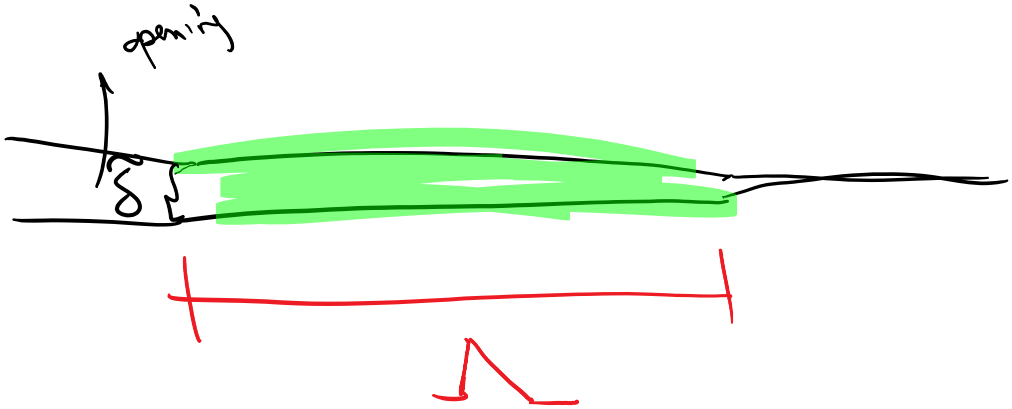
$\delta = \frac{K^2}{\sigma_y E'}$

$\delta \ll r_p$

$$\boxed{\frac{\delta}{r_p} \propto \frac{\sigma_y}{E}}$$

$$\boxed{\frac{\tilde{\delta}}{\Lambda} \propto \frac{\tilde{\tau}}{E}}$$

Λ is much larger than $\tilde{\delta}$



In fracture $\frac{\Lambda}{\tau} \propto \frac{E}{\tau}$ TSR, LEFM

friction (Earthquake)

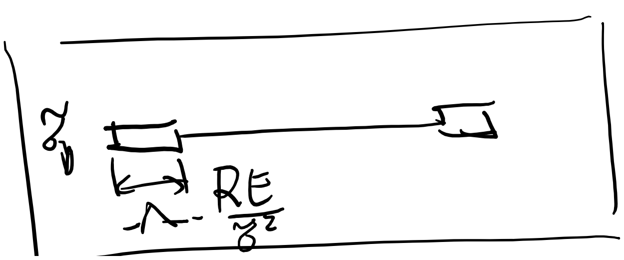
Estimates for Λ

I expect to have $\Lambda \propto \tilde{\delta} \frac{E}{\tilde{\tau}} = \frac{\tilde{\delta}\tilde{\tau} E}{\tau^2} = \frac{\tilde{\phi} E}{\tau^2}$

LEFM

In fact this is the case!

we can prove it, for example for strip yield model



$\Lambda \sim \frac{RE}{\tilde{\sigma}^2}$

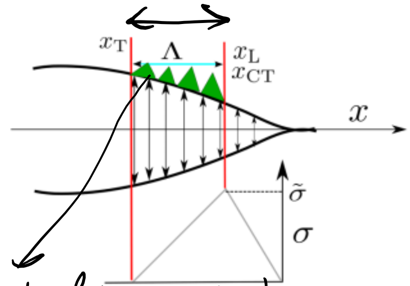
Why process zone size is important?

- Importance of process zone size Λ
  - Static estimate:

$$\Lambda = \varsigma \frac{\mu}{1-\nu} \frac{\tilde{\phi}}{\tilde{\sigma}^2} \propto \tilde{L}$$

$$\varsigma = \begin{cases} \frac{1}{4} & \text{Dugdale model} \\ \frac{9}{16} & \text{Potential-based TSRs} \end{cases}$$

$h \approx \frac{\Lambda}{6}$



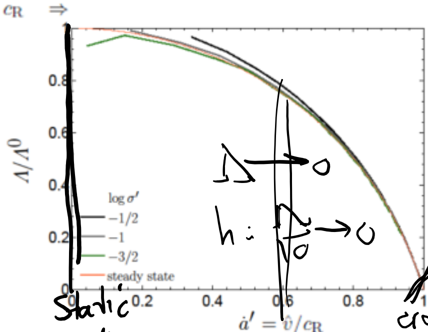
want to have at least

4 to 6 elements in FPZ

  - Minimum number of elements in process zone size: There should be at least 4-10 elements along the PZ
  - Dynamic estimate: PZS decreases as crack speed $\hat{v}$ approaches Rayleigh wave speed c_R

$$\Lambda(\hat{v}) = \frac{\Lambda}{A(\hat{v})}, \quad A(\hat{v}) \to 0 \text{ as } \hat{v} \to c_R \quad \Rightarrow$$

Smaller elements are needed in PZT as crack accelerates!



$\Lambda \to 0$

$h: \frac{\Lambda}{6} \to 0$

Static crack speed

crack speed $= c_R$

Rayleigh wave speed

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Smaller elements are needed in PZT as crack accelerates!

Cohesive scales

http://rezaabedi.com/wp-content/uploads/2013/11/2011_Reza_Abedi_Dimensional_Analysis_TSR.pdf

Cohesive scales

$\tilde{\phi} = \tilde{\sigma}\tilde{\delta},$ energy (39a)

$\boxed{\tilde{\tau} = \frac{\rho c_d \tilde{\delta}}{\tilde{\sigma}},}$ time (39b)

$\tilde{\delta} \quad \tilde{\sigma}$ velocity

$$\boxed{\tilde{\tau} = \frac{\rho c_{\mathrm{d}} \tilde{\delta}}{\tilde{\sigma}},} \qquad \text{time} \tag{39b}$$

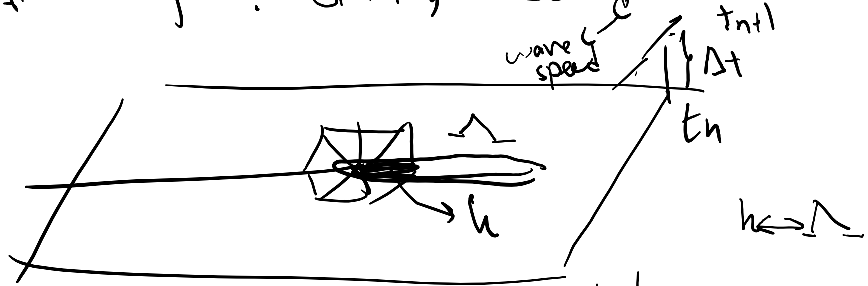
$$\tilde{v} = \frac{\tilde{\delta}}{\tilde{\tau}} = \frac{\tilde{\sigma}}{\rho c_{\mathrm{d}}}, \qquad \text{velocity} \tag{39c}$$

$$\tilde{E} = \frac{\tilde{v}}{c_{\mathrm{d}}} = \frac{\tilde{\sigma}}{\rho c_{\mathrm{d}}^2} \propto \frac{\tilde{\sigma}}{\|\mathbf{C}\|}, \qquad \text{strain} \tag{39d}$$

$$\tilde{p} = \rho \tilde{v} = \frac{\tilde{\sigma}}{c_{\mathrm{d}}}, \qquad \text{lin momentum} \tag{39e}$$

$$\tilde{L} = c_{\mathrm{d}} \tilde{\tau} = \frac{\rho c_{\mathrm{d}}^2 \tilde{\delta}}{\tilde{\sigma}} \propto \frac{\|\mathbf{C}\| \tilde{\phi}}{\tilde{\sigma}^2}. \qquad \text{Length} \tag{39f}$$

Explicit time marching : stability $\Delta t \propto \frac{h_{min}}{c}$



accuracy : how accurately we model cohesive fracture

$$\Delta t \ll \tilde{\tau}$$

the time scale for decohesion "debonding"

Z eq'd impedance

longitudinal wave speed

density

$$\tilde{\tau} = \frac{\rho c_d \tilde{\delta}}{\tilde{\sigma}}$$

$$\Delta t \to (0.1 \to .5)\, \tilde{\tau}$$

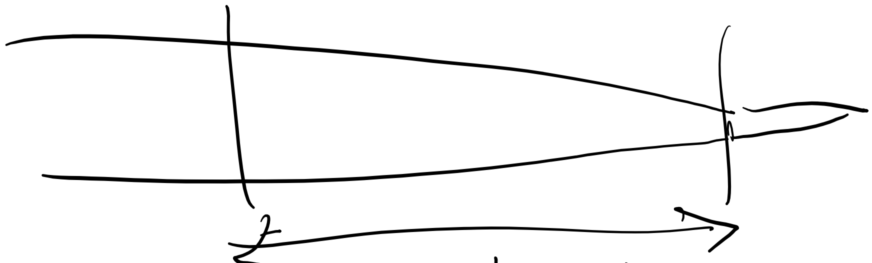
E: stiffness

ρ

$$\tilde{L} = c_{\mathrm{d}} \tilde{\tau} = \frac{\rho c_{\mathrm{d}}^2 \tilde{\delta}}{\tilde{\sigma}} \propto \frac{\|\mathbf{C}\| \tilde{\phi}}{\tilde{\sigma}^2}.$$

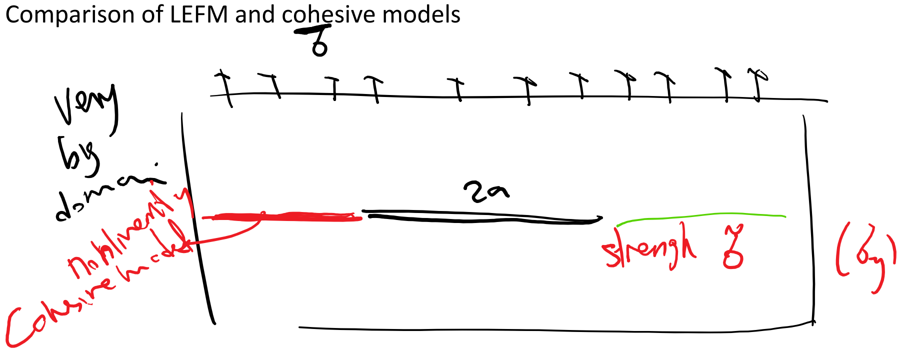
$$\tilde{L} \propto \frac{E \tilde{\phi}}{\tilde{\sigma}^2} \propto \left(\frac{E}{\tilde{\sigma}}\right) \tilde{\delta}$$

fracture process size scale



$$\Lambda \propto \tilde{L} = \frac{\Phi E}{\hat{\sigma}^2}$$

Comparison of LEFM and cohesive models



SSY $\quad r_p \ll$ all length scales

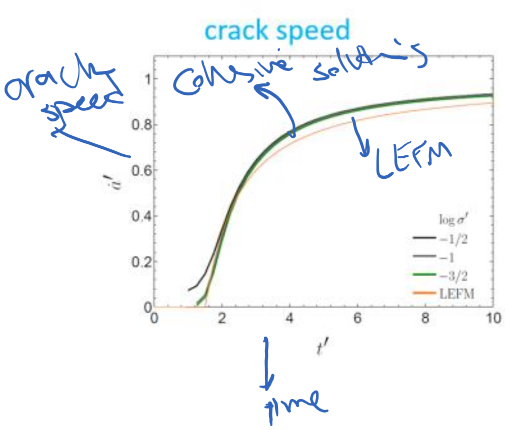
$r_p \to \Lambda$

r_s

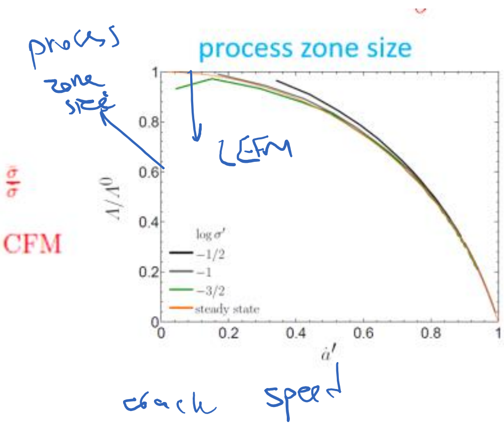
$$\frac{r_p}{r_s} \propto \left(\frac{\bar{\sigma}}{\hat{\sigma}}\right)^2$$

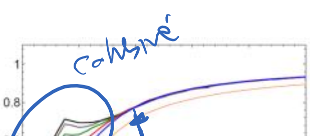
$\hat{\sigma} \to \sigma_y$

$\bar{\sigma} \ll .3\sigma_y \to$ LEFM is ok.



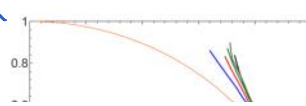
Low $\sigma' = \frac{\bar{\sigma}}{\hat{\sigma}}$
SSY
LEFM $\approx$ CFM

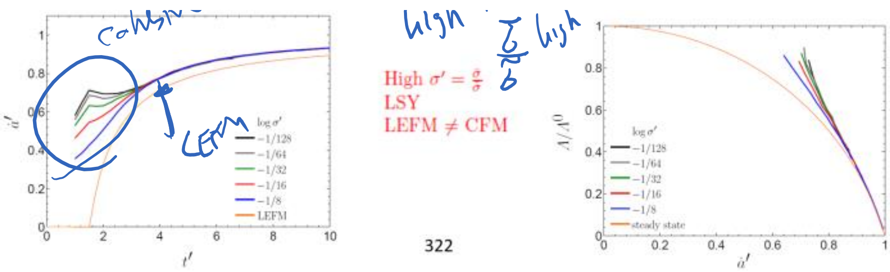




high load
$\bar{\sigma}$ high
$\hat{\sigma}$

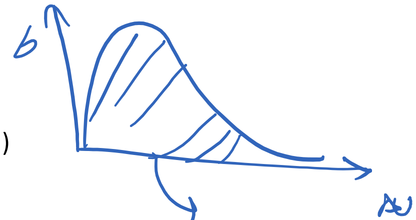
High $\sigma' = \frac{\bar{\sigma}}{\hat{\sigma}}$
LSY





Last point

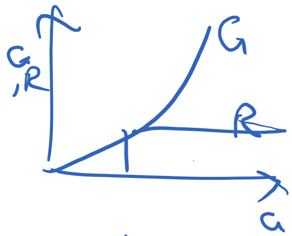
Relation between
- work of separation (Cohesive model)
- Resistance (LEFM)



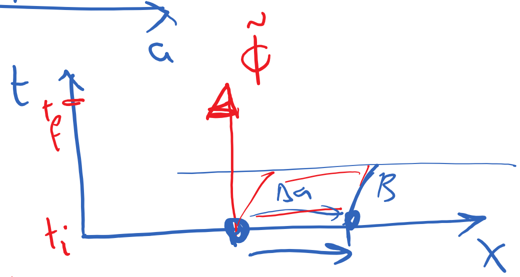
$\tilde{\Phi}$ work of separation

$$G(a) = R(a) = R_0$$

energy release rate

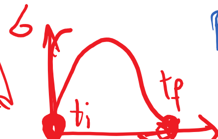


for



region $\Delta A = \Delta a\, B$

energy released from perfectly bonded to perfectly debonded $= \tilde{\Phi}$



R is resistance for one unit of crack advance in time

$\Delta a\, B\, R$

steady state crack propagation

$$\tilde{\Phi} \longleftrightarrow R$$

http://www.rezaabedi.com/pb/2016_Abedi_comparitive_cohesive_LEFM.pdf

Next it is shown that, under certain conditions, the ratio $\tilde{\phi}_{(k)}/G_{(k)}$ is well approximated by unity to obtain a useful estimate for the normalized CPZ size from (14). If it is assumed that the SSY assumption holds, then the *modal dynamic energy release rates* for an extrinsic cohesive model are given by Freund (1990),

$$G_{(k)} = \frac{1}{\tilde{v}} \int_{-\Lambda_{(k)}}^{0} \tilde{t}^k(\delta_k \boldsymbol{e}^k) \frac{\partial \delta_k}{\partial t} \, \mathrm{d}x_2 + \tilde{\phi}_{(k)} = I_{(k)} + \tilde{\phi}_{(k)}, \qquad (16)$$

energy release rate

transient part

for steady state crack growth

$\dot{V}_c \to C_R$

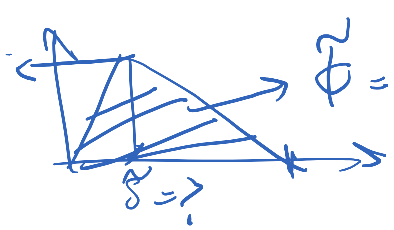
crack speed

practical lesson

LEFM parameter R given to you

→ want to use cohesive model

$\tilde{\delta}$ ✓



$\tilde{\phi} = R \to \tilde{\delta}$

CFM $\tilde{\phi}$ ⟶ LEFM $R = \tilde{\phi}$