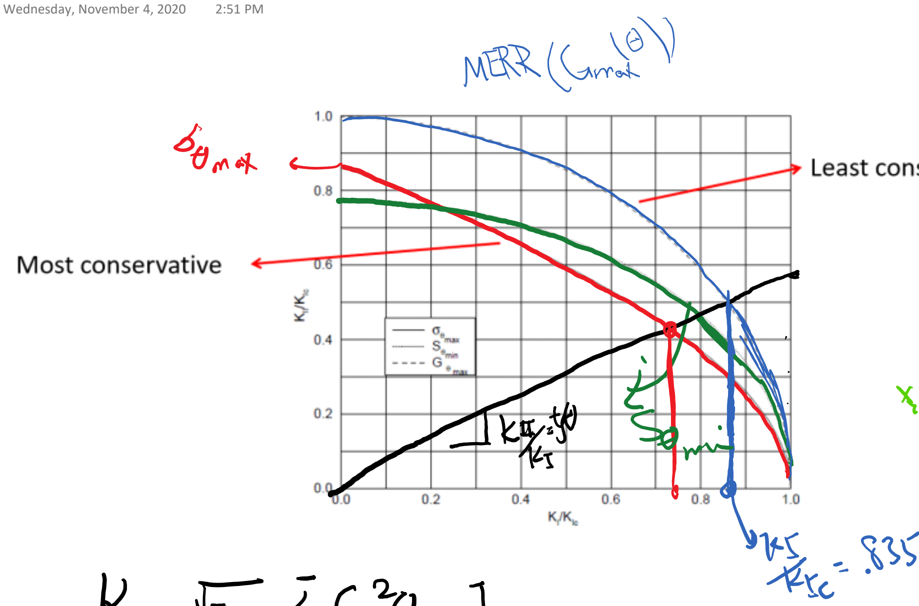
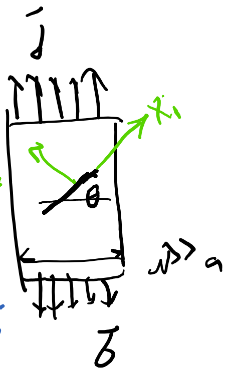
2020/11/04

Wednesday, November 4, 2020 2:51 PM





$$K_I = \sqrt{\pi a}\,\bar{\sigma}\cos^2\theta$$
$$K_{II} = \sqrt{\pi a}\,\bar{\sigma}\sin\theta\cos\theta \;\Big]\rightarrow\; \frac{K_{II}}{K_I} = tg\,\theta$$

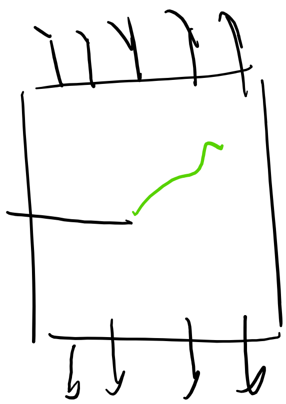
$\theta = 30^\circ \qquad tg\,\theta = \frac{1}{\sqrt{3}} = .577$

For Max. circum. criterion crack propagates when

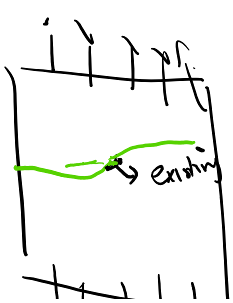
$$\frac{K_I}{K_{Ic}} = .75 \rightarrow \sqrt{\pi a}\,\bar{\sigma}\cos^2 30 = .75\,K_{Ic} \rightarrow \bar{\sigma}_{max} = \frac{.75}{3/4}\frac{K_{Ic}}{\sqrt{\pi a}} = \frac{K_{Ic}}{\sqrt{\pi a}}$$

MERR $\frac{K_I}{K_{Ic}} = .835 \longrightarrow$ — — $\bar{\sigma}_{max}$ MERR

Crack nucleation criterion:



crack Nucleation



existing crack that is not explicitly modeled or defect

We can formulate appropriate crack nucleation models for each of the three crack propagation criteria:

- Cracks nucleate from microscopic material defects under high stress/ strain loads.
- For each crack propagation criterion there can be a corresponding nucleation criterion.
- For example for maximum circumferential tensile stress, a crack nucleates when the maximum principle stress σ_1 at a point reaches material strength σ_0:

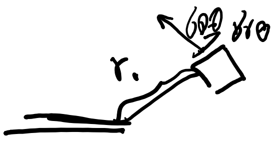
$$\max_{-\pi<\theta<\pi} \sigma_\theta(r \to 0^+, \theta) = \sigma_1 = \sigma_0, \quad \text{crack nucleates}$$

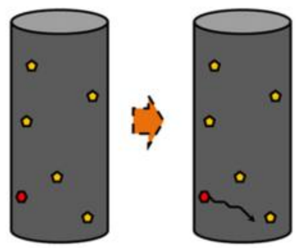
  Although we assume that there is no initial crack tip, we can measure r relative to the potential nucleation point.

- Same concept applies to modified maximum circumferential tensile stress criteria:

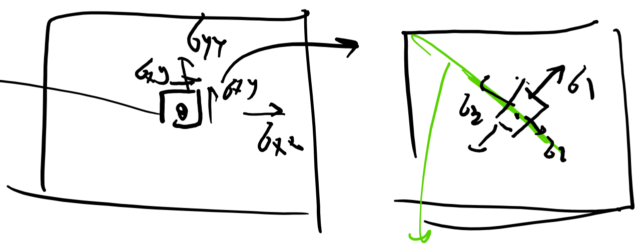
$$\max_{-\pi<\theta<\pi} \sigma_{\text{eff}}(r \to 0^+, \theta) = \sigma_0, \quad \text{crack nucleates}$$

σ_{eff} (eg $\sigma_{\theta\theta}$) $= \sigma_0$





does a crack propagate



$\sigma_1 \geq \sigma_0$

a crack is nucleated

early crack propagation direction

A nucleation model that can go well with MERR

Crack nucleation criterion

- For Maximum Energy Release Rate Criterion if we assume there are no defects, there will be no crack nucleation. However, assuming that local stress field generates a tensile maximum principal stress of σ_1 a "microscopic" initial crack (defect) of length a_{ini} perpendicular to σ_1 direction generates,

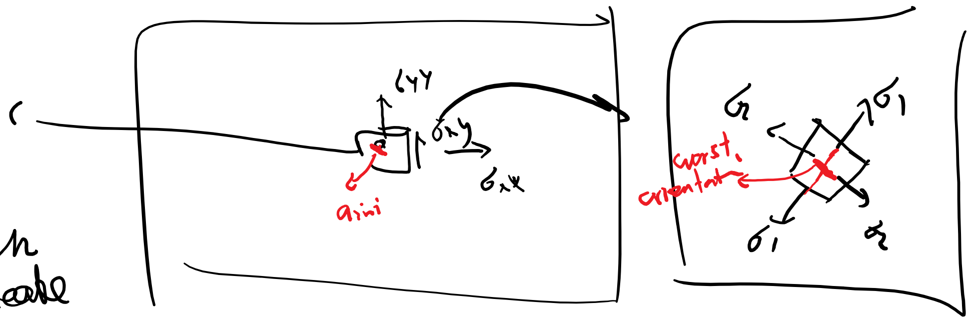
$$G = \frac{K_I^2 + K_{II}^2}{E'} = \pi a_{\text{ini}} \sigma_1^2$$

so the microcrack propagates (*i.e.*, a "macroscopic" crack nucleates) when,

$$G = G_c \quad \Leftrightarrow \quad \sigma_1 = \sqrt{\frac{G_c}{\pi a_{\text{ini}}}}$$

  - Initial crack direction perpendicular to σ_1 is chosen to maximize G.
  - We have assumed the initial crack to be small enough to use the infinite domain SIF formula of $K_I = \sqrt{\pi a}\,\bar{\sigma}$.

? does a crack nucleate



$K = \sigma_1 \sqrt{\pi a_{ini}}$ large domain approximation

what value to choose for a_{ini}?

— SHM, NDE length scale of crack that can be detected

OR

— a length below which we don't directly resolve material defects

$$G = \frac{K_I^2}{E'} = \frac{\sigma_1^2 \pi a_{ini}}{E'} = G_c$$

$$\rightarrow \quad \sigma_1 \geq \sqrt{\frac{E' G_c}{\pi a}}$$

a crack can be nucleated of size a_{ini} normal to σ_1 direction

8. Fatigue

8.1. Fatigue regimes
8.2. S-N, P-S-N curves
8.3. Fatigue crack growth models (Paris law)
- Fatigue life prediction

8.4. Variable and random load
- Crack retardation due to overload

Fatigue happens when the applied loads cycle in time

Fatigue fracture is prevalent!

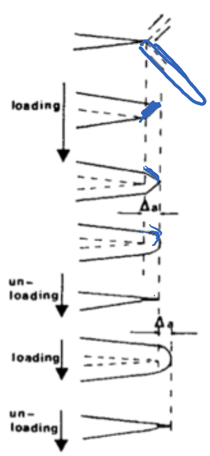
- Deliberately applied load reversals (e.g. rotating systems)
- Vibrations (machine parts)
- Repeated pressurization and depressurization (airplanes)
- Thermal cycling (switching off electronic devices)
- Random forces (ships, vehicles, planes)

(source: Schreurs fracture notes 2012)

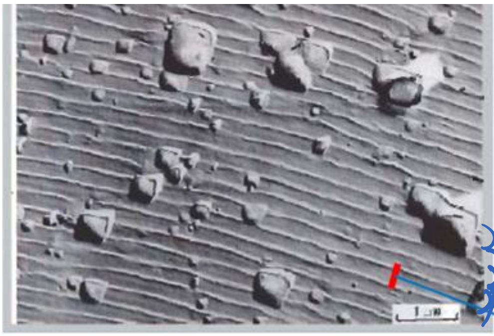
Fatigue occurs always and everywhere and is a major source of mechanical failure

blunting

resharpening



(a)

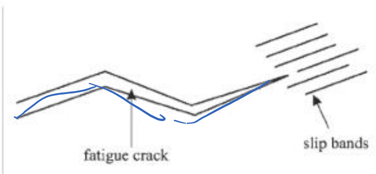


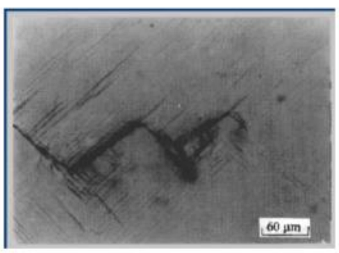
Fracture surface of a 2024-T3 aluminum alloy
(source S. Suresh MIT)

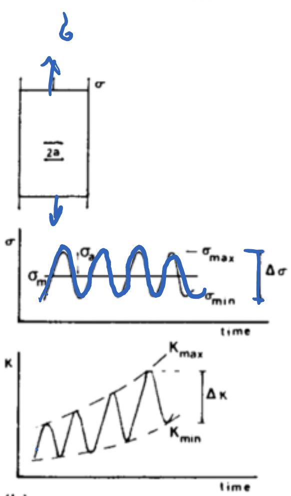
1

incremental crack propagation
– Each time after sharpening crack propagates a little and goes through blunting & sharpening again

Striation caused by individual microscale crack advance incidents







(b)

$K = \sqrt{\pi a}\,\sigma$

for a given a $\Delta K = \sqrt{\pi a}\,\Delta\sigma$

$\Delta\sigma$ is fixed

$\Delta K \uparrow$ $(\sqrt{\pi a}\,\Delta\sigma)$

because $a \uparrow$

a_1

$a_2 > a_1$ ...

Types of fatigue:

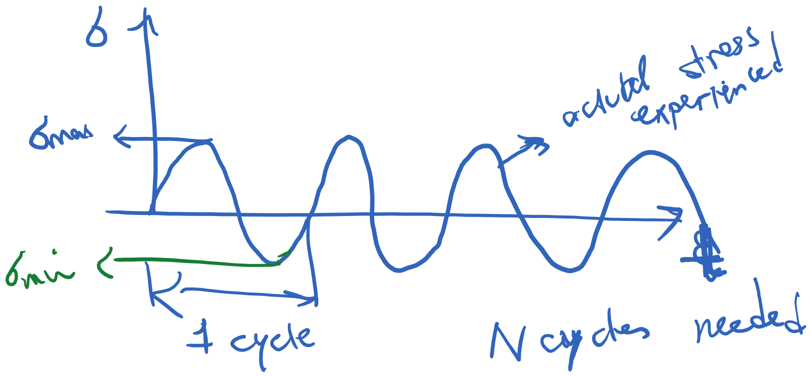
Table 7.1 Classification of fatigue damage

Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p / \Delta\varepsilon^e$	Energy ratio $\Delta W^p / \Delta W^e$
Very high cycle fatigue	$> 10^7$	$< \sigma_E$	≈ 0	≈ 0

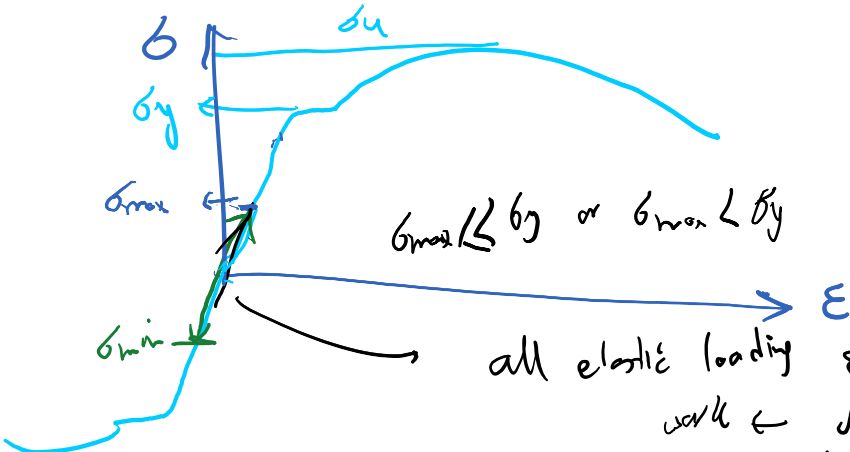
Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$
Very high cycle fatigue	$> 10^7$	$< \sigma_F$	≈ 0	≈ 0
High cycle fatigue	10^5 to 10^6	$< \sigma_Y$	≈ 0	≈ 0
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

Source: Dufailly and Lemaitre (1995)

# of cycles before final failure is in the order of M to 10 or higher Ms.



N cycles needed for final failure



$\sigma_{max} \ll \sigma_y$ or $\sigma_{max} < \sigma_y$

all elastic loading $\varepsilon_p \approx 0$

work ← $W_p \approx 0$

HCF, V HCF
- $N \sim 10^6 - 10^7$
- Almost all elastic deformation
- LEFM theory is appropriate
- stress (profile/history) is generally used to study fatigue

We only discuss this

	N	σ_{max}	$\Delta\varepsilon_p/\Delta\varepsilon_e$	$\Delta W_p/\Delta W_e$
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

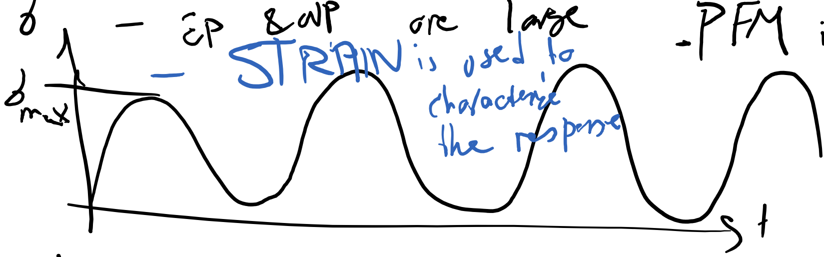
Source: Dufailly and Lemaitre (1995)

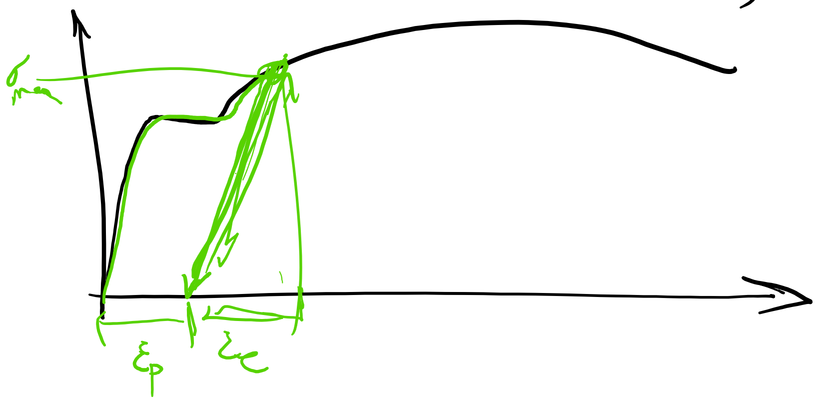
LCF & VLCF
- σ_{max} can get to or exceed σ_y

– N ranges from a few (≥10) to few hundred or thousands

– εp & σp are large – PFM is appropriate

– STRAIN is used to characterize the response

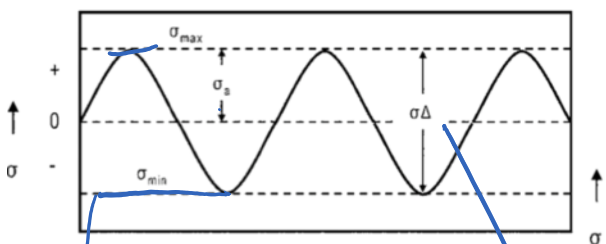




Some terminology:

Base line fatigue loading

$$\sigma_{\max} = -\sigma_{\min}$$



σ_{min}, σ_{max} →

$\Delta\sigma = \sigma_{max} - \sigma_{min}$

$\sigma_a = \frac{\Delta\sigma}{2}$ (for the figure on the right $\sigma_a = \sigma_{max}$) ∴ $\sigma_m = 0$

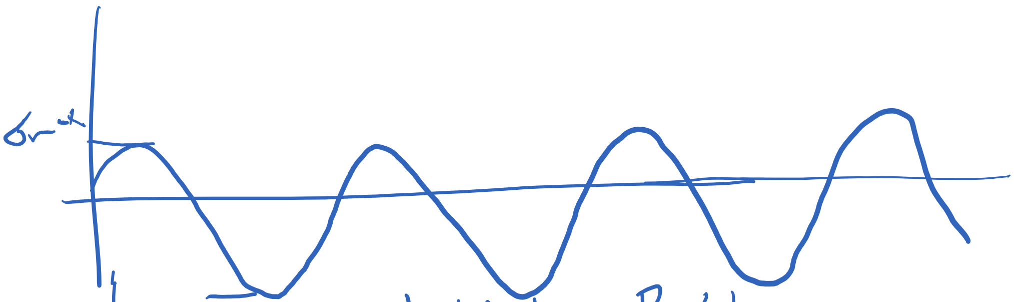
$\sigma_m = \frac{\sigma_{min} + \sigma_{max}}{2}$

$R = \frac{\sigma_{min}}{\sigma_{max}}$

$R = -1$

$R \in (-\infty, 1)$

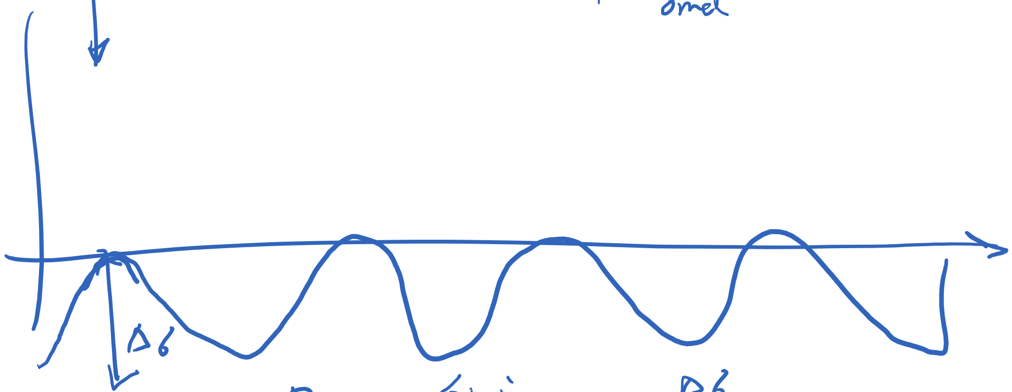
make the loading more compressive



σ_{min} $|\sigma_{min}| > |\sigma_{max}|$

$R < -1$

$R = \frac{\sigma_{min}}{\sigma_{max}}$

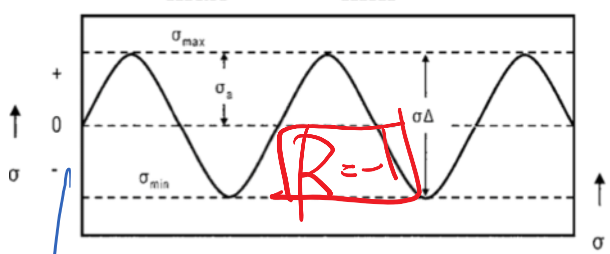


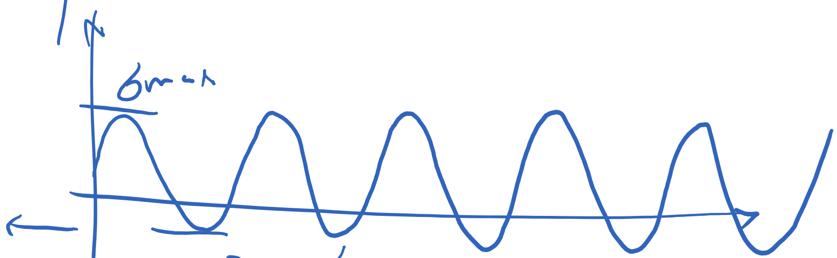
$R = \frac{\sigma_{min}}{\sigma_{max}} \rightarrow \frac{-\Delta\sigma}{\sigma} \rightarrow -\infty$

$\boxed{R \rightarrow -\infty}$ all compressive loading

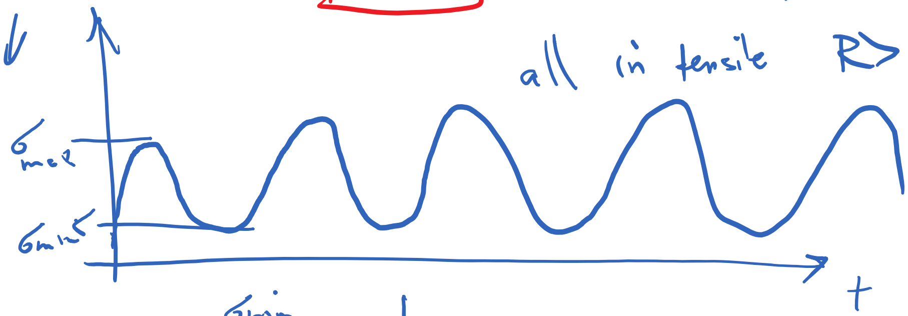
Let's make it more tensile:

$$\sigma_{max} = -\sigma_{min}$$





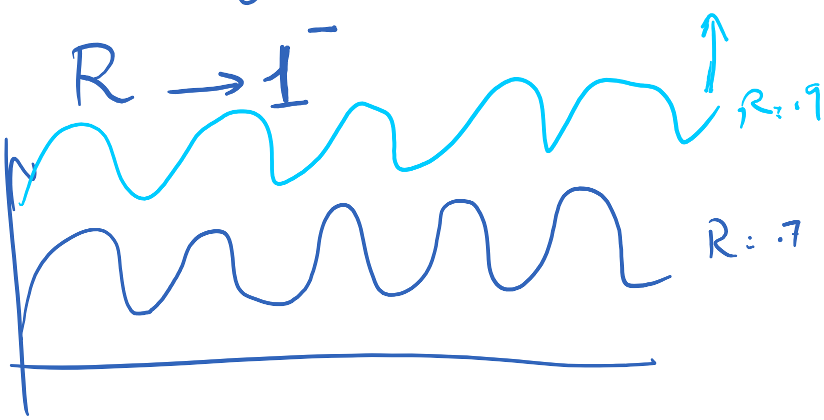
σ_{min} $-1 < R = \frac{\sigma_{min}}{\sigma_{max}} < 0$



$$\frac{\sigma_{min}}{\sigma_{max}} = \frac{1}{3} > 0$$

for a fixed $\Delta\sigma$ as $\sigma_m \nearrow$

$$R \rightarrow 1^-$$



$$R \in (-\infty, 1)$$

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

$$\sigma_a = 0.5(\sigma_{\max} - \sigma_{\min})$$

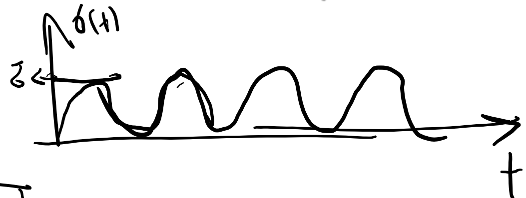
$$\sigma_m = 0.5(\sigma_{\max} + \sigma_{\min})$$

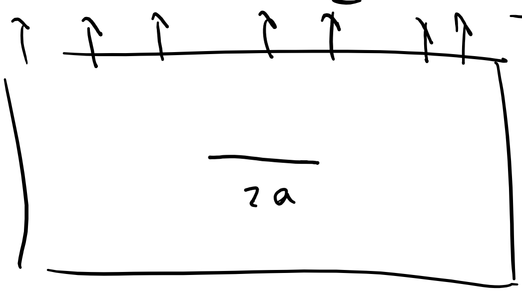
$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad \text{load ratio}$$

Cyclic vs. static loadings

- Static: Until K reaches K_c, crack will not grow
- Cyclic: K applied can be well below K_c, crack still grows!!!
- 1961, Paris et al used the theory of LEFM to explain fatigue cracking successfully.
- Methodology: experiments first, then empirical equations are proposed.

$\sigma(t) = \left(\frac{1+\cos\omega t}{2}\right)\bar{\sigma}$



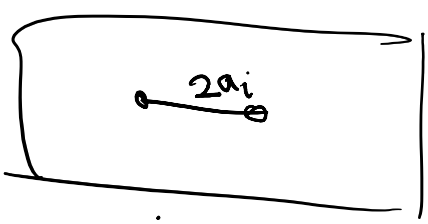


$K(t) = \sqrt{\pi a}\,\langle\sigma(t)\rangle_+$

$K_{max} = \sqrt{\pi a}\,\bar{\sigma}$

Compare K_{max} with K_{IC}

Even with $K_{max} \ll K_{Ic}$ the crack can still grow very slowly

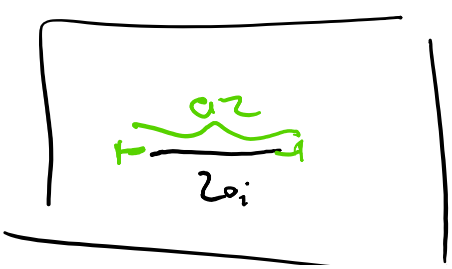


stage 1

$\Delta\sigma = \bar{\sigma}$

$\Delta K = \bar{\sigma}\sqrt{\pi a_i}$

later time



$\Delta\sigma = \bar{\sigma}$ const

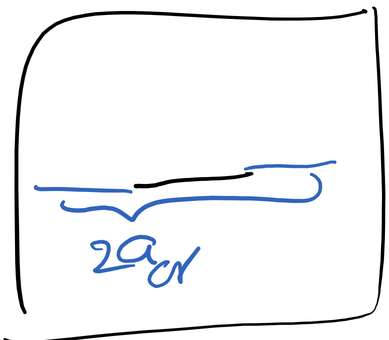
$K_{max} = \Delta K = \bar{\sigma}\sqrt{\pi a_2}\nearrow$

1 $\quad 2a_i$

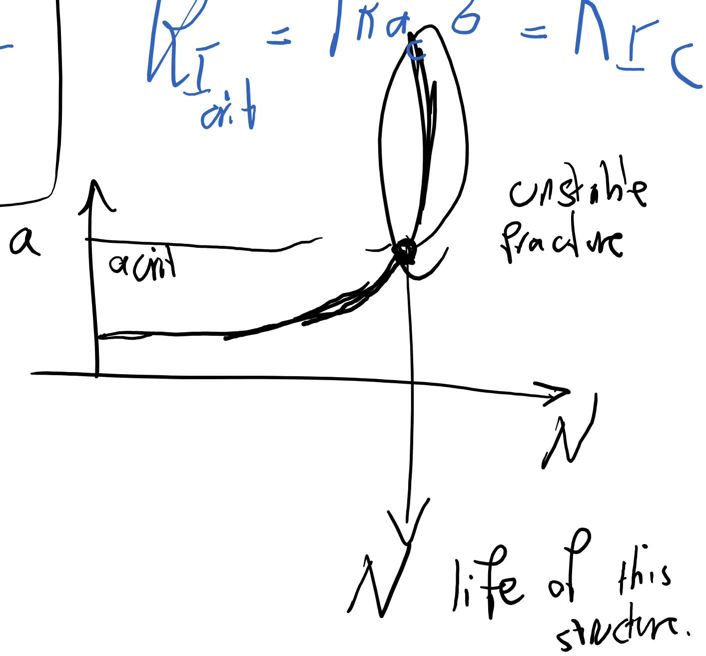
$K_{max} = \Delta K = \sigma\sqrt{\pi a_i}$
$< K_{Ic}$

gradually grows

UNTIL a is large enough that the crack can propage in an unstable manner



$$K_{I_{crit}} = \sqrt{\pi a_c}\,\sigma = K_{Ic}$$



loading, geometry, ... $\longrightarrow$ $N = ?$

old approach	New approach
— Stress-based ($\Delta\sigma$)	Paris law
— Doesn't directly	— K-based

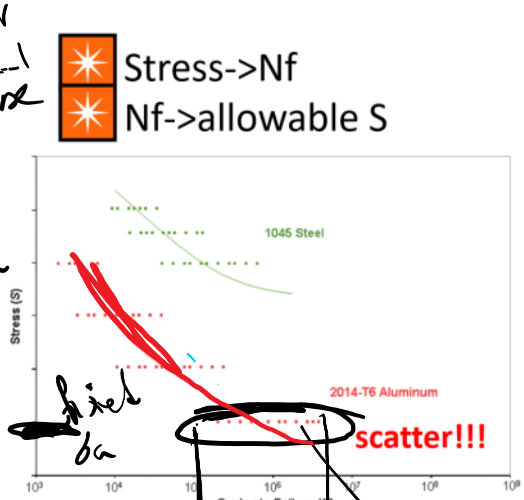
— Doesn't directly incorporate initial material flaws

— K-based

— we start with an initial crack length

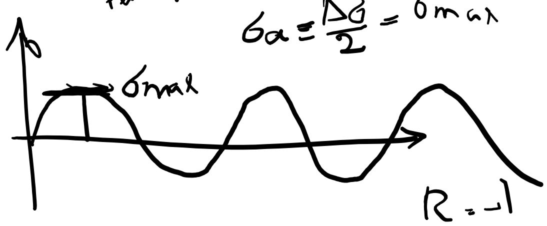
S–N plots, S–N–P plots

for R = −1 case



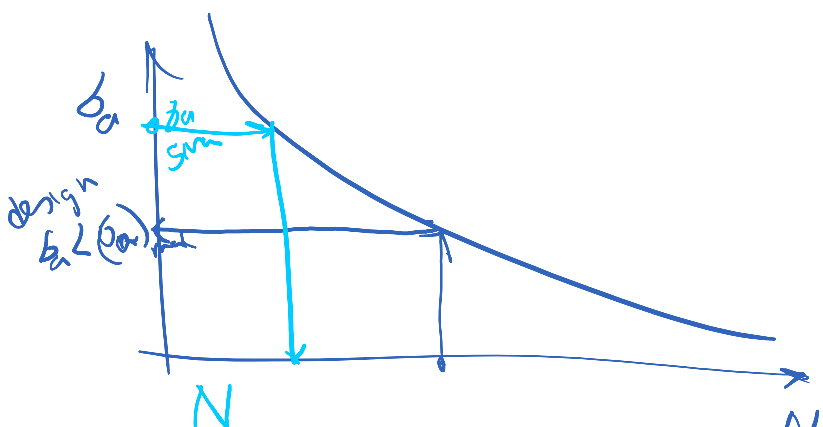
10^5 $\sim 5\times10^6$

for this case $\sigma_a = \frac{\Delta\sigma}{2} = \sigma_{max}$



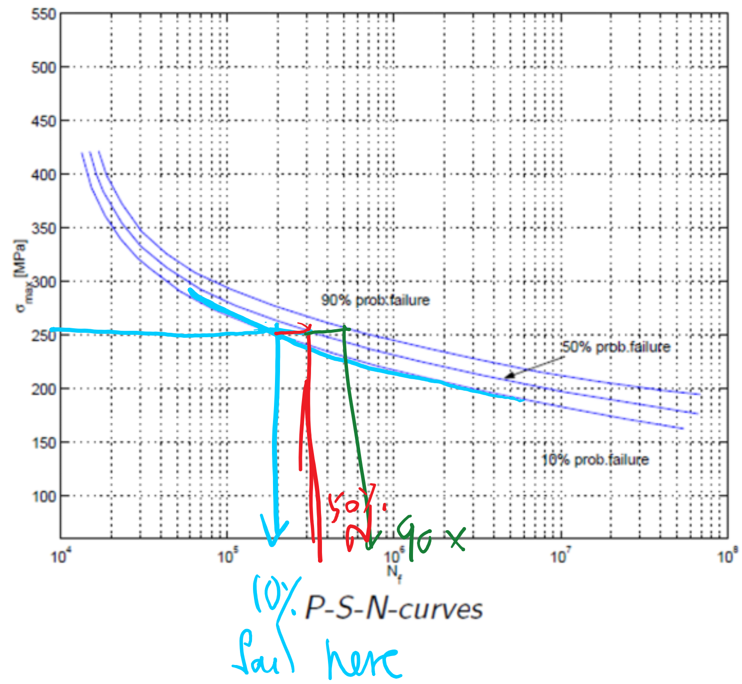
we observe a lot of scatter in N

greater than order of magnitude variation



specify the life of structure or helps

in monitoring time intervals .. to check the structure



P-S-N-curves

here