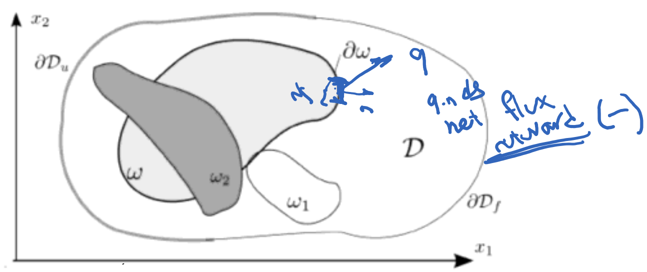
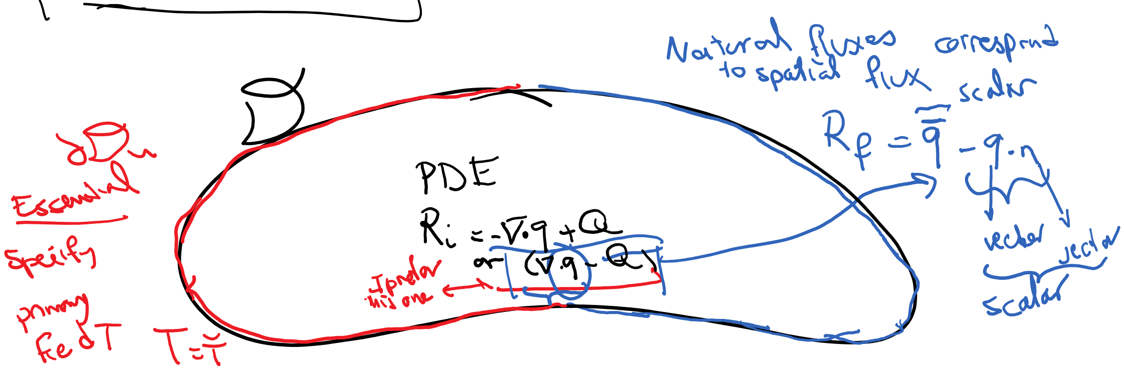
FEM2021/09/30-OH

Thursday, September 30, 2021 2:51 PM



$$\int_\omega Q\,d\omega - \int_{\partial\omega} q\cdot n\,ds = 0$$

$$\boxed{\text{PDE} \quad Q - \nabla\cdot q = 0}$$



PDE $\nabla\cdot q - Q = 0$
$q = -k\nabla T$

$\Big\}$ PDE $\overbrace{\nabla\cdot -k\nabla T}^{\text{2nd order}} - Q = 0 \quad M = 2$

Essential	order 0	$T = \bar{T}$ (specified temperature)	$R_u = \bar{T} - T$
natural	order 1	$q\cdot n = \bar{q}$	$R_f = \bar{q} - q\cdot n$

$$R_i = \overline{\nabla\cdot q} - Q$$

$$R_i(T) = \underbrace{-\nabla\cdot k\nabla T}_{} - Q$$

$$\underline{L_M}(\;) = +\nabla\cdot k\nabla(\;)$$

$$R_f(T) = \bar{q} - q\cdot n = \bar{q} + \underbrace{k\nabla T\cdot n}_{}$$

$$\underline{L_f}(\;) = k\nabla(\;)$$

$$T(x_1, x_2)$$

WRS

$R_i = \nabla\cdot q - Q$ → weakly

$R_f = \bar{q} - q\cdot n$ → weakly

I'll satisfy this strongly ← $R_u = \bar{T} - T$ $\boxed{\partial D_u}$

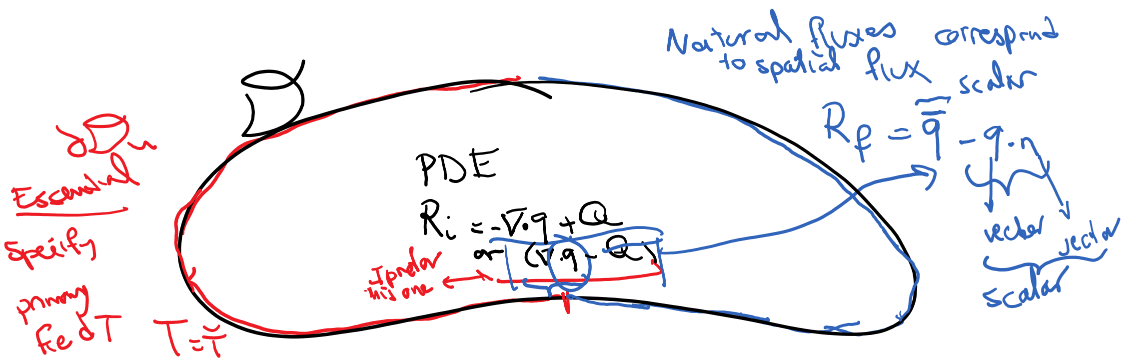
Discretization

I'll satisfy this strongly ← $R_u = \bar{T} - T$ $\boxed{\partial D_u}$

Discretization: $T^h = \phi_p + a_i \phi_i$ / the $\phi_p(x) = \bar{T}(x)$ on ∂D_u, $\phi_i(x) = 0$

$\rightarrow T^h(x) = \bar{T}(x)$

$q = -k\nabla T$



$$\int_{D} \underline{w} R_i \, dv + \int_{\partial D_f} w R_f \, ds = 0$$

Find $T \in \mathcal{D} = \{ C^2 \mid$ Must satisfy essential BC $\}$

$\ni \forall w \in W = \{ C^0 \mid \}$

$$\int_D w(\nabla \cdot q - Q)\, dv + \int_{\partial D_f} w(\bar{q} - q \cdot n)\, ds$$

$q = -k\nabla T$, $\underbrace{}_{\text{2 der on } T}$

0 on w

WRS to weak statement

$$\int_D w(\nabla \cdot q - Q) + \int w(\bar{q} - q \cdot n)\, ds = 0$$

move ... to w

cancelation

move grad to ω — cancelation

$q_{i,i} = \frac{\partial q_i}{\partial x_i}$

2 repeated index ⇒ there is a summation

$\omega \nabla \cdot q = \omega \frac{\partial q_i}{\partial x_i}$

$ab' = (ab)' - a'b$

$\omega q_{i,i} = \underbrace{(\omega q_i)_{,i}} - \omega_{,i} q_i$

$\omega \nabla \cdot q = \nabla \cdot \omega q - \nabla \omega \cdot q$

$\int_D \omega \nabla \cdot q \, dV = \int_D \nabla \cdot (\omega q) \, dV - \int_D \nabla \omega \cdot q$

Div theorem

$\int_D \nabla \cdot f \, dV = \int_{\partial D} f \cdot n \, ds$

$\int_D \omega \nabla \cdot q \, dV = \int_{\partial D} \omega q \cdot n \, ds - \int_D \nabla \omega \cdot q \, dV$

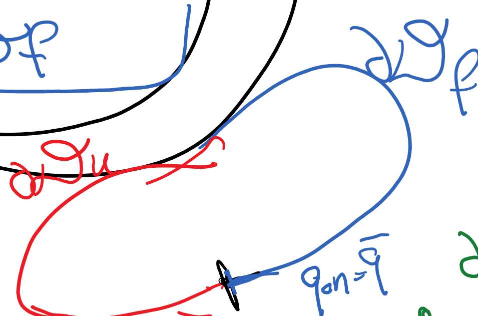
WRS

$\int_D \omega (\nabla \cdot q) \, dV - \int_D \omega Q \, dV + \int_{\partial D_f} \omega \bar{q} \, ds - \int_{\partial D_f} \omega q \cdot n \, ds = 0$

$-\int_D \nabla \omega \cdot q \, dV + \int_{\partial D} \omega q \cdot n \, ds$ → $\int_{\partial D_f} \omega q \cdot n \, ds + \underbrace{\int_{\partial D_u} \omega q \cdot n \, ds}$ this remains

$-\int_D \omega Q \, dV + \int_{\partial D_f} \omega \bar{q} \, ds - \int_{\partial D_f} \omega q \cdot n \, ds = 0$

cancel each other



$\partial D = \partial D_u \cup \partial D_f$

$-\int_D \nabla \omega \cdot q \, dV - \int_D \omega Q \, dV$

$+ \int_{\partial D_f} \omega \bar{q} \, ds + \int_{\partial D_u} \omega q \cdot n \, ds = 0$

$\int_{\partial D} f \, ds - \int_{\partial D_f} f \, ds$

$\int_{\partial D_f} \ldots + \int_{\partial D_u} \omega\, q \cdot n\, ds = 0$

$\int_{\partial D} f\,ds = \int_{\partial D_f} f\,ds + \int_{\partial D_u} f\,ds - \int_{\partial D_f} f\,ds$

how can we get rid of this term?

$\omega\, q\cdot n = 0$:)

$\boxed{q\cdot n}$ generally is nonzero

Choose ω such that $\underline{\omega = 0 \text{ on } \partial D_u}$

$$-\int_D \nabla\omega \cdot q\, dv - \int_D \omega Q\, dv + \int_{\partial D_f} \omega \bar{q}\, ds$$

good time to use

$q = -k\nabla T$

Find $T \in \mathcal{V} = \{$ satisfies $\bar{T}$ on $\partial D_u \}$

$\ni \forall \omega \in \mathcal{W} = \{ = 0$ on $\partial D_u \}$

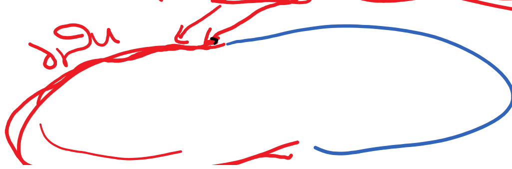
$$\int_D \nabla\omega \cdot k \nabla T\, dv = \int_D \omega Q\, dv + \int_{\partial D_f} \omega \bar{q}\, ds$$

$L_m(\,) = \nabla$

$L_m T = \nabla T$

$T^h = \phi + a_i \phi_i$ $\quad\Big|\quad \longrightarrow T^h(x) = \bar{T}(x)$ on ∂D_u :)

$\forall x \in \partial D_u$ $\quad \phi(x) = \bar{T} \quad \phi_i(x) = 0$



$w_i(x) = \phi_i(x)$ ☺

$\phi_i(x) = 0$ on $\partial\Omega_u$ so they're perfect choices for weights in weak statement

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$w = \phi$ Galerkin

we use Galerkin weight function for weak statement because $\phi_i = 0$ on $\partial\Omega_u$ means that $w_i = \phi_i = 0$ on $\partial\Omega_u$ which is the condition we needed to satisfy