2020/11/11

Wednesday, November 11, 2020 2:51 PM

From last time

For $m > 2$:

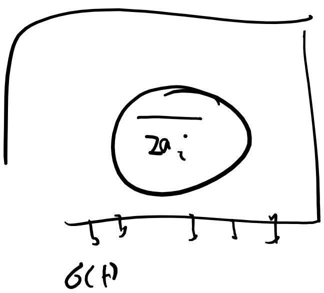
$$N_f = \frac{2}{(m-2)\,CY^m\,(\Delta\sigma)^m\,\pi^{m/2}}\left[\frac{1}{(a_0)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}}\right]$$

For $m = 2$:

$$N_f = \frac{1}{CY^2\,(\Delta\sigma)^2\,\pi}\ln\frac{a_f}{a_0}$$

$$K_{Ic} = Y\sqrt{\pi a_f}\,\sigma_{max} \rightarrow a_f = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\,\sigma_{max}}\right)^2$$

fixed



$$N_f(a_i) = D\left(\frac{1}{a_i^{m/2}} - \frac{1}{a_f^{m/2}}\right)$$

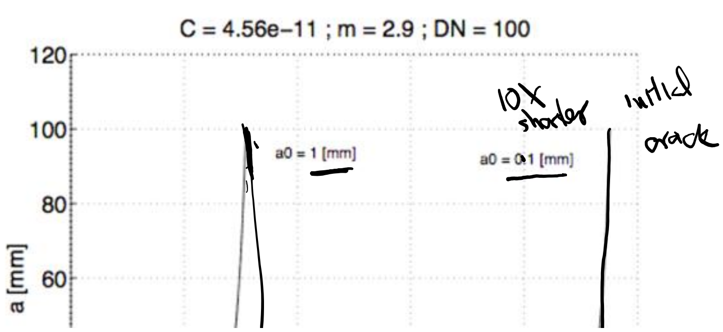
$$D = \frac{1}{(\frac{m}{2}-1)\,C\,\pi^{m/2}\,Y^m\,\Delta\sigma^m}$$

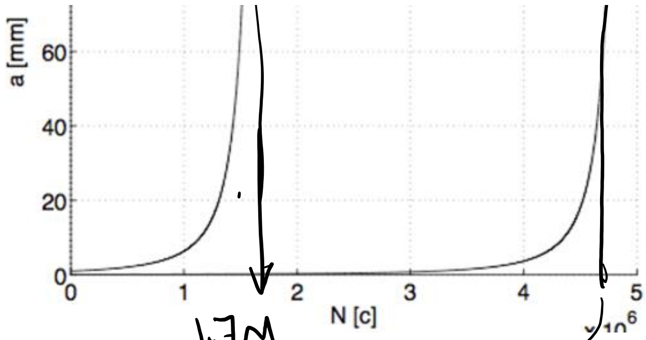
$a_i \rightarrow a_f$ $\quad N_f \rightarrow 0$ $\quad$ makes sense

$a_i \rightarrow 0$ $\quad N_f \rightarrow \infty$

$a_i \nearrow$ $\quad N_f(a_i) \searrow$

As shown below, ai has a significant influence on Nf (number of cycles to failure):

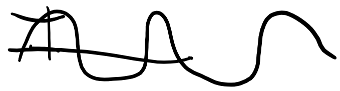




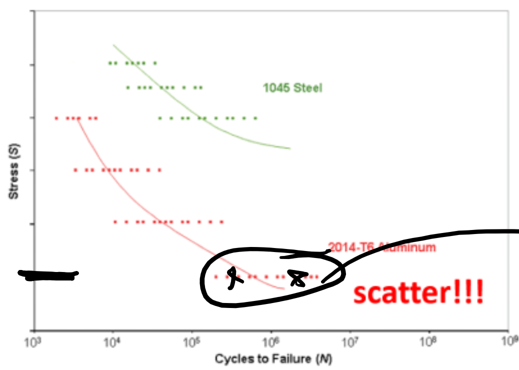
1.7M

4.7M

~ 3X decrease in life for a 10X longer crack



σ_a



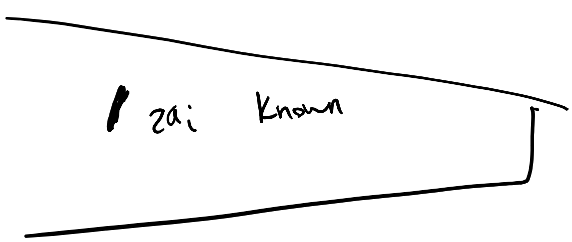
the scatter comes from the probabilistic nature of defects in material

S-N approach does not know much about the actual distribution of defects in a material.

If instead, we know the distribution of defects, we can analyze their fatigue growth in time and make sure it doesn't become unstable.

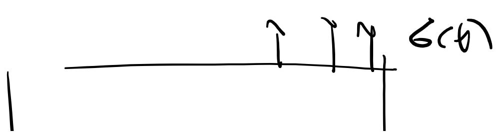
How do we choose ai?

Case I: We know the initial crack length because it's visible and we can see it.

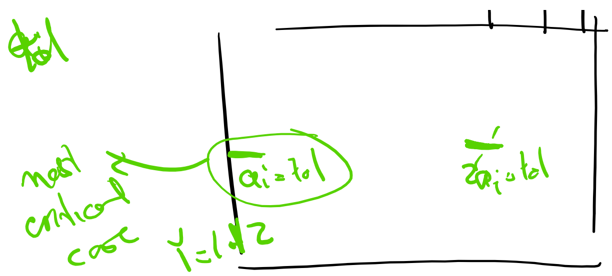


$2a_i$ known

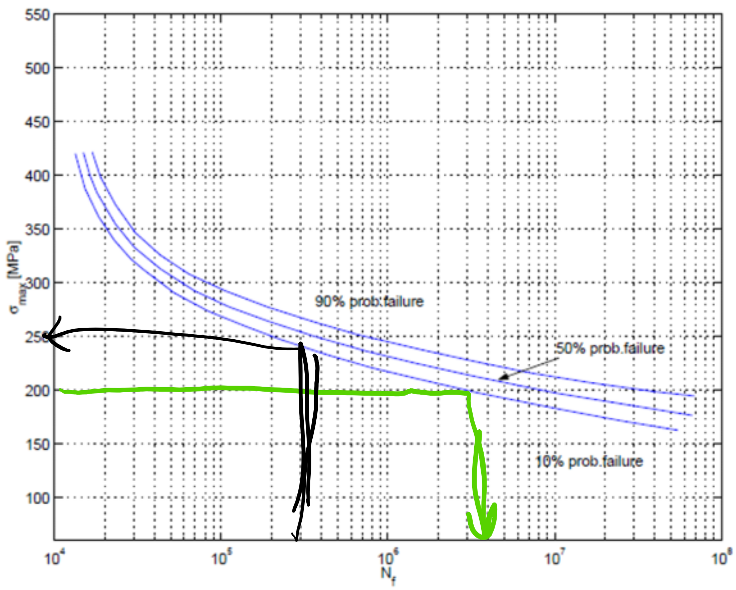
Case II: We cannot detect a crack. What should we do in this case?
We need to use the TOLERANCE of the measurement system to choose a worse-case scenario initial crack



$\sigma(t)$ $\Delta\sigma$ ✓

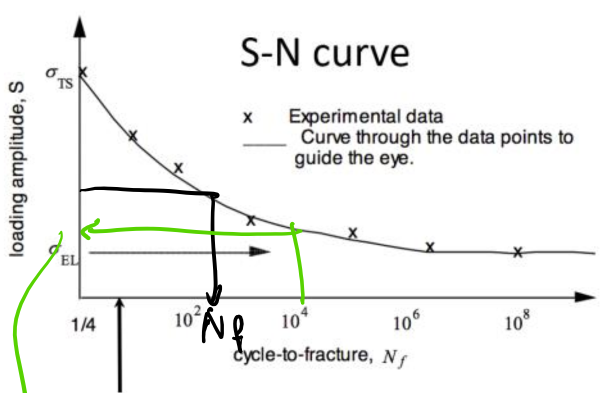


How do we build safety in our designs?
S/N plots



P-S-N-curves

Work with 10% prob of failure



use $\frac{N_f}{\text{Factor of safety}}$

use $\frac{\sigma_a}{FS}$

Paris law.

How can we incorporate safety in our designs

NDT: provides input (e.g. crack size) to fracture analysis

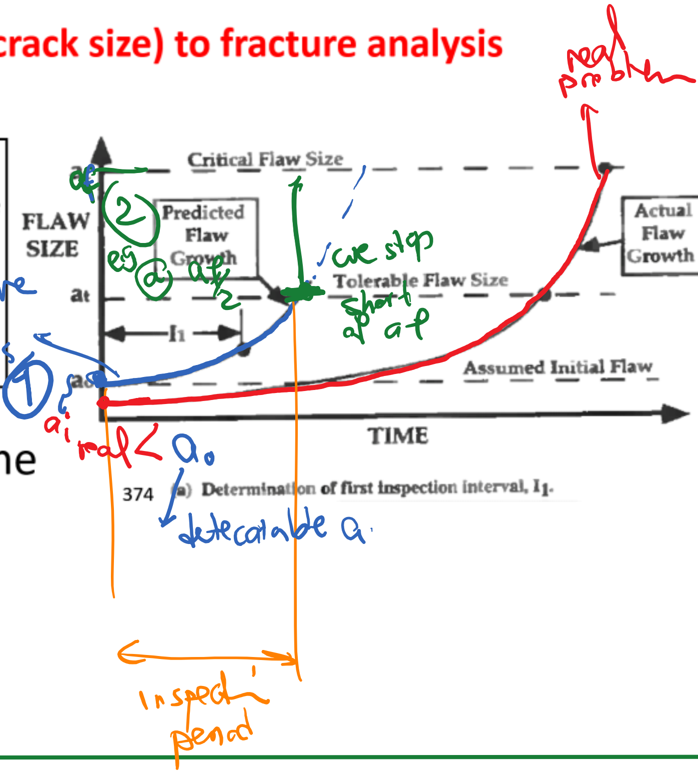
safety factor s

$K(a,\sigma) = K_c \rightarrow a_c -> a_t * s$

$\text{NDT} \rightarrow a_o$

$t: \quad a_o \rightarrow a_t$ (Paris)

inspection time



374 (a) Determination of first inspection interval, I_1.

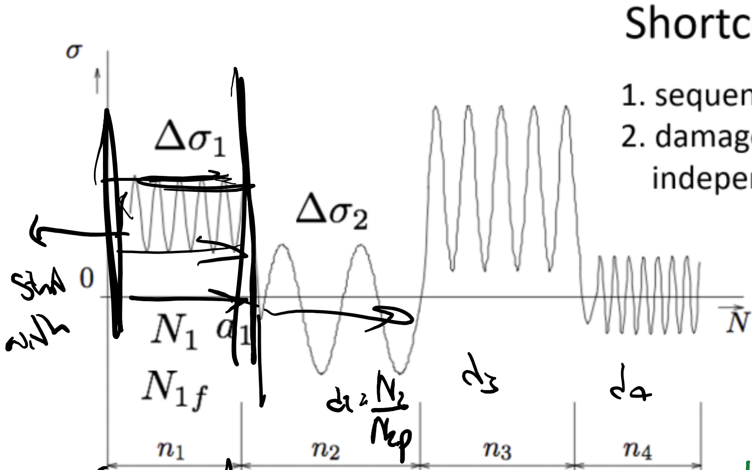
Variable load amplitudes

Miner's rule for variable load amplitudes

1945

Shortcomings:

1. sequence effect not considered
2. damage accumulation is independent of stress level



$d = \sum d_i = \sum \frac{N_i}{N_{fi}} \leq 1$

N_i/N_{if} : damage

Crude approach to see if the part is safe or not

a_i is given

$(\Delta\sigma)_i$ is known $\rightarrow$ $\underline{N_{if}}$ the number of cycles needed for fatigue failure

$d_1 = \frac{N_1}{N_{1f}}$ how much damage introduced through loading in stage 1

$d_1 < 1$

$\ll 1$ safe

do the same thing with load scenario 2

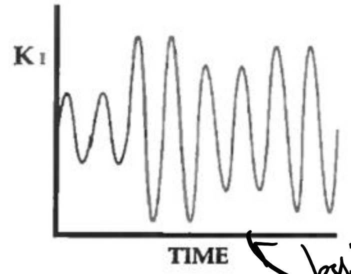
Influence of sequence of loading

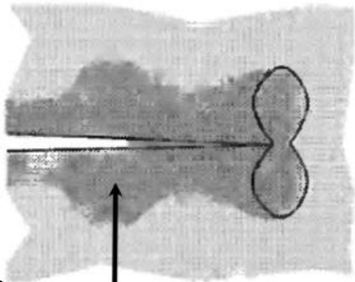
The component is assumed to fail when the total damage becomes equal to 1, or

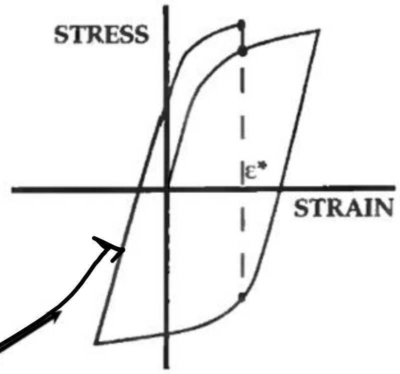
$$\sum_i \frac{n_i}{N_{fi}} = 1$$

It is assumed that the **sequence** in which the loads are applied has no influence on the lifetime of the component. In fact, the sequence of loads *can* have a large influence on the lifetime of the component.

Variable amplitude cyclic loadings







builds by loading history & local strain-stress history

$$\frac{da}{dN} = f_2(\Delta K, R, H)$$

history variables

plasticity: history dependent

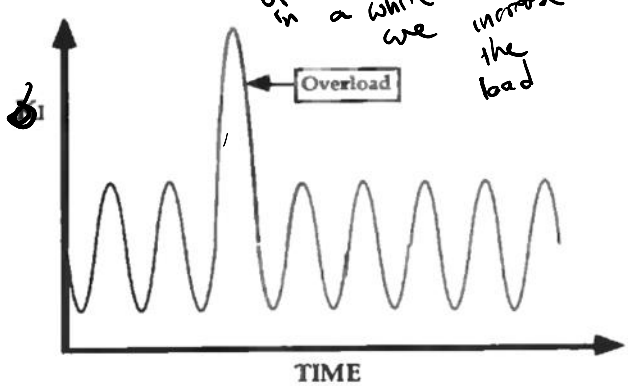
plastic wake

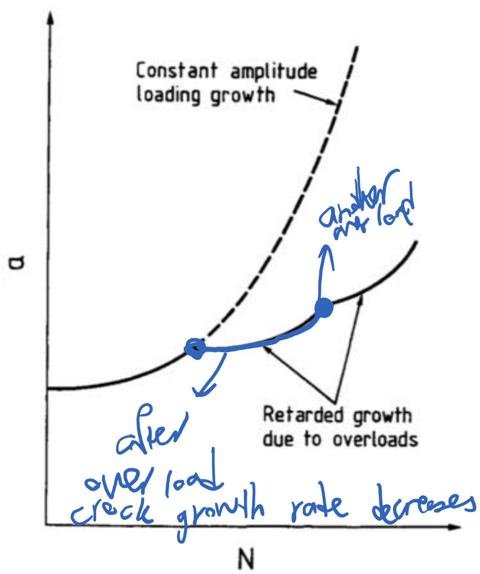
Approaches to increase fatigue life

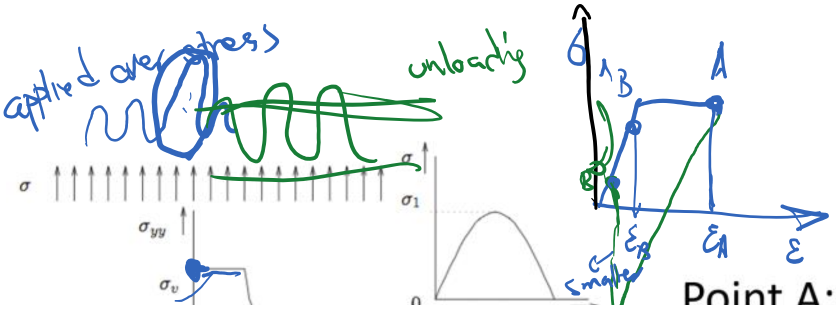
Overload and crack retardation

It was recognized empirically that the application of a tensile overload in a constant amplitude cyclic loa[d] leads to crack retardation following the overload; that is, the crack growth rate is smaller than it would have been under constant amplitude loading.

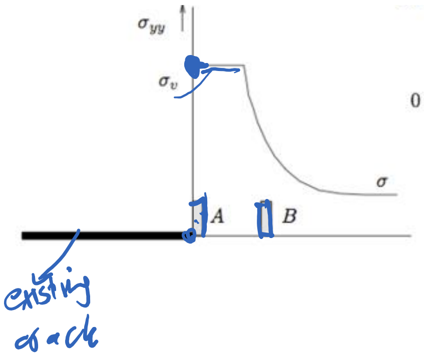
once in a while we increase the load

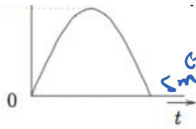


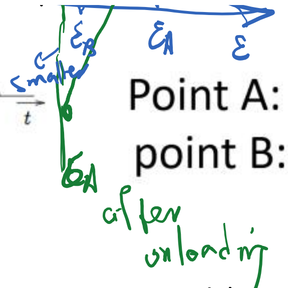




Point A: plastic





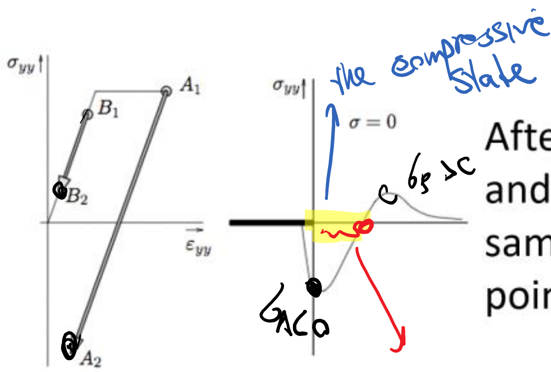


Point A: plastic
point B: elastic

after unloading A, B more or less have the same strain

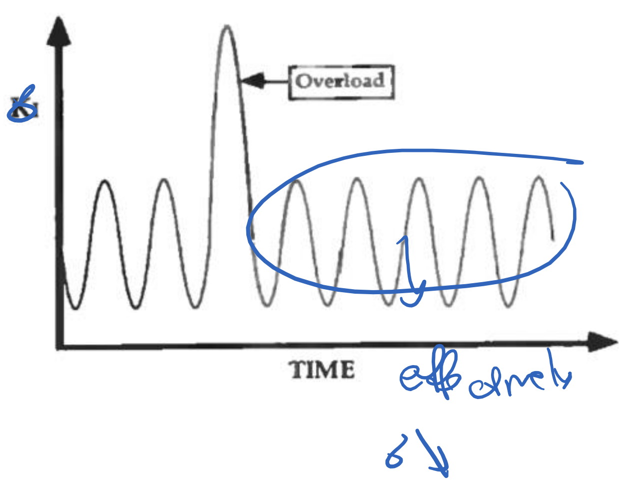
σ_A becomes compressive
σ_B " tensile
base line state

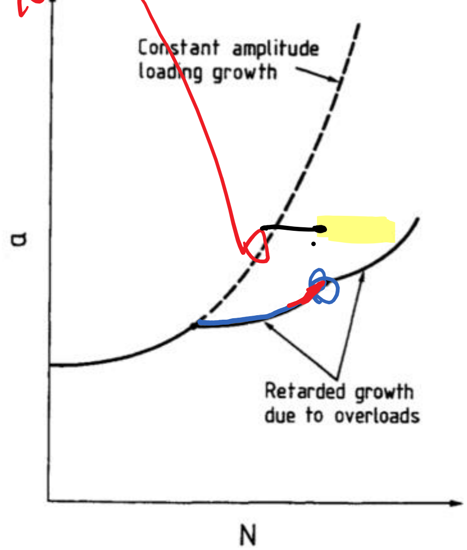
we have introduced residual stress where around the crack tip the stress state is compressive

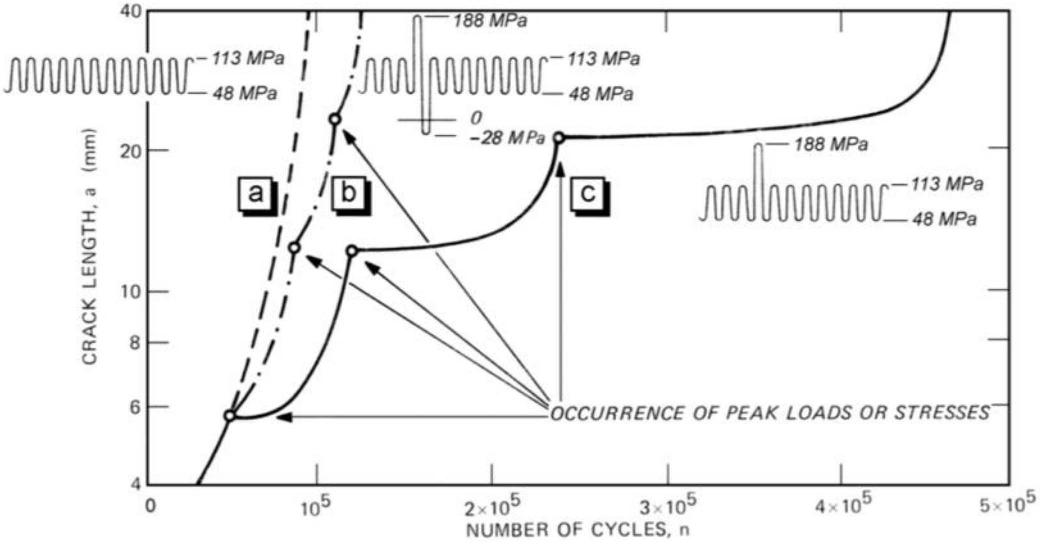


After unloading: point A and B has more or less the same strain ->
point A : compressive stress.

when the grows propagates beyond this zone we lose this effect

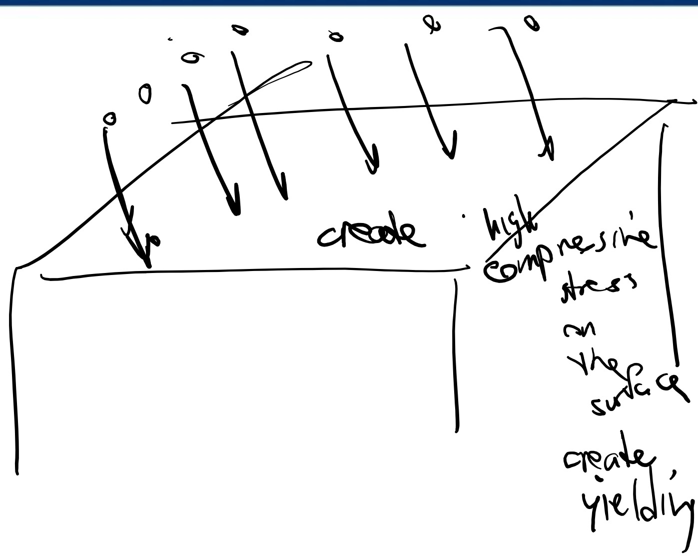






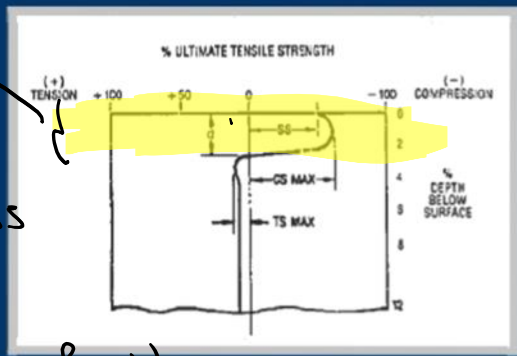
Fatigue crack inhibition: Shot-peening

Shot peening is a cold working process in which the surface of a part is bombarded with small spherical media called *shot*. Each piece of shot striking the surface acts as a tiny peening hammer, imparting to the surface a small indentation or dimple. The net result is a layer of material in a state of residual compression. It is well established that cracks will not initiate or propagate in a compressively stressed zone.



A typical residual stress profile created by shot peening is shown below:

below the surface we create compressive residual stress state (even as high as 50% of σ_y)



Examples for Fatigue

A large plate contains a crack of length $2a_0$ and is subjected to a constant-amplitude tensile cyclic stress normal to the crack which varies between 100 MPa and 200 MPa. The following data were obtained: for $2a_0 = 2$ mm it was found that $N = 20,000$ cycles were required to grow the crack to $2a_f = 2.2$ mm, while for $2a = 20$ mm it was found that $N = 1000$ cycles were required to grow the crack to $2a_f = 22$ mm. The critical stress intensity factor is $K_c = 60$ MPa $\sqrt{m}$. Determine the constants in the Paris (Equation (9.3)) and Formam (Equation (9.4)) equations.

$\sigma_{min} = 100$ MPa

$\sigma_{max} = 200$ MPa

$K_c = 60\ MPa\sqrt{m}$

$C = ?$

$m = ?$

what are Paris law parameters

case a

$2a_0 = 2mm$

$a_0 = 1mm = 1e\text{-}3\ m$

$N = 20{,}000$ $\quad 2a_f = 2.2mm$

$a_f = 1.1e\text{-}3$

case b

$2a_i = 20mm$

$a_i = 1e\text{-}2\ m$

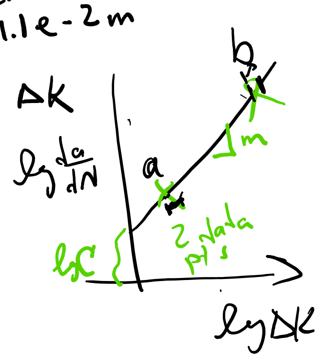
$N = 1000$

$2a_f = 22mm$

$a_f = 1.1e\text{-}2\ m$

$\frac{da}{dN} = C\Delta K^m \rightarrow \log\frac{da}{dN} = \log C + m\log\Delta K$

Find 2 data pts for ΔK & $\frac{da}{dN}$



$K = \sigma\sqrt{\pi a} \rightarrow \Delta K = \Delta\sigma\sqrt{\pi a}$

$(200-100)$ MPa

case a $\quad$ (…) $\quad 100\ MPa\sqrt{\pi(1e\text{-}3)m} = 5.6\ MPa\sqrt{m}$

MPa

case a $\quad (\Delta K)_a = 100\,MPa \sqrt{\pi (1e\text{-}3)\,m} = 5.6\,MPa\sqrt{m}$

$$\left(\frac{da}{dN}\right)_a \approx \frac{\Delta a}{\Delta N} = \frac{1.1e\text{-}3 - 1e\text{-}3}{20000} = 5e\text{-}9 \ m$$

Case b $\quad (\Delta K)_b = 100\,MPa \sqrt{\pi (1e\text{-}2\,m)} = 17.72\,MPa\sqrt{m}$

$$\left(\frac{da}{dN}\right)_b = \frac{1.1e\text{-}2 - 1e\text{-}2}{1000} = 10^{-6}$$

$$\log \frac{da}{dN} = \log C + m \log \Delta K$$

$\left(\frac{da}{dN}\right)_a = 5e\text{-}9\,m \qquad \Delta K_a = 5.6\,MPa\sqrt{m}$

$\left(\frac{da}{dN}\right)_b = 10^{-6}\,m \qquad (\Delta K)_b = 17.72\,MPa\sqrt{m}$

$$-8.3 = \log C + .748\,m$$

$$-6 = \log C + 1.248\,m$$

$$\boxed{m = 4.6 \qquad C = 1.82e\text{-}12 \ \frac{m}{(MPa\sqrt{m})^{4.6}}}$$

$$\underbrace{\frac{da}{dN}}_{m} = C\,\underbrace{(\Delta K)}_{(MPa\sqrt{m})}{}^{m}$$

$m \to$ power

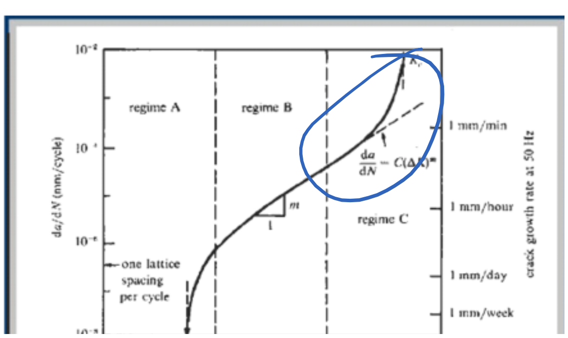
$\to$ length

What if we use Forman's correction?

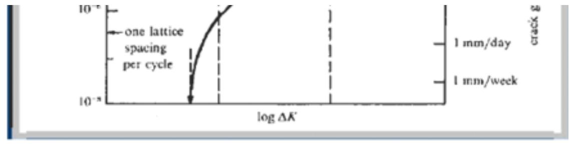
multiply & take log after

$$\frac{da}{dN} = \frac{C\,\Delta K^m}{\underbrace{[(1-R)K_c - \Delta K]}_{\text{correction}}}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{100\,MPa}{200\,MPa} = .5$$



$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{100\ MPa}{200\ MPa} = .5$



$K_c = 60\ MPa\sqrt{m}$

$$\Delta k = \begin{cases} 5.6\ MPa\sqrt{m} & \text{case a} \\ 17.72\ MPa\sqrt{m} & \text{case b} \end{cases}$$

$$[(1-R)K_c - \Delta k]\frac{da}{dN} = C\Delta k^m$$

$$[(1-.5)60 - 5.6]\ 5e{-9} = C(5.6)^m \qquad \text{case a}$$

$$[(1-.5)*60 - 17.72]\ 1e{-6} = C(17.72)^m \qquad \text{〃 b}$$

then take the log

$$-6.914 = \log C + .748\,m$$

$$-4.911 = \log C + 1.248\,m$$

$$\boxed{\begin{aligned} C &= 1.22 - — \\ m &= 4.006 \end{aligned}}$$

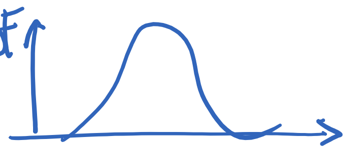
with Forman's correction

4.6 with Forman's correction

Next time

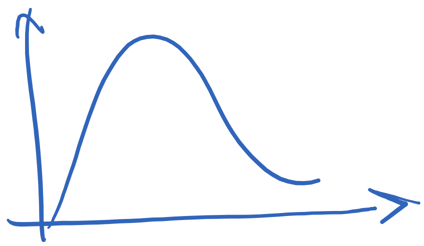
PDF

ai



ai

PDF



Nf