2021/08/26Theory

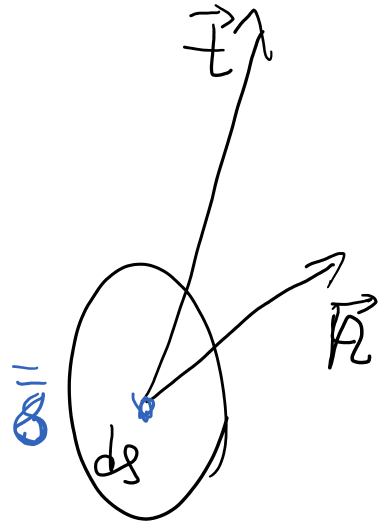
Thursday, August 26, 2021 3:42 PM

From the first session, formulation of elastostatics:

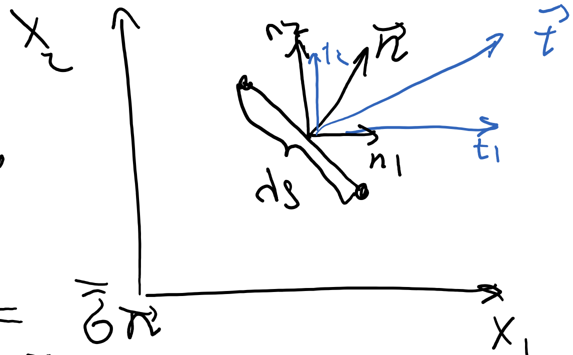
$$\int_{\omega} \rho b \, dV + \int_{\partial\omega} \bar{t} \, dS =$$

$$\vec{t} = \underbrace{\sigma}_{\text{stress tensor}} \cdot \vec{n}$$

$$d\vec{F}_s = \vec{t}\, dS$$



2D elasticity

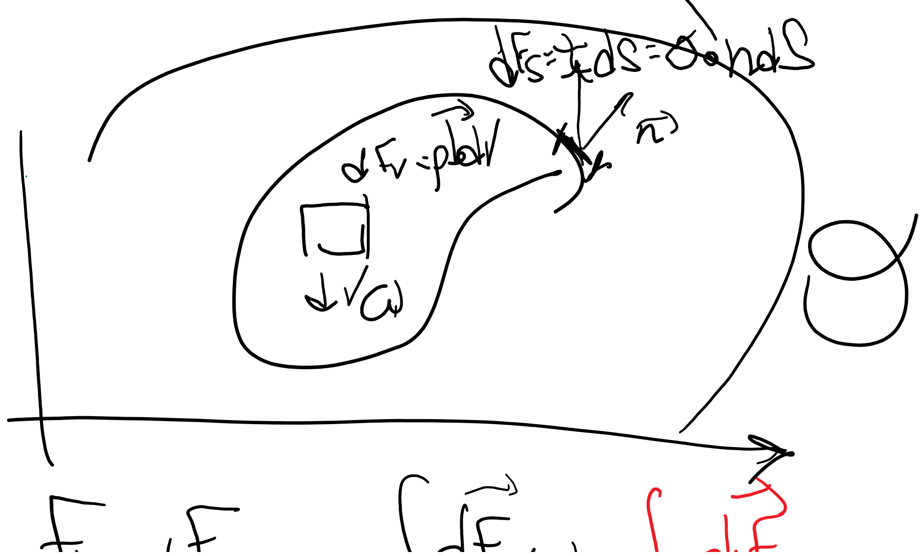


$$\vec{t} = \bar{\sigma}\vec{n}$$

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}}_{2\times 2 \text{ stress tensor}} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$d\vec{F}_s = \vec{t}\, dS = (\bar{\sigma}\vec{n})\, dS = \bar{\sigma}\, d\vec{S}$$

dS $\vec{n}$ $d\vec{S} = (dS)\vec{n}$



$F = F_v + F_s$ $\int d\vec{F}_v + \int d\vec{F}_s$

for this course

$\boxed{\int \sigma \quad \text{static}}$

linear momentum

$$F = F_V + F_S = \int_{\omega} d\vec{F}_V + \int_{\partial\omega} d\vec{F}_S = \int \frac{dP}{dt} = \frac{d\int \rho \vec{v}\, dv}{dt}$$

linear momentum

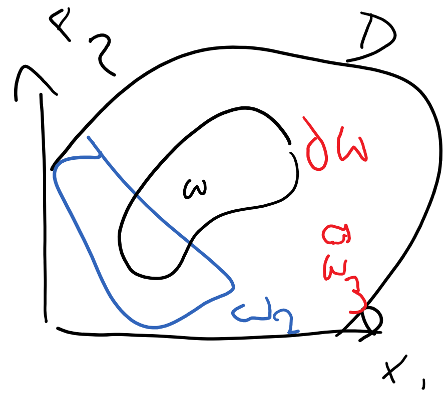
This is discussed in detail in my course notes,
I am not going to cover any dynamic concept during the class.

$$\Sigma F = ma = \frac{dmv}{dt}$$
$$0$$

Static:

$$\int_{\omega} d\vec{F}_V + \int_{\partial\omega} d\vec{F}_S = 0$$

$$\int_{\omega} \rho b\, dV + \int_{\partial\omega} \sigma \cdot \vec{n}\, dS = 0$$



Apply the divergence theorem

$$\int_{\partial\omega} \sigma \cdot n\, dS = \int_{\omega} \nabla \cdot \sigma\, dV$$

$$\int_{\omega} \rho b\, dV + \int_{\omega} \nabla \cdot \sigma\, dV = 0$$

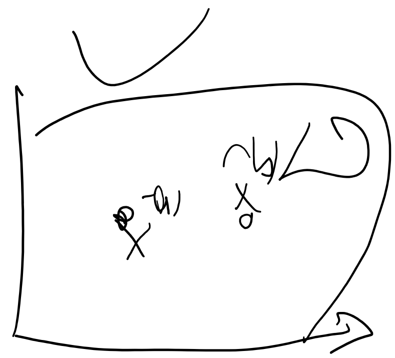
$$\boxed{\int_{\omega} (\nabla \cdot \sigma + \rho b)\, dV = 0}$$

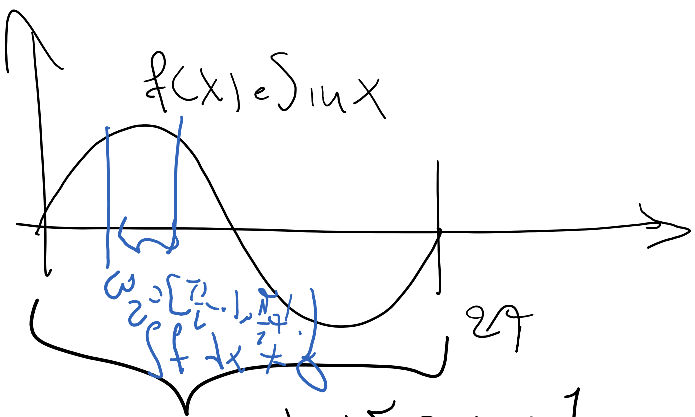
weak

Ω

PDE $\nabla \cdot \sigma + \rho b = 0$

Strong form $\qquad \forall x$





$\omega_2 = [\frac{\pi}{2} - 1, \frac{\pi}{2} + 1]$

$\int f\, dx \neq 0$

$\Omega = [0, 2\pi]$

$$\int_0^{2\pi} \sin y\, dx = 0$$

$$\Downarrow$$

$\sin x = 0$!