HW4-problem5Hint

Sunday, November 24, 2024 12:47 PM

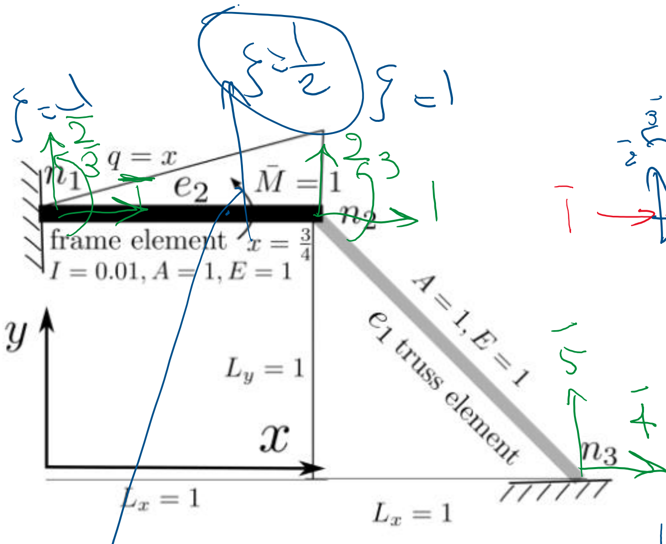
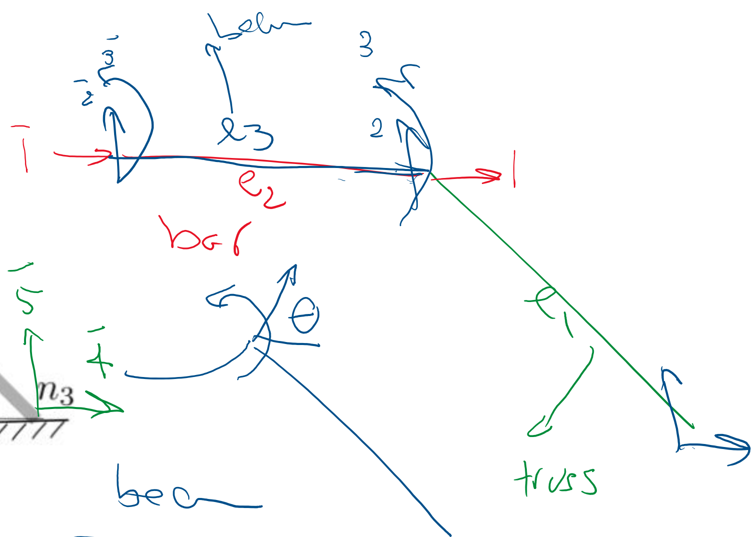


Figure 3: Frame and truss example.



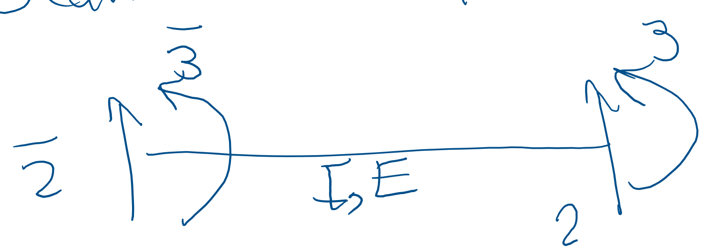
$$f_r^M = M \frac{d}{dx}\begin{bmatrix}N_1\\N_2\\N_3\\N_4\end{bmatrix}$$

$$= M \frac{d}{d\xi}\begin{bmatrix}N_1\\ \vdots \\N_4\end{bmatrix} \times \frac{1}{L/2}$$

@ $\xi = .5$

see the end of page

Beam Element



has 4 dofs

$$f^{beam} = f_r^{\bar{M}} + f_r^q + \cancel{f_N^{beam}} - \cancel{f_D^{beam}}$$

$f = f_r + f_r + \cancelto{0}{f_N} - \cancelto{0}{f_D}$

which is a 4×1 vector

Assemble it to global $(F_e)_{3\times1}$

using the map $[\bar{2}\ \bar{3}\ 2\ 3]$

Assemble the $(K^{beam})_{4\times4}$ to global $(K)_{3\times3}$ again using the map $[\bar{2}, \bar{3}, 2, 3]$

---

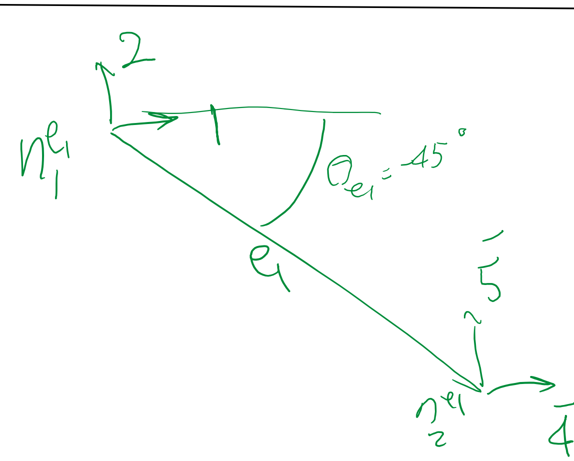
bar element

Assemble $K^{bar}_{2\times2}$ to global $K_{3\times3}$

using the map $[\bar{1}, 1]$

Truss (bar) element

$f^{e_1} = \cancelto{0}{f_r^{e_1}} + \cancelto{0}{f_N^{e}} - \cancelto{0}{f_D^{e}} = 0$



$$(K^{e_1}) = \left[\begin{array}{c|c} (kb)_{2\times2} & -kb \\ \hline k & kb \end{array}\right]$$

$$(K^{e1})_{4\times4} = \left[\begin{array}{c|c} & 2\times2 \\ \hline -K_b & K_b \end{array}\right] \qquad n_2^{e1} \quad \bar{4}$$

Assemble 4×4 K^{e1} to global K using

$$M^{e1} = [1, 2, \bar{4}, \bar{5}]$$

Global system

$$K_{3\times3}\, a_{3\times1} = (F)_{3\times1} = (F_r)_{3\times1} + (F_e)_{3\times1}$$

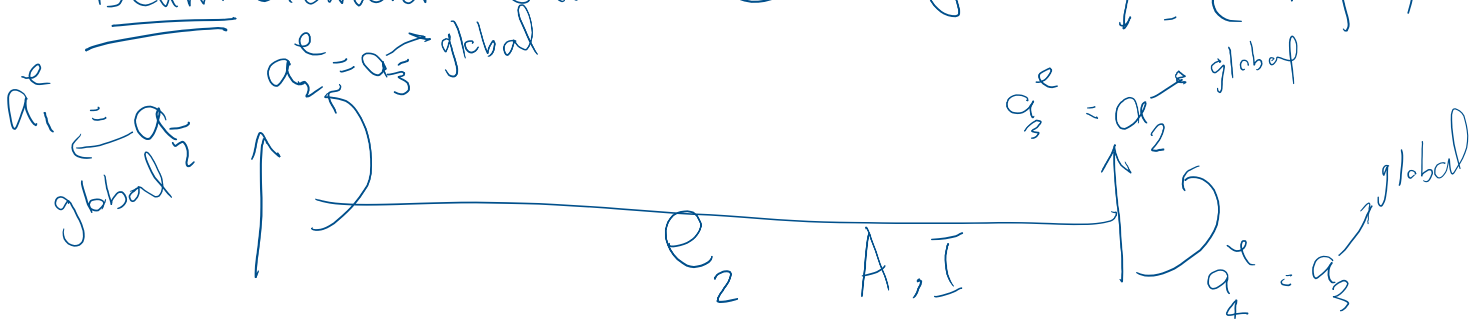
F_r: 0

F_e: only beam part of e_2 contributes

Solve

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

<u>Beam element</u> solution @ a given x^* (or ξ^*)



$$a^e = [a_1^e, a_2^e, a_3^e, a_4^e] = [0, 0, a_2, a_3]$$

use this in equations below

Use this in equations below

Beam Example: Calculation of y, θ, M, V within element

- After the solution of global free dofs, they are transferred to elements.
- Once element dofs are know, we have the Displacement in the entire elements:

$$y(\xi) = N_1^e(\xi)a_1^e + N_2^e(\xi)a_2^e + N_3^e(\xi)a_3^e + N_4^e(\xi)a_4^e.$$

element shape functions are given in (421).

- Rotation: Obtained by differentiating previous equation w.r.t. x & noting that $\frac{\mathrm{d}x}{\mathrm{d}\xi} = \frac{L^e}{2}$:

$$\theta(\xi) = \frac{\mathrm{d}y}{\mathrm{d}x}(\xi) = \frac{\frac{\mathrm{d}y}{\mathrm{d}\xi}(\xi)}{\frac{\mathrm{d}x}{\mathrm{d}\xi}(\xi)} = \frac{2}{L^e}\left\{\frac{\mathrm{d}N_1^e}{\mathrm{d}\xi}(\xi)a_1^e + \frac{\mathrm{d}N_2^e}{\mathrm{d}\xi}(\xi)a_2^e + \frac{\mathrm{d}N_3^e}{\mathrm{d}\xi}(\xi)a_3^e + \frac{\mathrm{d}N_4^e}{\mathrm{d}\xi}(\xi)a_4^e\right\}$$

- Moment is directly obtained by differentiating the above equation:

$$\begin{aligned} M(\xi) &= E(\xi)I(\xi)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}(\xi) = E(\xi)I(\xi)\mathbf{B}^e(\xi) \\ &= E(\xi)I(\xi)\left\{B_1^e(\xi)a_1^e + B_2^e(\xi)a_2^e + B_3^e(\xi)a_3^e + B_4^e(\xi)a_4^e\right\} \quad \textit{cf.}\ (424)\ \text{for } \mathbf{B}^e \end{aligned}$$

- Shear force is obtained by differentiating M w.r.t. x. It's a similar process to deriving θ from y with the difference that if EI are not constant we need to take it into account. For constant EI we have:

$$V(\xi) = \frac{\mathrm{d}M}{\mathrm{d}x}(\xi) = \frac{\frac{\mathrm{d}M}{\mathrm{d}\xi}(\xi)}{\frac{\mathrm{d}x}{\mathrm{d}\xi}(\xi)} = \frac{2EI}{L^e}\left\{\frac{\mathrm{d}B_1^e}{\mathrm{d}\xi}(\xi)a_1^e + \frac{\mathrm{d}B_2^e}{\mathrm{d}\xi}(\xi)a_2^e + \frac{\mathrm{d}B_3^e}{\mathrm{d}\xi}(\xi)a_3^e + \frac{\mathrm{d}B_4^e}{\mathrm{d}\xi}(\xi)a_4^e\right\}$$

- To obtain these fields for the entire beam we evaluate these equations for all elements.

$x = .75 \implies \xi = .5$ (needed for $\bar{M}$) $\quad dx/d\xi = .5$

$x = .5 \to \xi = 0$ for y, θ, V, M here

$$x = N_1^L(\xi)\underbrace{x_1^e}_{0} + \underbrace{N_2^L(\xi)}_{\frac{1+\xi}{2}}\underbrace{x_2^e}_{1} = \frac{1+\xi}{2} \iff \boxed{x = \frac{1+\xi}{2} \iff \xi = 2x-1}$$

$x_1^e = 0$ $\qquad$ $x_2^e = 1$

$\xi_1^e = -1$ $\qquad$ $\xi_2^e = 1$