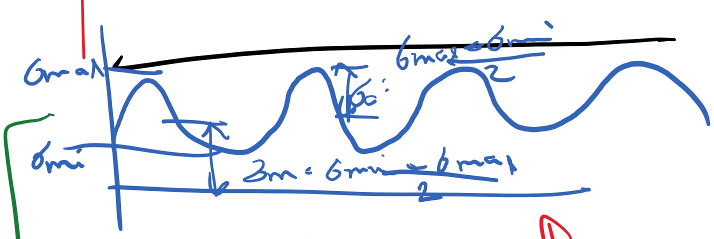
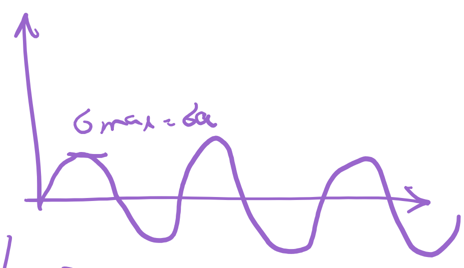
2020/11/09

Monday, November 9, 2020 2:51 PM

Continuation of S-N curve approach

S-N curves are represented for $R = -1$ case





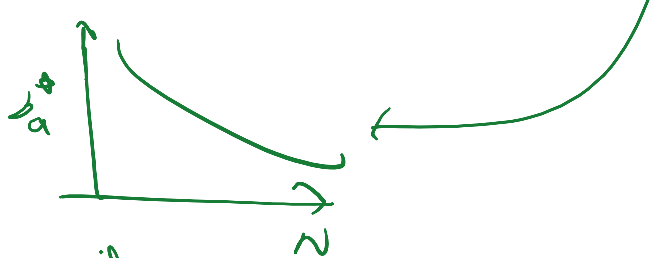
$R = \frac{\sigma_{min}}{\sigma_{max}}$

$R = -1$

Q: how can we get N for $R = -1$ $(\sigma_m \neq 0)$?

worse $(\sigma_m > 0)$ because we have higher σ_{max}

Idea modify $\sigma_a \rightarrow \sigma_a^*$ then go back to



$\sigma_m \rightarrow \sigma_y$

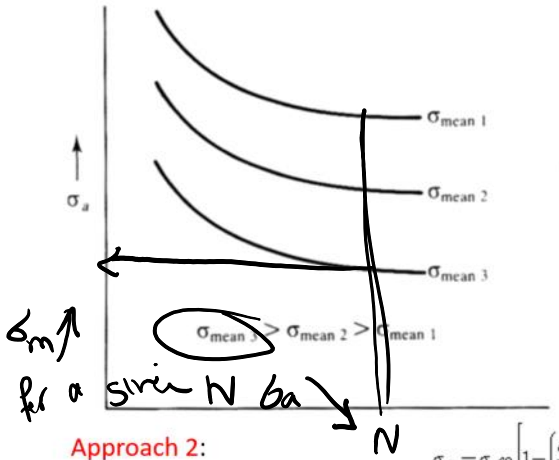
$N \rightarrow 0$ because material should fail by yielding

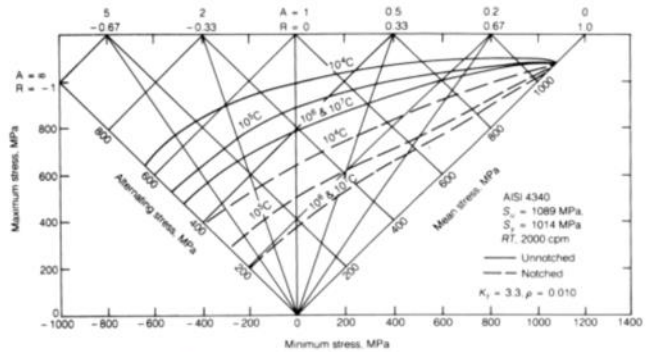
$$\sigma_a^* = \sigma_a \left[1 - \left(\frac{\sigma_m}{\sigma_u} \right)^{\alpha} \right]$$

σ_a^*: effective σ_a

σ_u: ultimate stress, or σ_y

Effect of mean stress





Approach 1: Master diagram

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

$$A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$$

Approach 2: Correction-factor formulas

$$\sigma_a = \sigma_{f0}\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)^r\right]$$

where σ_a is the amplitude of allowable stress (alternating stress).

σ_{f0} is the stress at fatigue fracture when the material under zero mean stress cycled loading

σ_m is the mean stress of the actual loading.

σ_u is the tensile strength of the material.

r = 1 is called Goodman line which is close to the results of notched specimens.

r = 2 is the Gerber parabola which better represents ductile metals.

Other correction factor

Gerber (1874)	$\frac{\sigma_a^*}{\sigma_a} = 1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2$
Goodman (1899)	$\frac{\sigma_a^*}{\sigma_a} = 1 - \frac{\sigma_m}{\sigma_u}$
Soderberg (1939)	$\frac{\sigma_a^*}{\sigma_a} = 1 - \frac{\sigma_m}{\sigma_{y0}}$

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allowed stress variation goes to zero

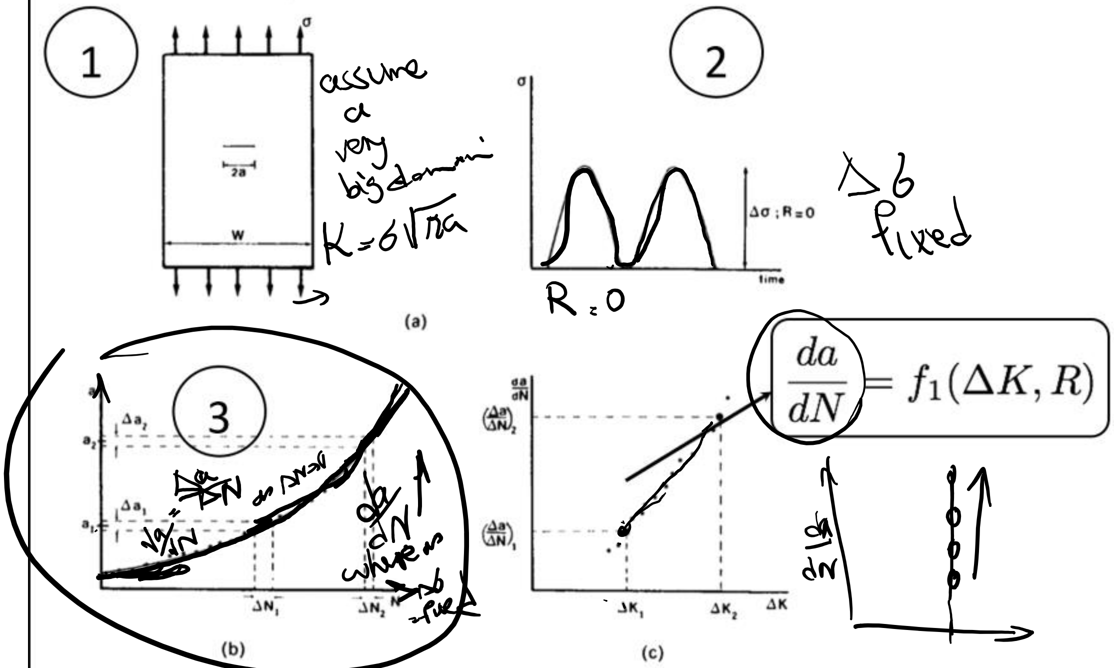
$\sigma_m \to \sigma_u$ or σ_{y0}

$\sigma_a^* \to 0$

New approach

Paris law

$$K = \sigma\sqrt{\pi a}$$



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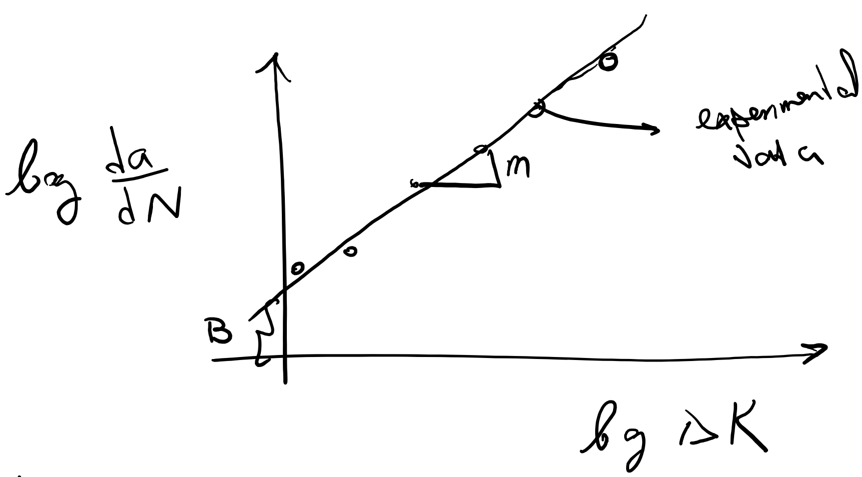
$K = \sigma\sqrt{\pi a}$ $\quad$ $\Delta K = \Delta\sigma\sqrt{\pi a}$ $\quad$ as $a \nearrow$ ΔK increases

$\frac{da}{dN}$ is it only a function of $\Delta\sigma$?

$$\frac{da}{dN} = f(\Delta\sigma)$$

fixed



$$\log \frac{da}{dN} \simeq B + m \log \Delta K$$

$$e^{(\quad)} \qquad e^{(\quad)}$$

$$\frac{da}{dN} = e^{B + m \log \Delta K} = (e^{B})(e^{m \log \Delta K})$$

$$\frac{da}{dN} = e^{B + m \log \Delta K} = \underbrace{(e^{B})}_{C} \underbrace{(e^{m \log \Delta K})}_{\Delta K^m}$$

$$(e^{\log \Delta K})^m$$

$$\boxed{\frac{da}{dN} = C \Delta K^m} \quad \text{Paris law}$$

We must be careful about units

$$\frac{da}{dN} : \frac{L}{1} = L \qquad \Delta K = [\sigma]\sqrt{L}$$

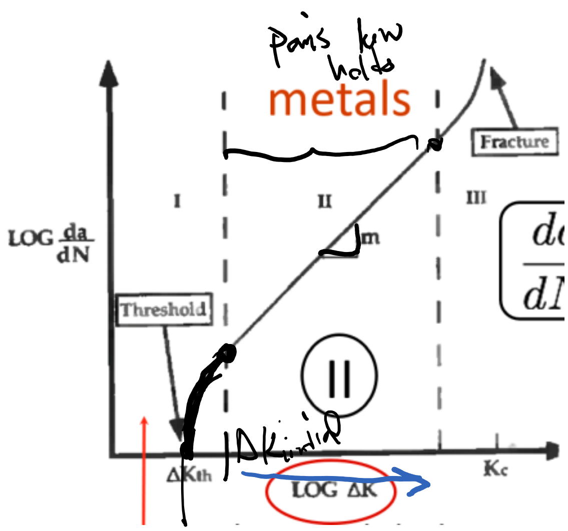
units : MPa$\sqrt{m}$ ...

C is not dimensionless

& its value changes from one system to another.

m is fixed for a material

In fact, Paris law ONLY holds for stage II fatigue crack propagation as shown below

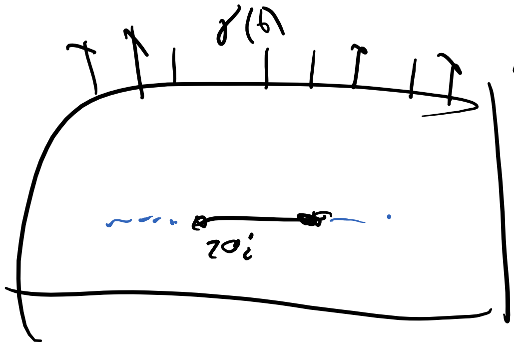


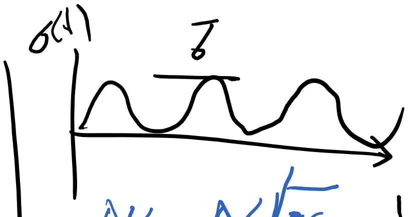
growth

$\Delta K < \Delta K_{th}$ no fatigue crack growth

$\Delta K = \Delta\sigma\sqrt{\pi a}$

$\Delta\sigma \nearrow$





$\Delta K = \Delta\sigma\sqrt{\pi a}$

$= \bar{\sigma}\sqrt{\pi a} \nearrow$

$K_{max} = \sigma_{max}\sqrt{\pi a}$

fixed $a \nearrow$

$= \bar{\sigma}$

$$K_{max} = K_t$$

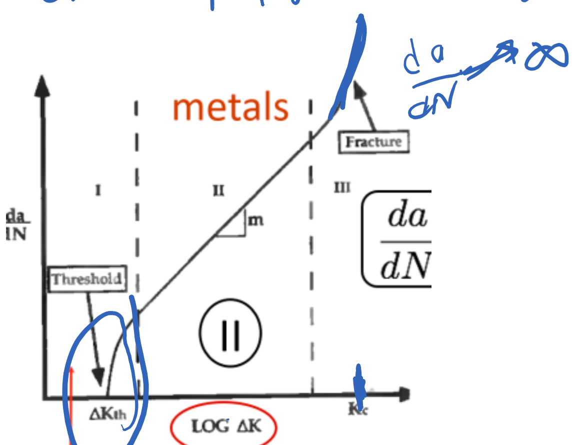
crack propagates right away

$$\sigma_{max}\sqrt{\pi a_f} = K_{Ic} \rightarrow a_f \text{ when } a \rightarrow a_f$$

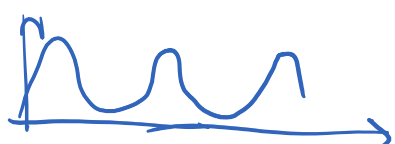
$(K \rightarrow K_{Ic})$

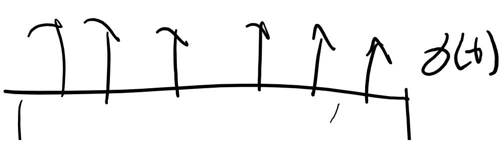
crack propagates in an unstable fashion

$\frac{da}{dN} \rightarrow \infty$

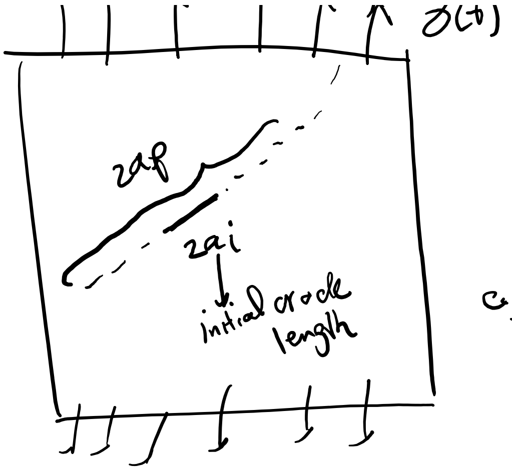


$\Delta K = K_{max}$ $(R = 0)$





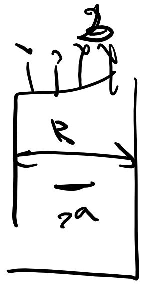
$K = \sigma\sqrt{\pi a}\, Y(a)$



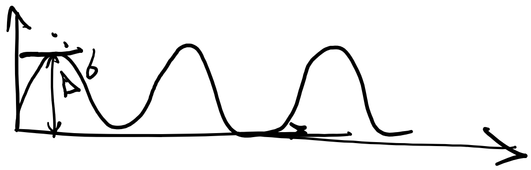
$K = \sigma\sqrt{\pi a}\, Y(a)$

$Y(a)$ → geometry correction factor

e.g. $Y(a) = \sec\left(\frac{a}{B}\right) = \frac{1}{\cos\left(\frac{a}{B}\right)}$



$\sigma_{min} = 0$



$$\Delta K = \Delta\sigma\sqrt{\pi a}\, Y(a)$$

$$\frac{da}{dN} = C\Delta K^{m}$$

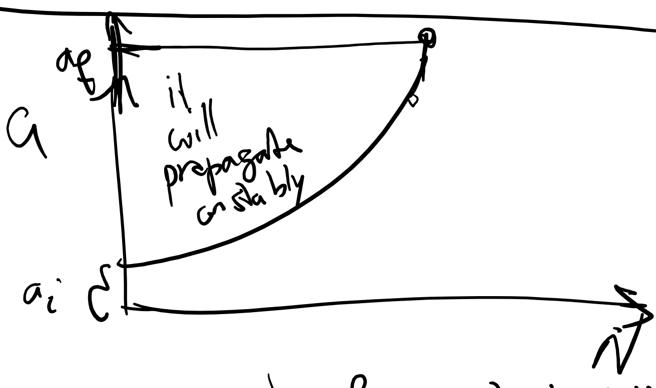
Paris law

This is an ODE that should be integrated in time

$$\frac{da}{dN} = C\Delta\sigma^{m}\, \pi^{m/2}\, a^{m/2}\, Y(a) \quad \text{ODE}$$

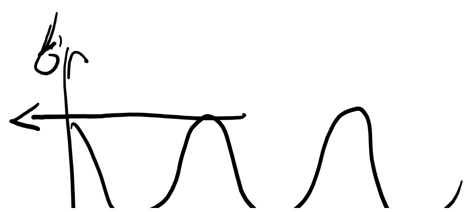
$$a(N=0) = a_i \qquad a_i < a \leq a_f$$

(1)

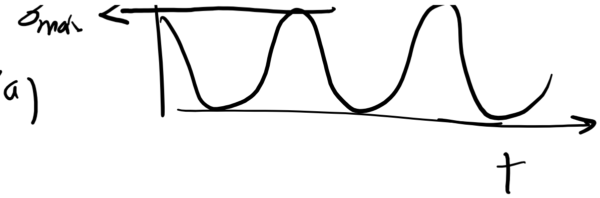


a_f is obtained from instantaneous crack propagation criterion

$K = \sigma\sqrt{\pi a}\, Y(a)$



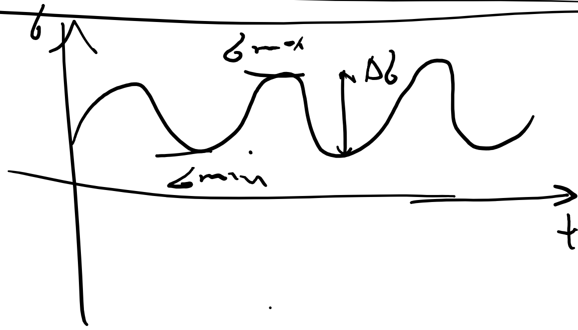
$$K_{max} = \sigma_{max}\sqrt{\pi a}\, Y(a)$$



$$= K_{Ic} \Longrightarrow$$

Solve a_f from $\boxed{Y(a_f)\sqrt{\pi a_f} = \frac{K_{Ic}}{\sigma_{max}}}$

Summary



$$\frac{da}{dN} = C\Delta K^m \quad (*)$$

$\Delta K = \Delta\sigma\sqrt{\pi a}\, Y(a)$

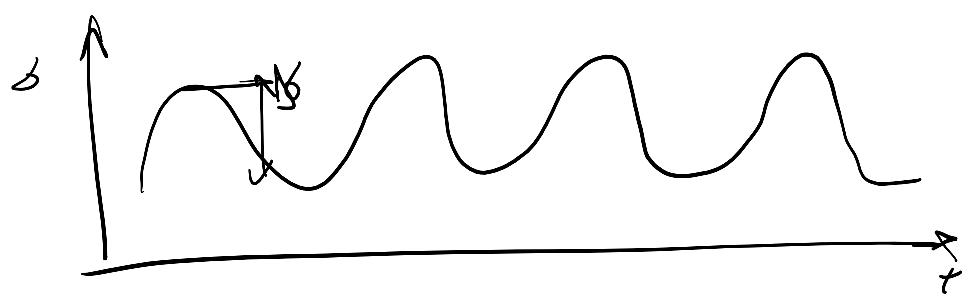
IC, $a = a_i$ @ $N = 0$

Final condition is $a = a_f$: $\sigma_{max}\sqrt{\pi a_f}\, Y(a_f) = K_{Ic}$

(2)

integrate (*) from $a = a_i$ to $a = a_f$
to get N_f (the life of this structure)

What about the case where $Y(a) \simeq 1$ is acceptable?



$$\frac{dN}{da} = C(\Delta K)^m = C\left(\pi^{1/2}\Delta\sigma^{1/2}\, a^{1/2}\right)^m$$

$\Delta K = \sqrt{\pi a}\,\Delta\sigma\, \cancel{Y}^{1}$

$$\longrightarrow \frac{dN}{da}\ \underbrace{\left[C' \pi^{m/2} \Delta\sigma^{m/2}\right]}_{A}\ a^{m/2}$$

$$\boxed{\frac{da}{dN} = A a^{m/2}, \quad A = C \pi^{m/2} \Delta\sigma^{m/2}} \qquad (3)$$

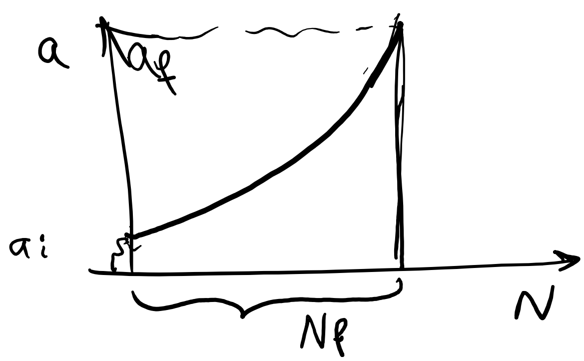
$$\frac{da}{a^{m/2}} = A\,dN \ : \ a^{-m/2}\,da = dN \qquad m > 2$$

OK typical of all materials

$$\longrightarrow \text{integrate} \quad \underbrace{\frac{1}{1-\frac{m}{2}}}_{-\frac{1}{\frac{m}{2}-1}} \ \underbrace{a^{1-\frac{m}{2}}}_{\frac{1}{a^{\frac{m}{2}-1}}} \Bigg|_{a_i}^{a} = N \Bigg|_{N=0}^{N}$$

$$-\frac{1}{\frac{m}{2}-1}\,\frac{1}{a^{\frac{m}{2}-1}} + \frac{1}{\frac{m}{2}-1}\,\frac{1}{a_i^{\frac{m}{2}-1}} = N(a)$$

$$\boxed{N(a) = \frac{1}{\left(\frac{m}{2}-1\right) C \pi^{m/2} \Delta\sigma^{m}} \left(\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a^{m/2}} \right)} \qquad (4)$$



$a_f = ?$

$$K_{max} = \sigma_{max}\sqrt{\pi a_f} = K_{IC}$$

$$\rightarrow \boxed{a_f = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_{max}}\right)^2} \quad (5)$$

4,5 →

$$N_f = \frac{1}{\left(\frac{m}{2}-1\right) C \pi^{m/2} \Delta\sigma^{m}} \left(\frac{1}{a_i^{m/2-1}} - \frac{1}{a_f^{m/2}} \right) \quad (5)$$

For $m > 2$:

$$N_f = \frac{2}{(m-2)\, C Y^m (\Delta\sigma)^m \pi^{m/2}} \left[\frac{1}{(a_0)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}} \right]$$

For $m = 2$:

$$N_f = \frac{1}{C Y^2 (\Delta\sigma)^2 \pi} \ln \frac{a_f}{a_0}$$

(source Course presentation S. Suresh MIT)

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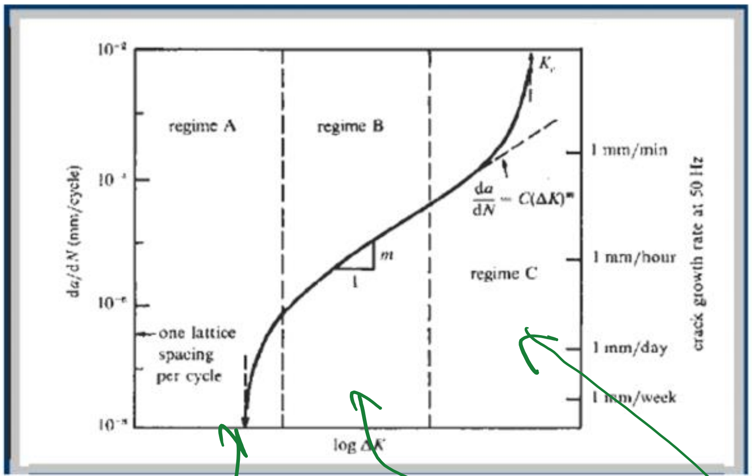
Some notes about fatigue



Regime	A	B	C
Terminology	Slow-growth rate (near-threshold)	Mid-growth rate (Paris regime)	High-growth rate
Microscopic failure mode	Stage I, single shear	Stage II, (striations) duplex slip	Additional static modes
Fracture surface features	Faceted or serrated	Planar with ripples	Additional cleavage or microvoid coalescence
Crack closure levels	High	Low	—
Microstructural effects	Large	Small	Large
Load ratio effects	Large	Small	Large
Environmental effects	Large	*	Small
Stress state effects	—	Large	Large
Near-tip plasticity†	$r_c \leq d_g$	$r_c \geq d_g$	$r_c \gg d_g$

*large influence on crack growth for certain combinations of environment, load ratio and frequency.

† r_c and d_g refer to the cyclic plastic zone size and the grain size, respectively.

(source Course presentation S. Suresh MIT)

not depends on load ratio R

$$\frac{da}{dN} = C(\Delta K)^m, \quad \Delta K = K_{\max} - K_{\min}$$

Table 1: Numerical parameters in the Paris equation.

alloy	m	A
Steel	3	10^{-11}
Aluminum	3	10^{-12}
Nickel	3.3	4×10^{-12}
Titanium	5	10^{-11}

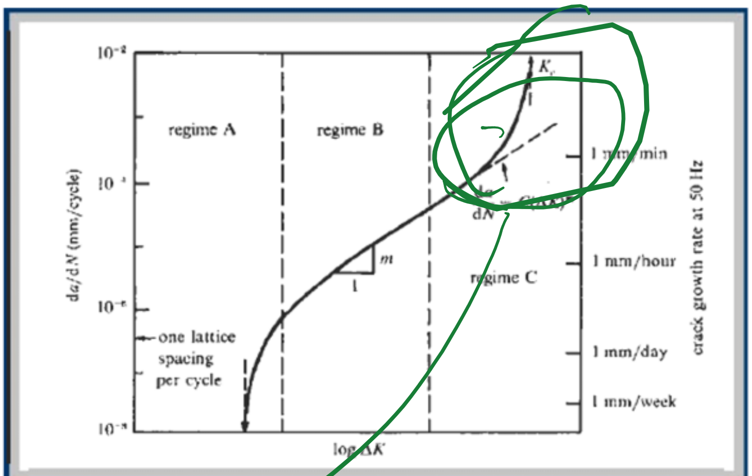
C, m

C, m

are material properties that must be determined experimentally from a log(delta K)-log(da/dN) plot.

m	
2-4	metals
4-100	ceramics/ polymers

much more fatigue sensitive



Improvements to Paris law to also model region III & accurate representation for transition from stage II to III

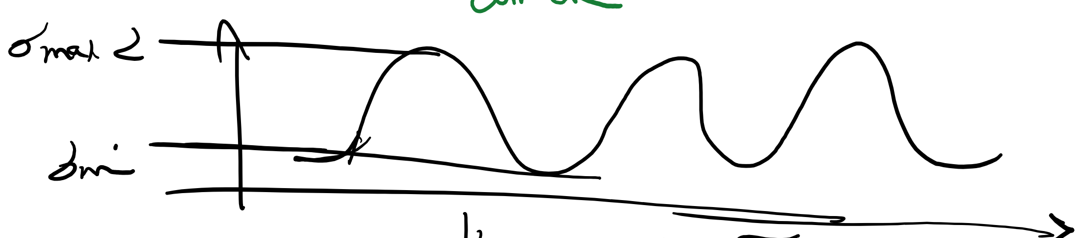
Forman's model (stage II-III)

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{C(\Delta K)^n}{(1-R)K_c - \Delta K}$$

correction

Paris' model

$$\frac{da}{dN} = C(\Delta K)^m$$



$R = \frac{\sigma_{min}}{\sigma_{max}}$

$K_{min} = \sigma_{min}\sqrt{\pi a}$

$K_{max} = \sigma_{max}\sqrt{\pi a}$

$\rightarrow R = \frac{K_{min}}{K_{max}}$

denominator = $(1-R)K_{Ic} - \Delta K =$

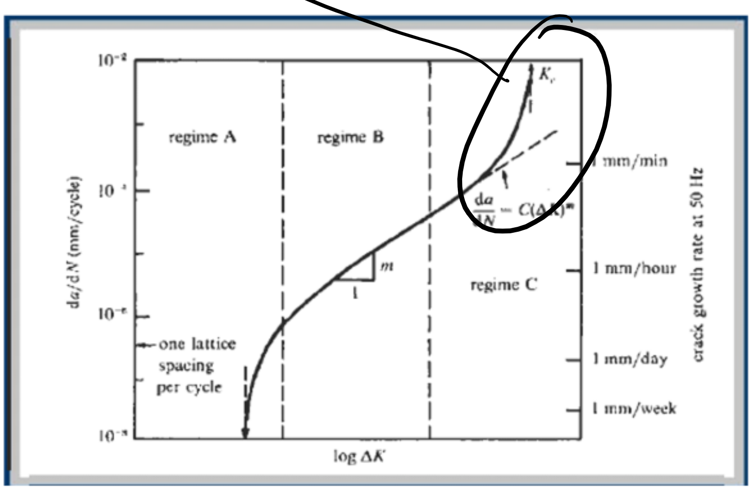
$$\left(1 - \frac{K_{min}}{K_{max}}\right)K_{Ic} - (K_{max} - K_{min})$$

$$= \underbrace{(K_{max} - K_{min})}_{\Delta K}\left(\frac{K_{Ic}}{K_{max}} - 1\right)$$

$$(6) \quad \boxed{\frac{da}{dN} = \frac{C\,\Delta K^m}{\Delta K\left(\frac{K_{Ic}}{K_{max}} - 1\right)}}$$ Forman's model

$K_{max} \uparrow \quad K_{max} = Y\sigma_{max}\sqrt{\pi a}$ because $a \uparrow$

$$K_{max} \rightarrow K_{Ic} \implies \frac{da}{dN} \rightarrow \infty$$



This smoothens the transition of $\frac{da}{dN} \sim C\Delta K^m$ from zone II (Paris law) to $\approx$ III (fast crack growth)

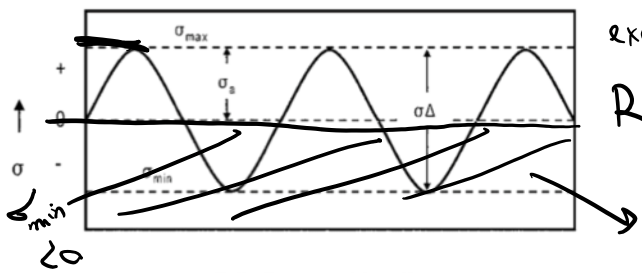
Tension-compression loading

$$R = \frac{\sigma_{min}}{\sigma_{max}} < 0$$

($\sigma_{min} < 0$, $\sigma_{max} > 0$)

$$K_{max} = \sigma_{max}\sqrt{\pi a}$$

$$K_{min} = \begin{cases} 0 & ? \\ \sigma_{min}\sqrt{\pi a} < 0 & ? \end{cases}$$



Fully Reversed Loading

example

$R = -1$

$\sigma_{min} < 0$

not doing anything for fatigue

$$\Delta K = \left(\sigma_{max} - \max(\sigma_{min}, 0)\right)\sqrt{\pi a}$$

correction