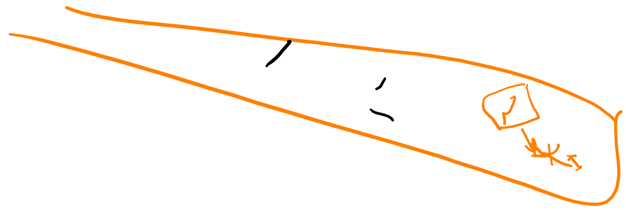
2020/11/16

Monday, November 16, 2020 2:53 PM

wing



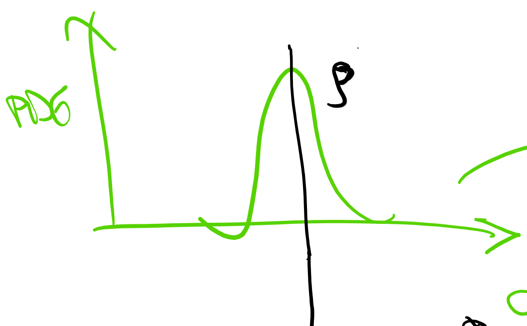
For $m > 2$:

$$N_f = \underbrace{\frac{2}{(m-2)\,C Y^m (\Delta\sigma)^m \pi^{m/2}}}_{D}\left[\frac{1}{(a_0)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}}\right]$$

$$N_f(a_i) = D\left[\frac{1}{a_i^{\,m/2-1}} - \frac{1}{a_f^{\,m/2-1}}\right]$$

$$Y\sqrt{\pi a_f}\,\sigma_{max} = K_{IC}$$

random variable



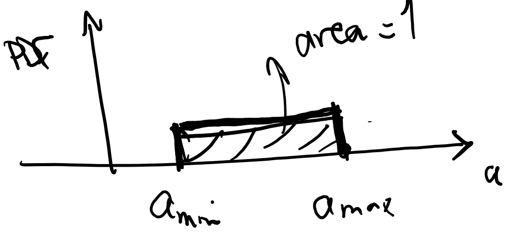
$$E(a_i) = \int_{-\infty}^{+\infty} a_i \underbrace{\rho(a_i)}_{\text{PDF of } a_i}\, da_i$$

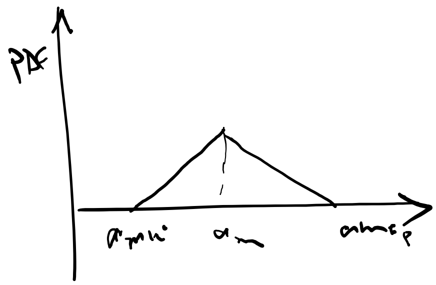
mean crack length

$$N_f\left(E(a_i)\right)$$

average initial crack length

Sample PDFs for a_i



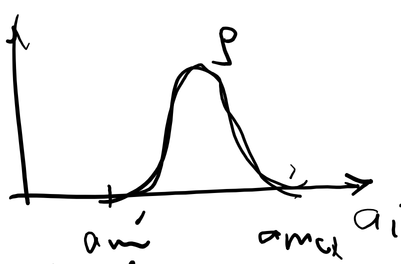


$$PDF(a) = U(a_{min}, a_{max}) = \frac{1}{a_{max} - a_{min}}$$

lognormal, normal

Expected value

mean value of something



$f(a_i)$

↓ some parameter that is a function of a_i

$$N(a_i) = D\left[\frac{1}{a_i^{m/2-1}} - \frac{1}{a_f^{m/2-1}}\right]$$

$$\underbrace{E(f)}_{\text{mean value of } f} = \bar{f} = \int_{-\infty}^{+\infty} f(a_i)\, \underset{\text{PDF}}{p(a_i)}\, da_i$$

for example for fatigue life,

$$E(N_f) = \int_{-\infty}^{+\infty} p(a_i)\, N_f(a_i)\, da_i$$

for example for uniform distribution

$-\infty$

$$\rho(a_i) = \frac{1}{a_{max}-a_{min}} \quad a_{min} < a_i < a_{max}$$

0 elsewhere

$$\boxed{E(N_f) = \int_{a_{min}}^{a_{max}} \frac{1}{a_{max}-a_{min}} \left(D\left(\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a_f^{\frac{m}{2}-1}} \right) \right) da_i} \quad (1)$$

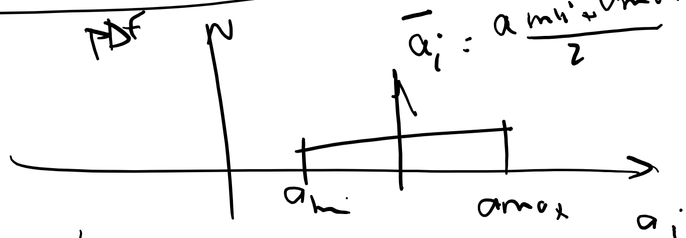
mean of life

What about life of mean cycle length

$$E(a_i) = \int_{-\infty}^{+\infty} a_i \, \rho(a_i) \, da_i = \int_{a_{min}}^{a_{max}} a_i \cdot \frac{1}{a_{max}-a_{min}} \, da_i$$

$$\boxed{E(a_i) = \frac{a_{min}+a_{max}}{2}}$$ for this case

$$\bar{a}_i = \frac{a_{min}+a_{max}}{2}$$



$$E(N_f) \neq N_f(E(a_i)) \quad ?$$

$$\frac{1}{a_{max}-a_{min}} \int_{a_{min}}^{a_{max}} D\left(\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a_f^{\frac{m}{2}-1}}\right) da_i \neq D\left(\frac{1}{\left(\frac{a_{min}+a_{max}}{2}\right)^{\frac{m}{2}-1}} - \frac{1}{a_f^{\frac{m}{2}-1}}\right)$$

(1)

We often do deterministic analysis

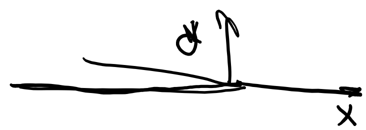
by using the average values (N_f / $E(\sigma_{1})$)

but as seen this is not equal to

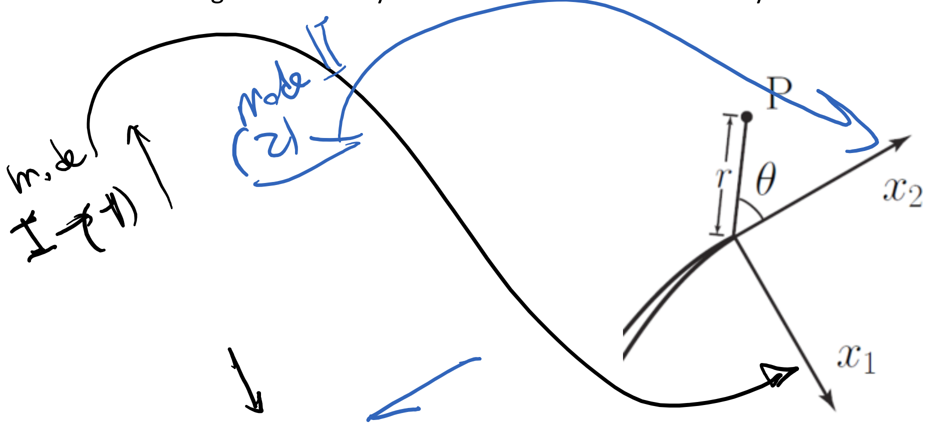
the actual average!

Dynamic stress intensity factor

Earlier



It's easier to use the following coordinate system for local stress and velocity fields:



What is the generalization of the solutions below for dynamics:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$$

(mode I)

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \quad \Sigma^{II}_{xx}(\theta)$$

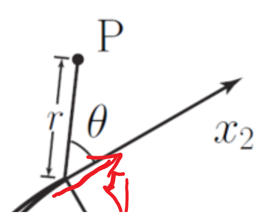
$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

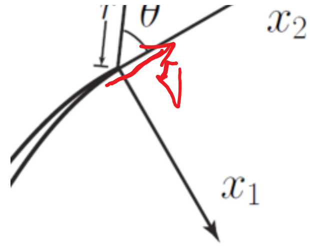
(mode II)

$$s^{ij}(r,\theta,t) = \frac{K_I(t)}{\sqrt{2\pi r}}\Sigma^{ij}_I(\theta,\hat{v}) + \frac{K_{II}(t)}{\sqrt{2\pi r}}\Sigma^{ij}_{II}(\theta,\hat{v}) \quad \text{as} \quad r \to 0.$$

crack speed



crack speed



$$K_I(t) = \lim_{x_2 \to 0} \sqrt{2\pi x_2}\, s^{11}(x_2, 0, t), \qquad K_{II}(t) = \lim_{x_2 \to 0} \sqrt{2\pi x_2}\, s^{12}(x_2, 0, t)$$

$$\Sigma_I^{11} = -\frac{1}{D}\left\{(1+\alpha_{II}^2)^2 \frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - 4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_I^{12} = \frac{2\alpha_I(1+\alpha_{II}^2)}{D}\left\{\frac{\sin\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - \frac{\sin\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_I^{22} = \frac{1}{D}\left\{(1+\alpha_{II}^2)(1+2\alpha_I^2-\alpha_{II}^2)\frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - 4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\}.$$

Mode I

$$\Sigma_{II}^{11} = \frac{2\alpha_{II}(1+\alpha_{II}^2)}{D}\left\{\frac{\sin\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - \frac{\sin\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_{II}^{12} = \frac{1}{D}\left\{4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - (1+\alpha_{II}^2)^2\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\},$$

$$\Sigma_{II}^{22} = -\frac{2\alpha_{II}}{D}\left\{(1+2\alpha_I^2-\alpha_{II}^2)\frac{\sin\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - (1+\alpha_{II}^2)\frac{\sin\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\}$$

Mode II

function of crack speed

$$\alpha_{(k)} = \sqrt{1 - \left(\frac{\hat{v}}{c_{(k)}}\right)^2}$$

crack speed

elastic wave speeds

k = 1 or 2

$$\gamma_{(k)} = \sqrt{1 - \left(\frac{\hat{v}\sin\theta}{c_{(k)}}\right)^2}$$

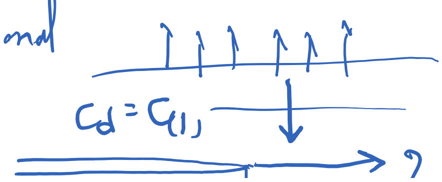
$$\theta_{(k)} = \alpha_{(k)}\, tg\,\theta \qquad k = 1, 2$$

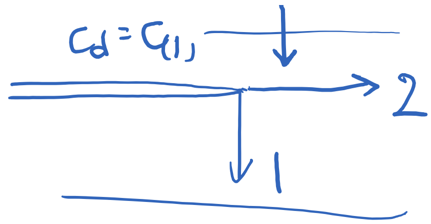
$$D(\hat{v}) = 4\alpha_I\alpha_{II} - (1+\alpha_{II}^2)^2$$

There is a crack speed for which D becomes zero. That means stress around the crack tip is basically infinity !!! So this is a forbidden crack speed.

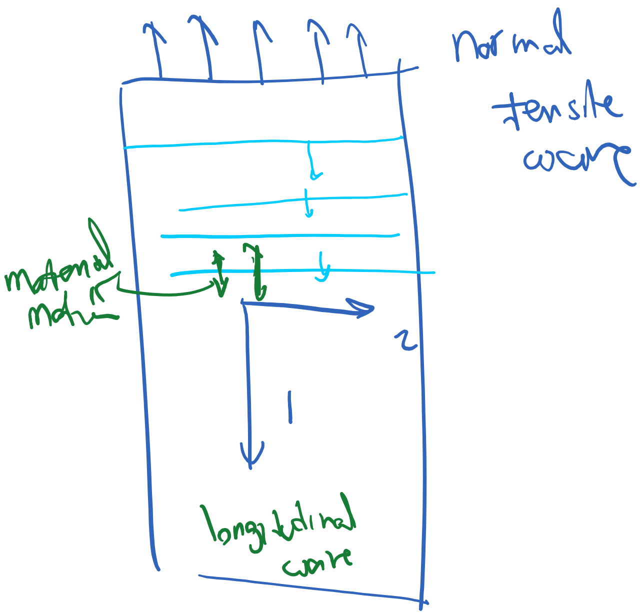
$$c_{(1)} = c_d = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

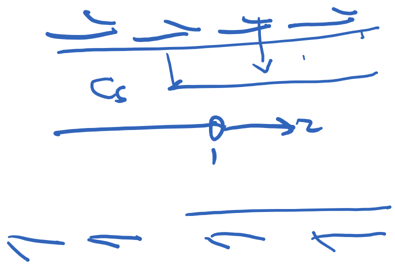
longitudinal/ dilational wave speed

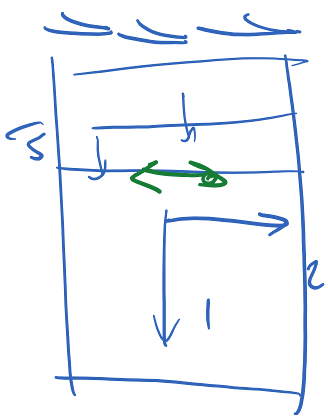


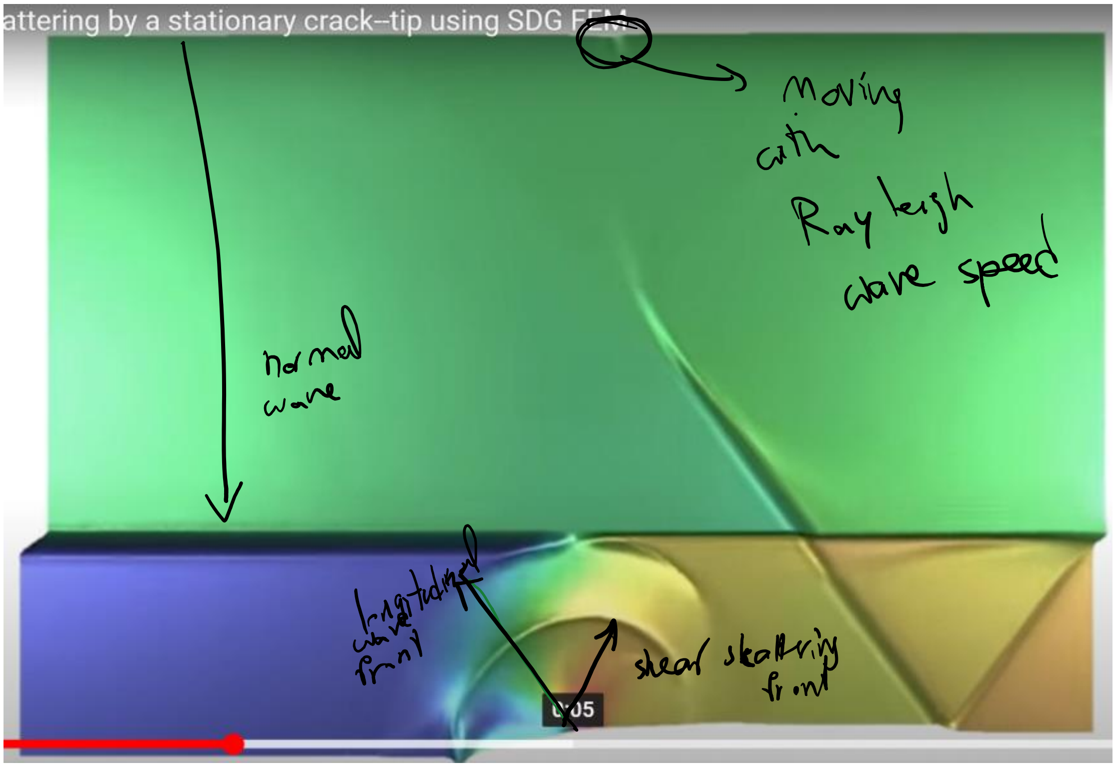


$$c_{(2)} = c_s = \sqrt{\frac{\mu}{\rho}}$$ shear wave speed









Rayleigh waves are surface waves

c_R

Rayleigh wave is the speed for which $D \neq 0$

$$\tilde{V} = c_R \Rightarrow D(\hat{v}) = 0$$

$$4\alpha_I(c_R)\,\alpha_{II}(c_R) - (1 + \alpha_{II}^2(c_R))^2 = 0$$

$\alpha_1 = \sqrt{1 - c_R^2/c_1^2}$ $\qquad c_2 = \sqrt{1 - c_R^2/c_2^2}$

Simplified approximate formula for Rayleigh wave speed:

$$\frac{c_R}{c_S} \approx \frac{.862 + 1.14\nu}{1+\nu}$$

$\hat{\nu} \longrightarrow c_R \Longrightarrow D(\hat{\nu}) \longrightarrow 0$

$\tilde{\sigma}^{ij} \longrightarrow \infty$ for all r!