2022/09/01

Thursday, September 1, 2022 2:22 PM

Size effect:

Bazant's size effect law

$$(\sigma_N)_u = Af_t\left(1+\frac{D}{B}\right)^{-1/2} = \frac{Af_t}{\sqrt{1+\frac{D}{B}}} \Rightarrow \frac{Af_t}{\sqrt{D/B}} \quad (14.8)$$

$D \to \infty$

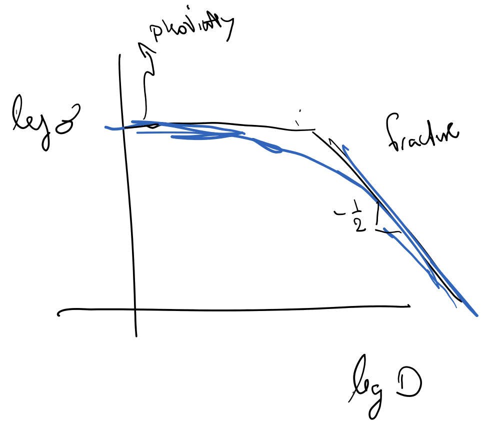
where

$(\sigma_N)_u$ = Nominal stress at failure of a structure of specific shape and loading condition.

W = Characteristic length of the structure.

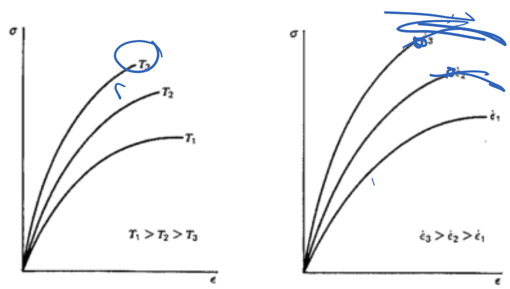
A, B = Positive constants that depend on the fracture properties of the material and on the shape of the structure, but not on the size of the structure.

f_t = Tensile strength of the material introduced for dimensional purposes.



7. Rate effects on ductility

- Same materials that show temperature toughness sensitivity (BCC metals) show high rate effect
- Polymers are highly sensitive to strain rate (especially for T > glass transition temperature)



Strain rate ↗ similar to T ↘

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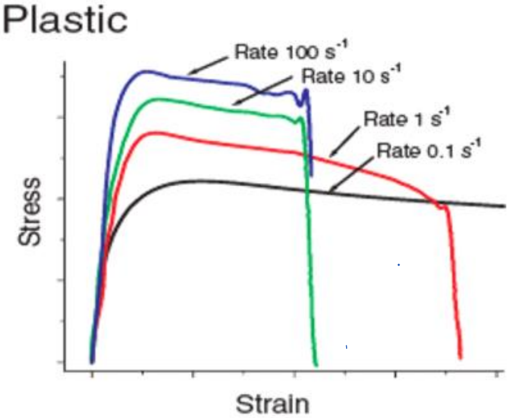
$\dot{\varepsilon}$ ↗ → strength increase

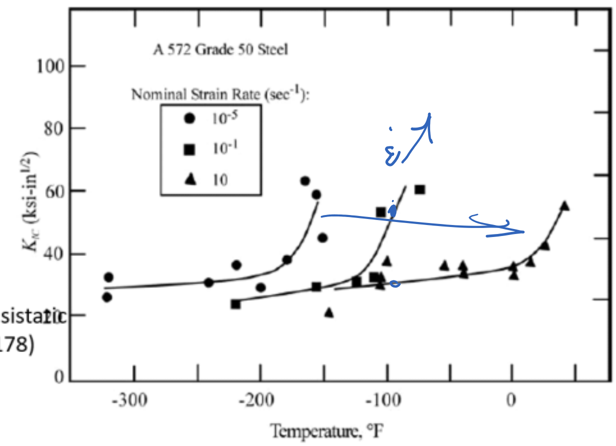
- Strain rate ↗
  1. Strength ↗
  2. Ductility ↘
  3. Toughness not clear

Plastic



Strain rate effects on Impact toughness

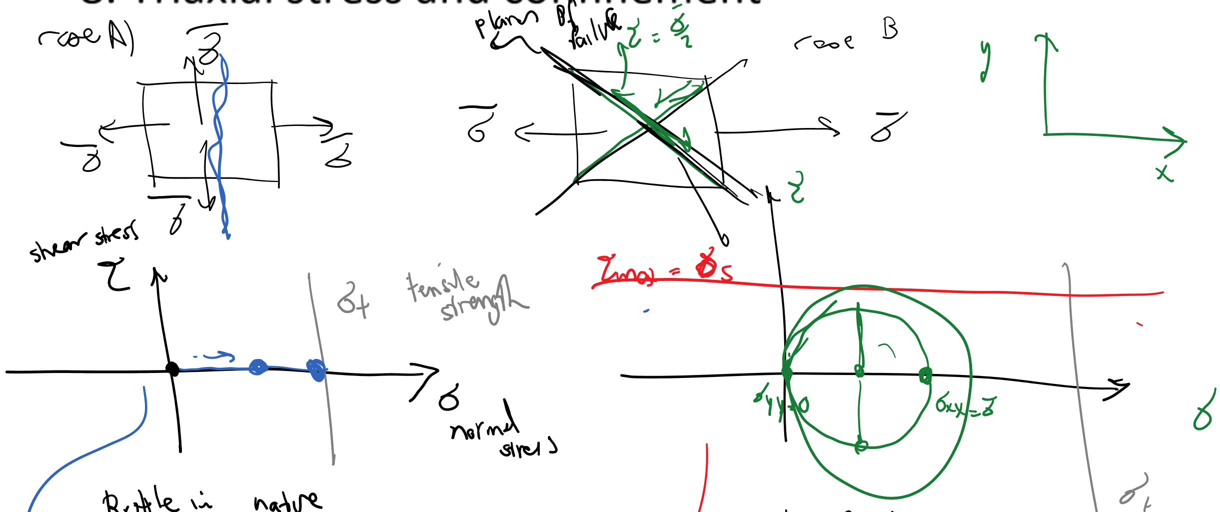
Strain rate ↗ ⇨
DBTT ↗ (more brittle in impact)



Ki Computed using quasistatic relations (Anderson p.178)

FIGURE 4.5 Effect of loading rate on the cleavage fracture toughness of a structural steel. Taken from Barsom, J.M., "Development of the AASHTO Fracture Toughness Requirements for Bridge Steels." *Engineering Fracture Mechanics*, Vol. 7, 1975, pp. 605–618.

8. Triaxial stress and confinement

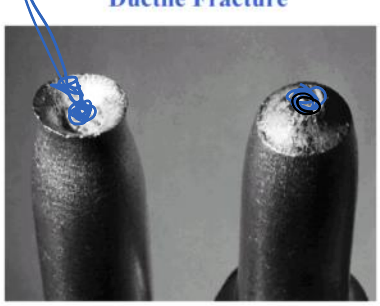


Brittle in nature
"triaxial" loading

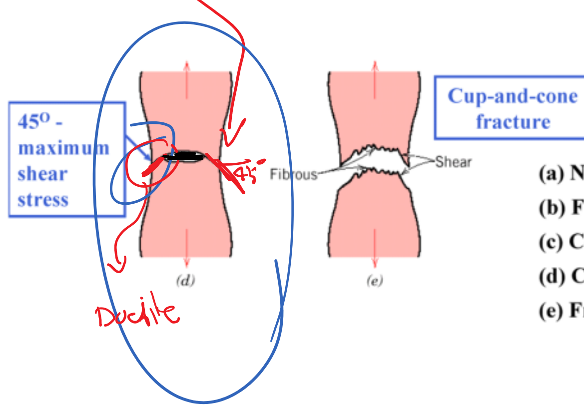
plastic failure
Ductile
Uniaxial

more confined & triaxial state

Ductile Fracture

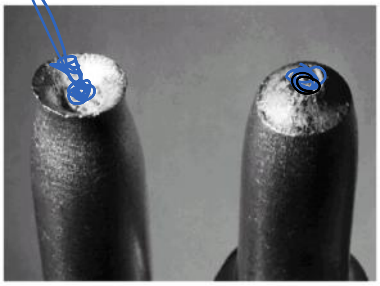


(Cup-and-cone fracture in Al)

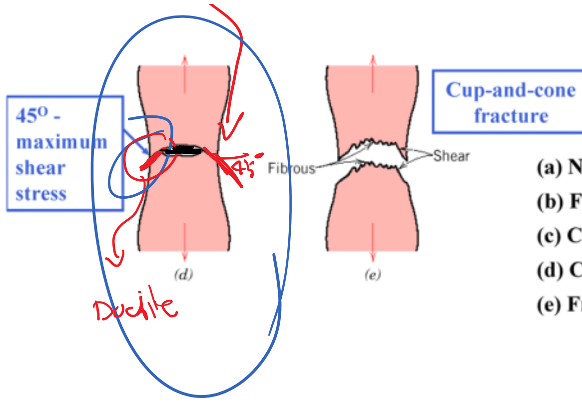


(a) Necking
(b) Formation of microvoids
(c) Coalescence of microvoids to form a crack
(d) Crack propagation by shear deformation
(e) Fracture

Ductile Fracture



(Cup-and-cone fracture in Al)



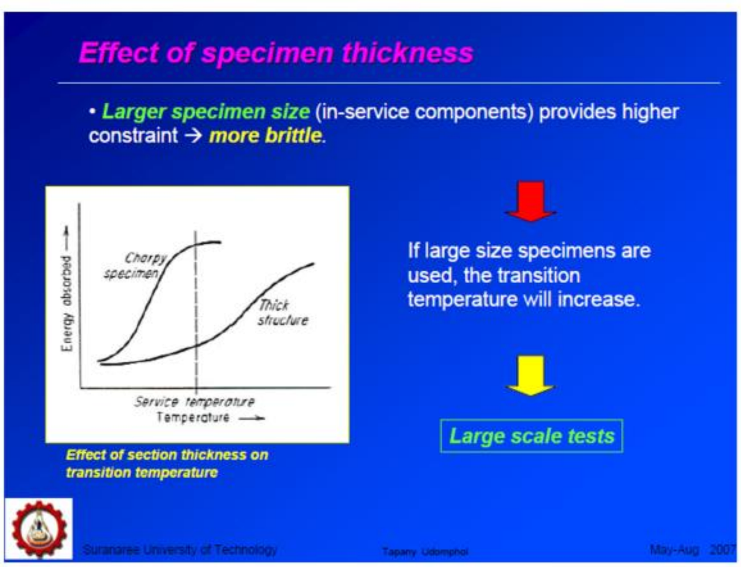
(a) Necking
(b) Formation of microvoids
(c) Coalescence of microvoids to form a crack
(d) Crack propagation by shear deformation
(e) Fracture

side note we can use other failure criteria
Mohr - Coulomb (MC)

σ_y = - -

Larger specimens have less of surface regions even for uniaxial loading and tend to have higher relative volume of "triaxial" stress state -> more prone to brittle fracture (this is another reason beside having higher probability of larger defects for brittle fracture)

8. Triaxial stress and confinement



Source: Tapany Udomphol, Suranaree University of Technology
http://eng.sut.ac.th/metal/images/stories/pdf/14_Brittle_fracture_and_impact_testing_1-6.pdf

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Often hardening (increasing strength) reduces ductility
Phenomena affecting ductile/brittle response

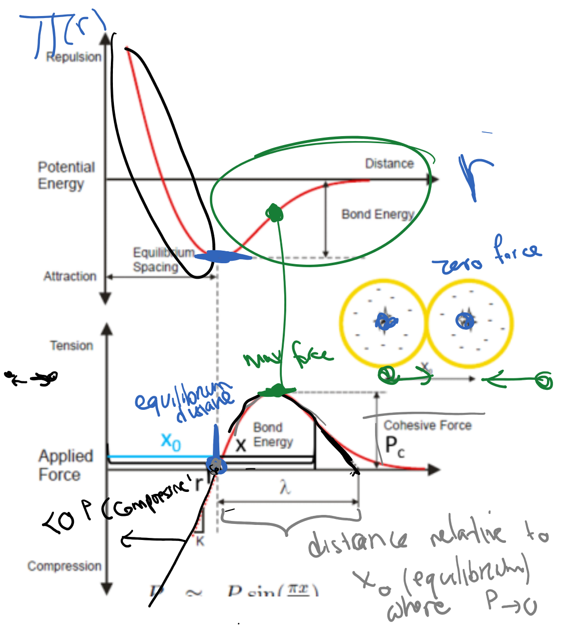
1. T (especially for BCC metals and ceramics)
2. Impurities and alloying
3. Radiation
4. Hydrogen embrittlement
5. Grain size
6. Size effect
7. Rate effect
8. Confinement and triaxial stress state

$d \searrow$ $\quad D \searrow$ $\quad \sigma_y \nearrow$ & toughness $\nearrow$

Decreasing grain size is the only mechanism that hardens and promotes toughness

4. Linear Elastic Fracture Mechanics (LEFM)

4.1Griffith energy approach



$\vec{r}$

$$\vec{P} = -\frac{\partial \Pi}{\partial \vec{r}}$$

$$\boxed{P = \frac{d\Pi}{dr}}$$

tensile

r

P tensile force P

force

position

$$P(x) = P_c \sin\left(\frac{\pi x}{\lambda}\right)$$

$$x = r - x_0$$

displacement beyond equilibrium

strain

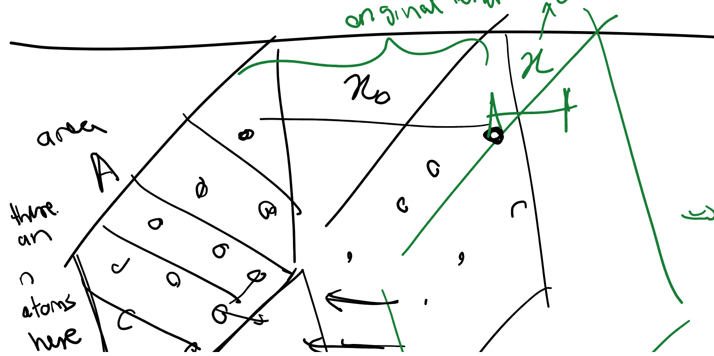
step1 $X \longrightarrow \varepsilon$

stress

step2 $P \longrightarrow \sigma$

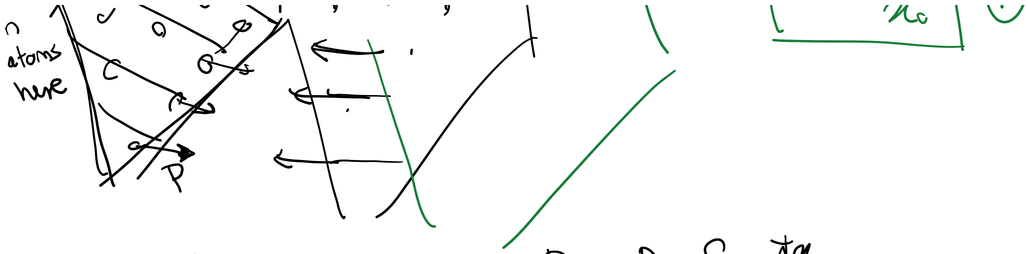
original length

displacement



strain $\varepsilon = \dfrac{\text{change of length}}{\text{original length}}$

$$\Rightarrow \boxed{\varepsilon = \frac{x}{x_0}} \quad (1)$$



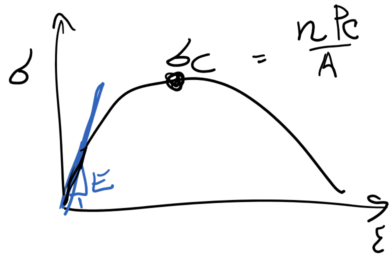
step 2 : $P \rightarrow \sigma$

$P = P_c \sin\frac{\pi x}{\lambda}$ of 1 atom ; $\lambda \rightarrow$ cut-off distance

$F = \sum P = n.P_c = nP_c \sin\frac{\pi x}{\lambda}$

$\sigma = \frac{F}{A} = \underbrace{\left(\frac{n}{A}\right)}_{\text{density of atoms}} P_c \sin\frac{\pi x}{\lambda}$

$x = \varepsilon x_0$ ①



$$\boxed{\sigma = \left(\frac{nP_c}{A}\right) \sin\frac{\varepsilon x_0 \pi}{\lambda}}$$

$$\boxed{\sigma = \sigma_c \sin\left(\frac{\varepsilon x_0 \pi}{\lambda}\right)} \quad ②$$

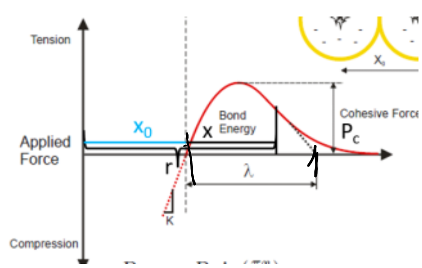
$\sigma_c \rightarrow$ strength

$E = \frac{d\sigma}{d\varepsilon}\Big|_{\varepsilon=0}$

what is the elastic modulus

$E = \frac{d\sigma}{d\varepsilon}\Big|_{\varepsilon=0} = \sigma_c \frac{x_0 \pi}{\lambda} \cos\left(\frac{\varepsilon x_0 \pi}{\lambda}\right)\Big|_{\varepsilon=0} \rightarrow \boxed{E = \sigma_c \frac{x_0 \pi}{\lambda}}$ ③

$\sigma_c \Leftrightarrow E$



$\frac{\lambda}{x_0} = O(1)$

$E \approx \sigma_c \pi \rightarrow \boxed{\sigma_c \approx \frac{E}{\pi}}$ ④

Steel $E = 200$ GPa, ④ predicts strength $\sigma_c \approx 60$ GPa !

theoretical estimate

$\sigma_y = 250$ MPa

practice

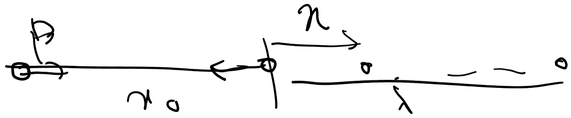
E often GPa

Strengths often MPa

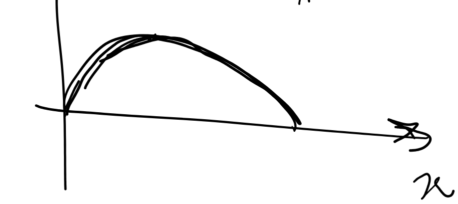
/ why equation ④ does not hold?

$E \longleftrightarrow \sigma_c$

let's also relate these to toughness

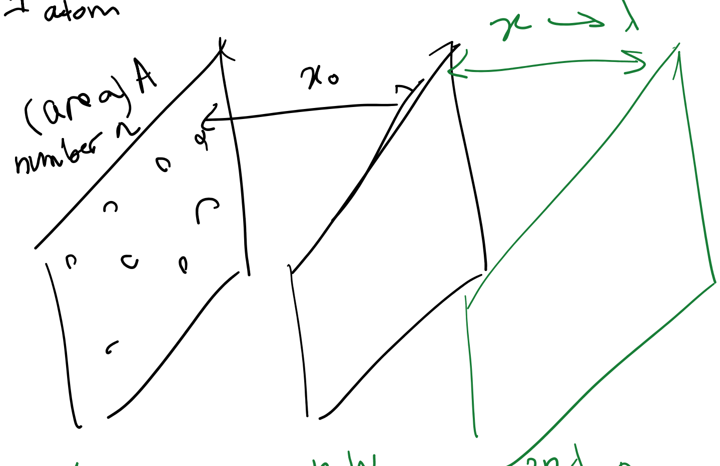


$P = P_c \sin\frac{\pi x}{\lambda}$



$$W_1 = \int_{x=0}^{\lambda} P(x)\,dx = \int_0^{\lambda} P_c \sin\frac{\pi x}{\lambda}\,dx = \frac{2\lambda}{\pi} P_c$$

1 atom



$$W_{\text{for this plane}} = n W_1 = \frac{2n\lambda}{\pi} P_c$$

energy needed to break the bond on this plane

$$\frac{W}{A} = \frac{2n\lambda P_c}{\pi A} = 2\gamma_s$$

σ_c

energy per unit area needed to cause fracture

fracture toughness

$(2A)\gamma_s$

$$\frac{nP_c}{A} = \sigma_c$$

$\gamma_s \longleftrightarrow \gamma_s$

$$\gamma_s = \frac{\lambda}{\pi}\sigma_c \rightarrow \boxed{\sigma_c = \frac{\pi}{\lambda}\gamma_s} \quad (5)$$

multiply these two

we also had $\sigma_c = \frac{E}{\pi}\frac{\lambda}{x_0}$ (3)

$E \longleftrightarrow \sigma_c$

$$\sigma_c^2 = \frac{\pi}{\lambda}\gamma_s \frac{E}{\pi}\frac{\lambda}{x_0} \longrightarrow \sigma_c = \sqrt{\frac{E\gamma_s}{x_0}}$$

Summary $\quad \sigma_c = \frac{E}{\pi}\frac{\lambda}{x_0} \quad \& \quad \sigma_c = \sqrt{\frac{E\gamma_s}{x_0}}$ (6)

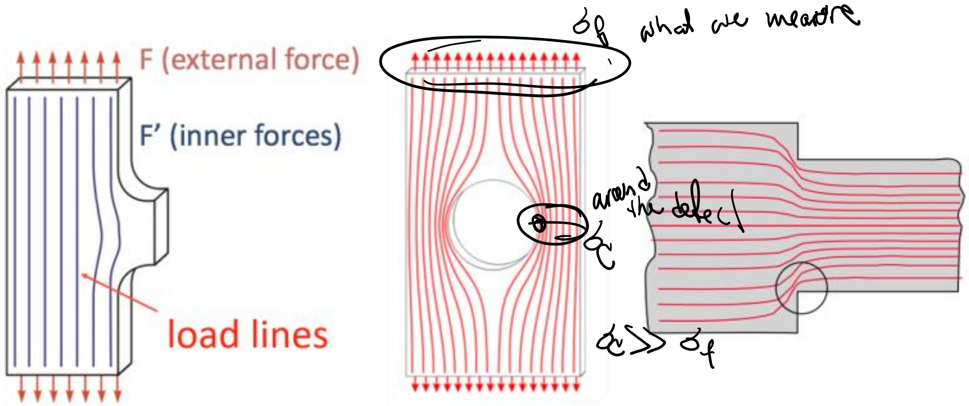
Summary $\sigma_c = \frac{E}{n} \frac{\lambda}{x_0}$ & $\sigma_c = \sqrt{\frac{E\gamma_s}{x_0}}$ (6)

The main reason why the stress is 100s to <1000 smaller than theoretical estimate is the presence of defects

We'll use two explanations for this discrepancy
Stress-based and energy-based

1. Stress-based (stress concentration around defects, …)

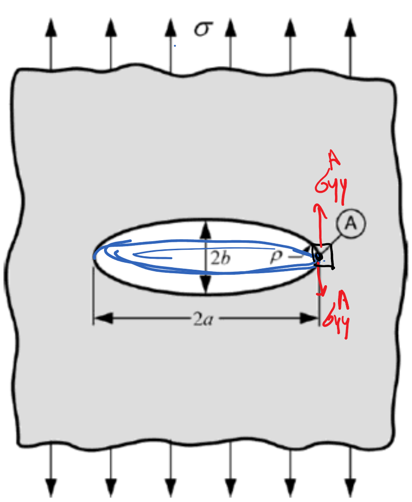
Stress concentration



Geometry discontinuities: holes, corners, notches, cracks etc: stress concentrators/risers

Elliptic hole

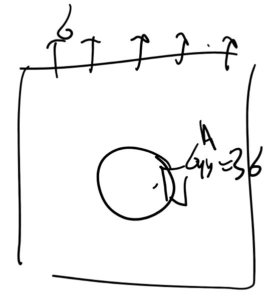
Inglis, 1913, theory of elasticity

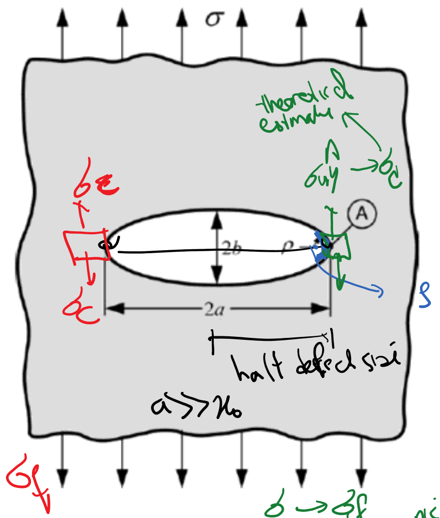


$\sigma_{yy}^A = \sigma \underbrace{\left(1 + \frac{2a}{b}\right)}_{\text{stress concentration factor}}$

$\frac{b}{a} \to 0 \quad \frac{\sigma_{yy}^A}{\sigma} \to \infty$

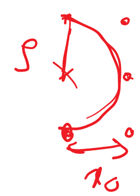
side note





$$\left.\begin{aligned}\rho &= \frac{b^2}{a}\\ \sigma_{yy}^A &= \sigma\left(1+\frac{2a}{b}\right)\end{aligned}\right\} \rightarrow \sigma_{yy}^A = \sigma\left(1+2\sqrt{\frac{a}{\rho}}\right)$$

ρ = radius of curvature



$\rho_{min} \sim x_0$

worse case scenario

"sharpest crack"

$$\sigma_{yy}^A = \sigma\left(1+2\sqrt{\frac{a}{x_0}}\right)$$

for the sharpest crack

$\sigma \to \sigma_f$

macroscopic strength

$$\sigma_{yy}^A \approx 2\sigma\sqrt{\frac{a}{x_0}}$$

Increase σ until

$\sigma_{yy}^A \to \sigma_c$

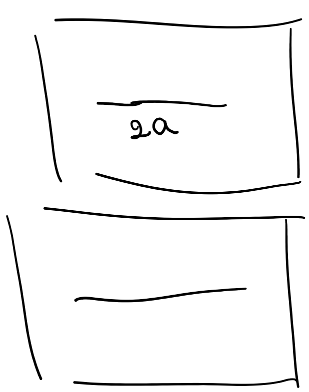
then we'll experience macroscopic strength ($\sigma \to \sigma_f$)

$$\sigma_c = 2\sigma_f\sqrt{\frac{a}{x_0}}$$

$$\boxed{\sigma_f = \sqrt{\frac{x_0}{4a}}\,\sigma_c} \quad (4)$$

$$\sigma_f \propto \frac{\sigma_c}{\sqrt{a}}$$

Stress concentration argument



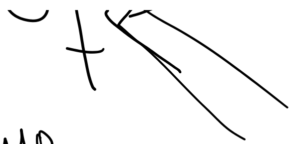
σ_f high

$\sigma_f \searrow$ as $\frac{1}{\sqrt{a}}$

$$a >> x_c \longrightarrow$$

$$\sigma_f << \sigma_c$$

$\approx E$



~MPa

GPa $\approx E/10$

Griffith's work (brittle materials)

FM was developed during WWI by English aeronautical engineer A. A. Griffith to explain the following observations:

- The stress needed to fracture bulk glass is around 100 MPa
- The theoretical stress needed for breaking atomic bonds is approximately 10,000 MPa
- experiments on glass fibers that Griffith himself conducted: the fracture stress increases as the fiber diameter decreases => Hence the uniaxial tensile strength, which had been used extensively to predict material failure before Griffith, could not be a specimen-independent material property.

Griffith suggested that the low fracture strength observed in experiments, as well as the size-dependence of strength, was due to the presence of **microscopic flaws** in the bulk material.

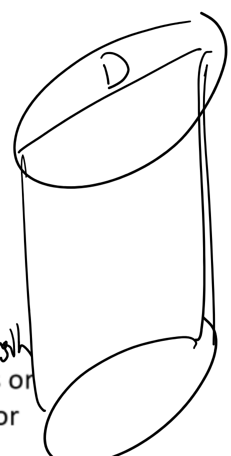
D

TABLE 1.1. Strength of glass fibers according to Griffith's experiments.

Diameter (10^{-3} in)	Breaking stress (lb/in^2)	Diameter (10^{-3} in)	Breaking stress (lb/in^2)
40.00 (thickest)	24 900	0.95	117 000
4.20	42 300	0.75	134 000
2.78	50 800	0.70	164 000
2.25	64 100	0.60	185 000
2.00	79 600	0.56	154 000
1.85	88 500	0.50	195 000
1.75	82 600	0.38	232 000
1.40	85 200	0.26	332 000
1.32	99 500	0.165	498 000
1.15	88 700	(thinnest) 0.130	491 000

~20x increase in strength

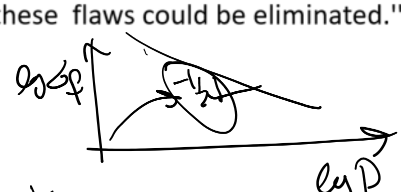
$$\sigma_f = \sqrt{\frac{\gamma_s}{4a}}\,\sigma_c$$



"the weakness of isotropic solids... is due to the presence of discontinuities or flaws... The effective strength of technical materials could be increased 10 or 20 times at least if these flaws could be eliminated.''

the larger D the larger potential a's

log D



if $a \propto D$ holds

$\sigma_f \propto \frac{1}{\sqrt{a}}$